Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/17-1.1.1.6-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [78]. This is test number [17].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (78)	0.00 (0)
Mathematica	100.00 (78)	0.00 (0)
Maple	100.00 (78)	0.00 (0)
Fricas	82.05 (64)	17.95 (14)
Giac	61.54 (48)	38.46 (30)
Mupad	50.00 (39)	50.00 (39)
Maxima	34.62 (27)	65.38 (51)
Sympy	5.13 (4)	94.87 (74)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

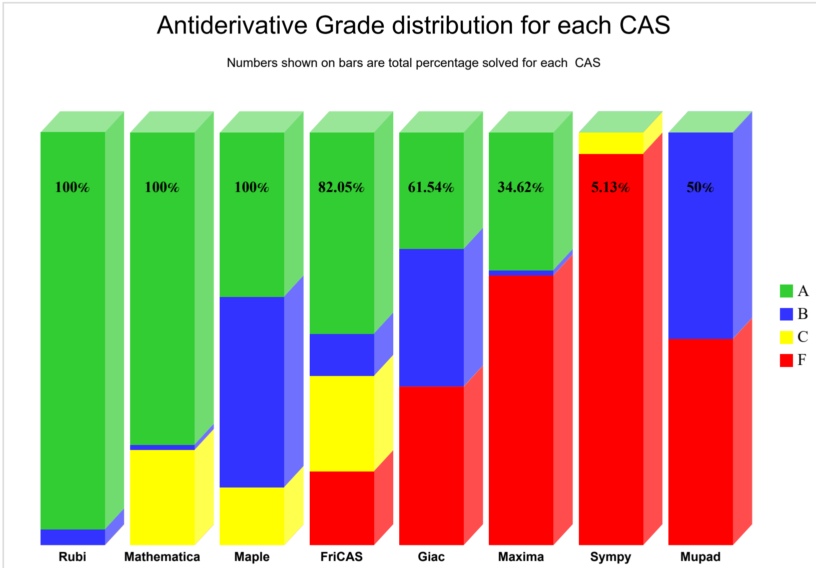
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

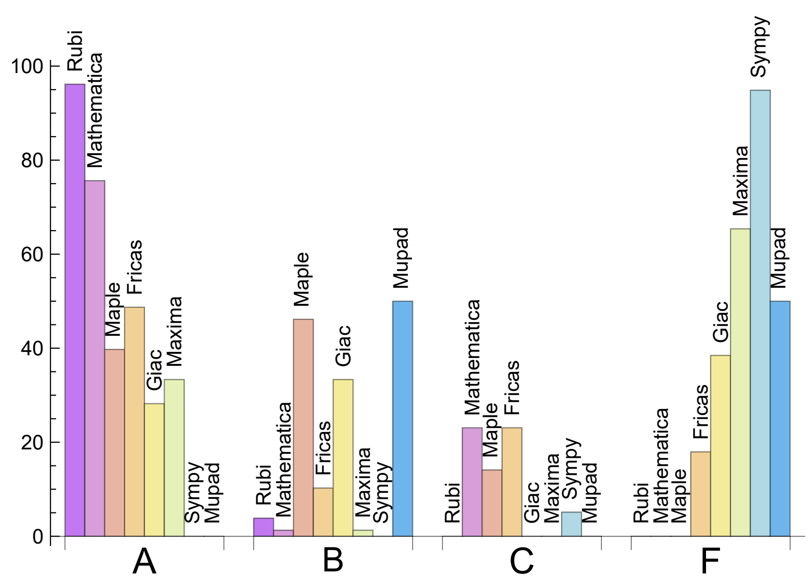
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.154	3.846	0.000	0.000
Mathematica	75.641	1.282	23.077	0.000
Fricas	48.718	10.256	23.077	17.949
Maple	39.744	46.154	14.103	0.000
Maxima	33.333	1.282	0.000	65.385
Giac	28.205	33.333	0.000	38.462
Mupad	0.000	50.000	0.000	50.000
Sympy	0.000	0.000	5.128	94.872

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	14	0.00	100.00	0.00
Giac	30	63.33	0.00	36.67
Mupad	39	0.00	100.00	0.00
Maxima	51	35.29	0.00	64.71
Sympy	74	67.57	32.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Rubi	0.71
Maple	2.41
Giac	4.31
Fricas	4.92
Mathematica	8.42
Sympy	28.64
Mupad	34.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	187.07	0.99	100.00	1.01
Sympy	230.50	4.50	230.50	4.48
Rubi	438.29	1.08	350.50	1.00
Mathematica	472.68	1.00	260.50	0.95
Fricas	1230.86	2.51	823.50	1.88
Maple	1883.79	3.85	880.00	1.97
Giac	2271.65	4.58	526.50	2.07
Mupad	3248.46	15.07	1732.00	6.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

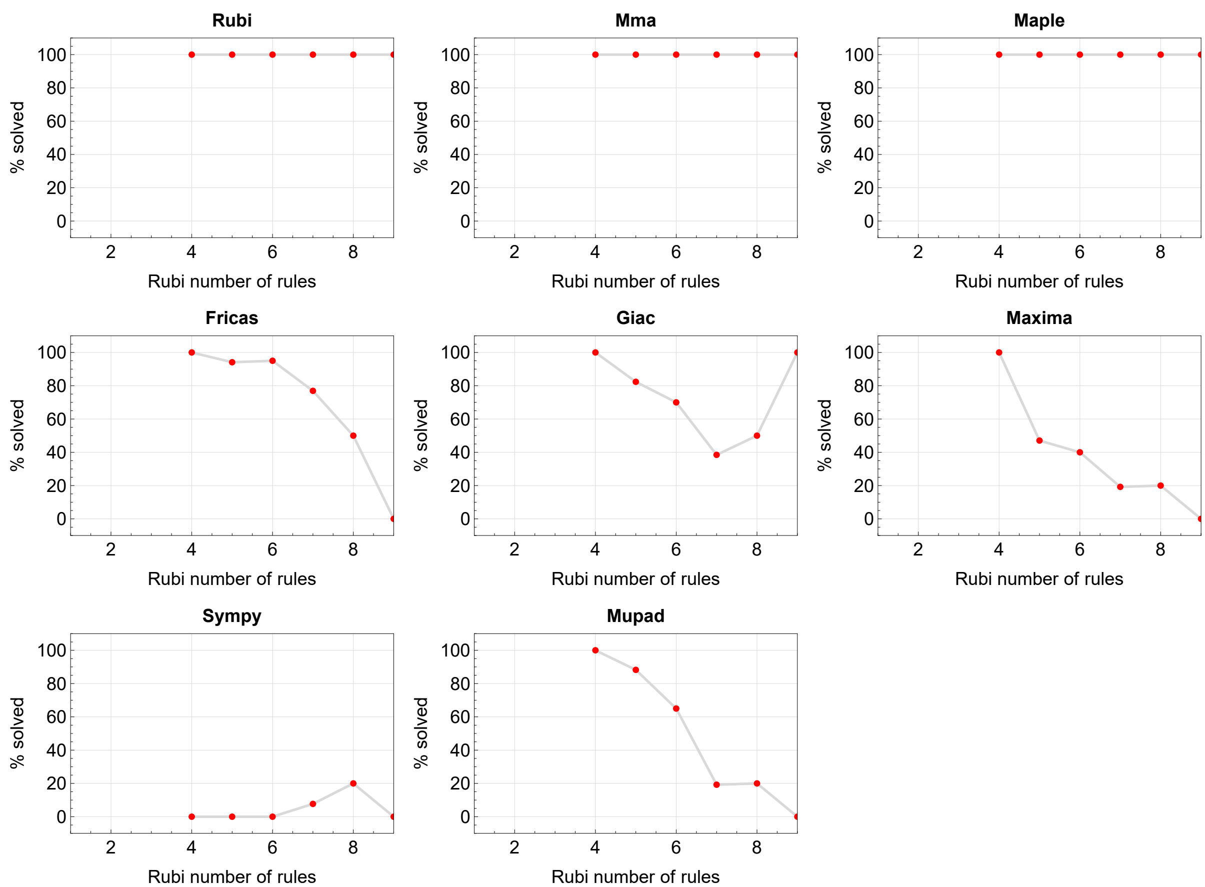


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

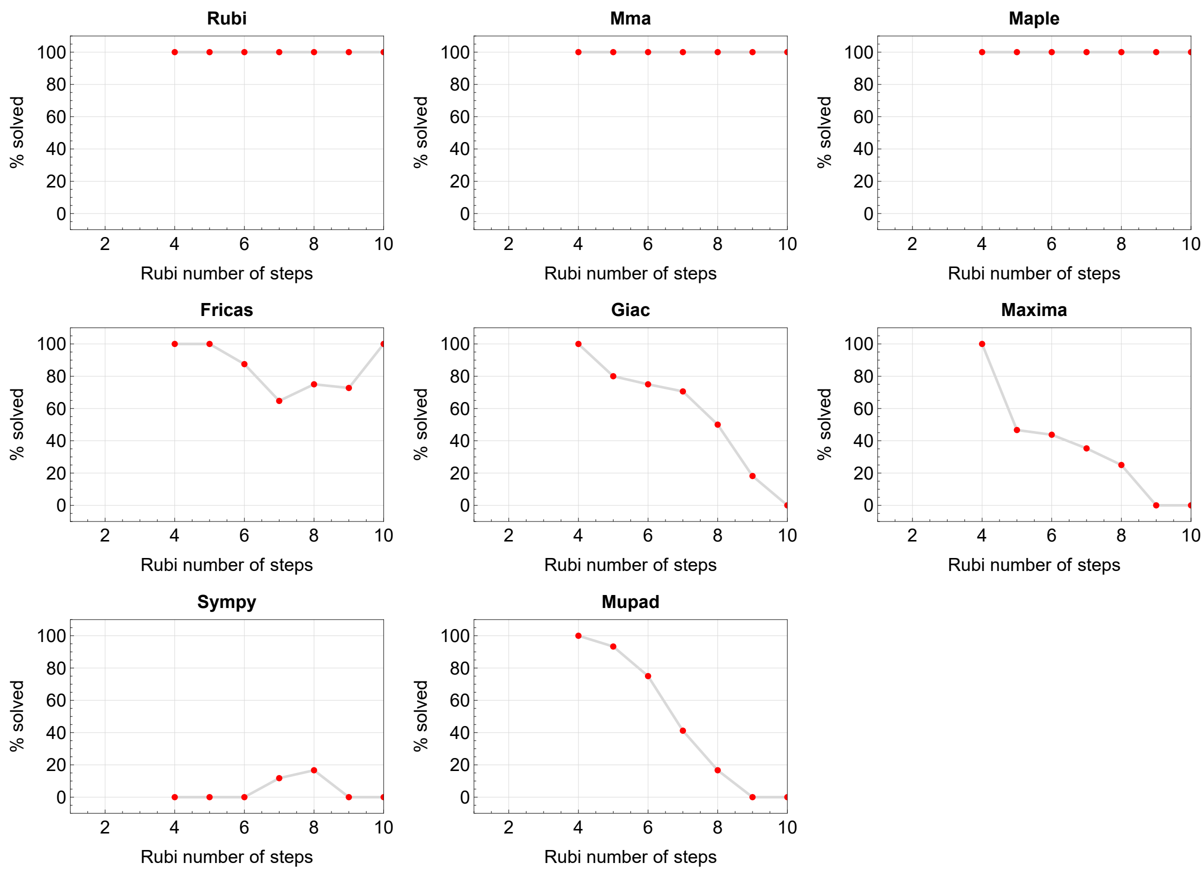


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

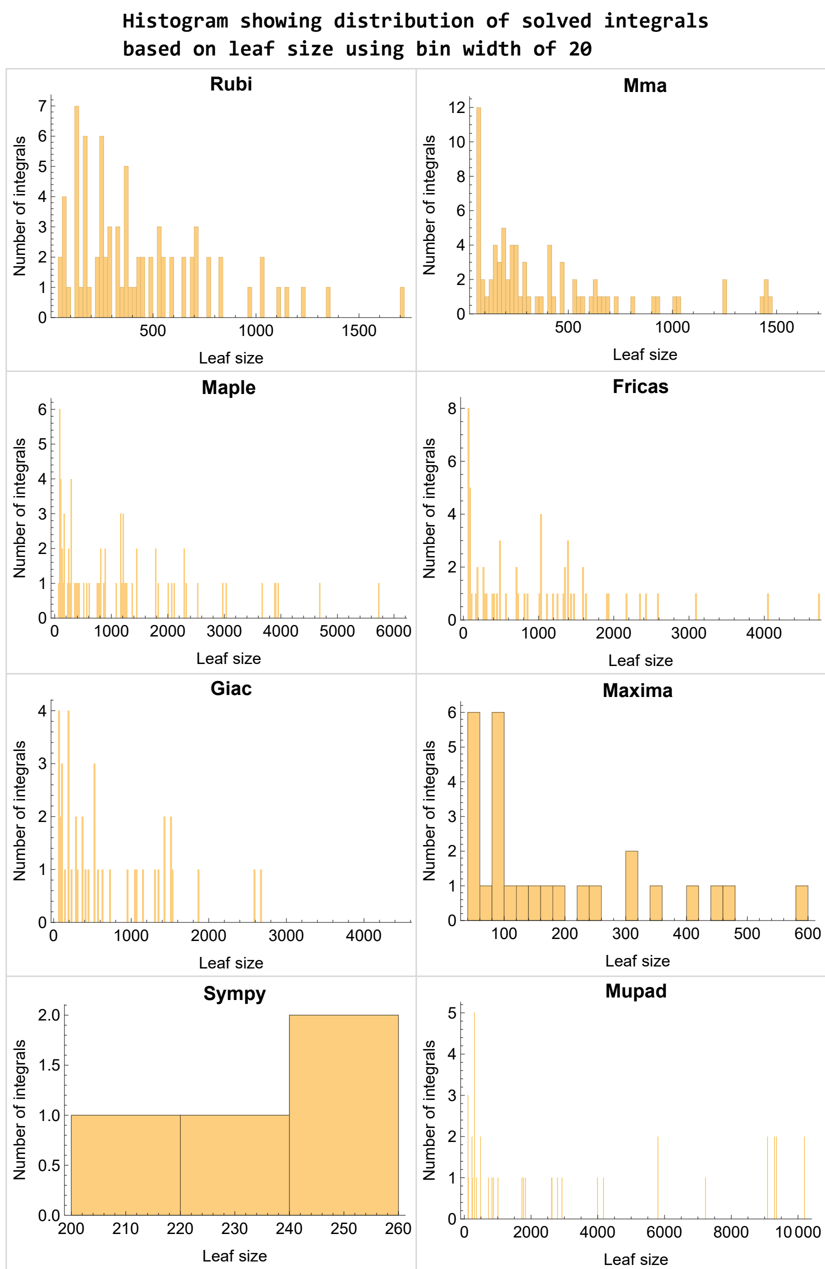


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

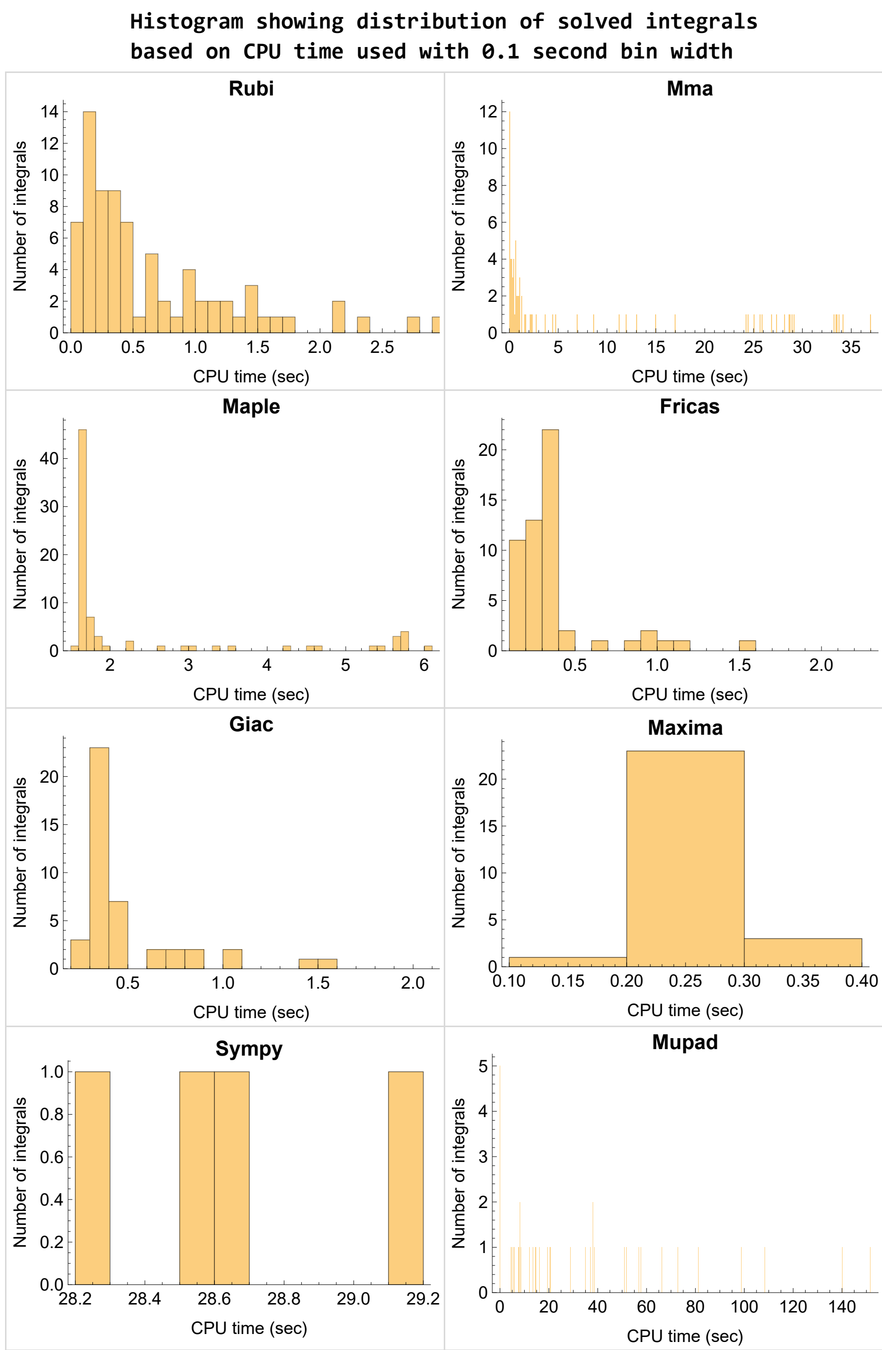


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

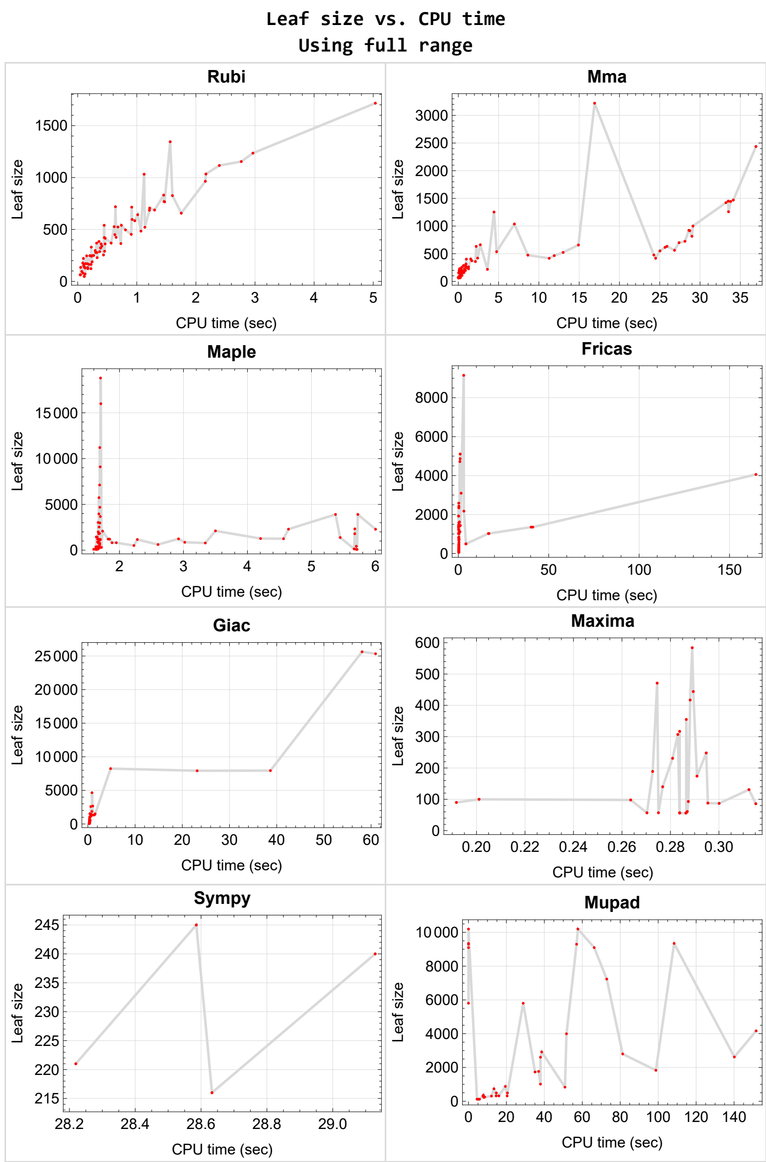


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{35, 36, 37, 38\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

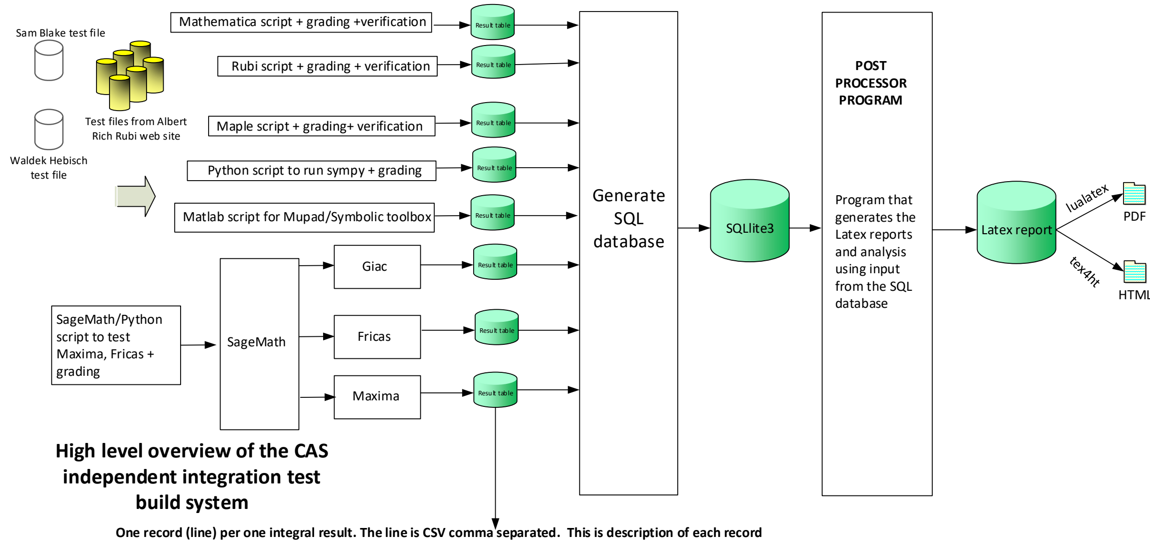
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

B grade { 35, 36, 37 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

B grade { 47 }

C grade { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 8, 9, 10, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 34, 37, 38, 39, 61, 62, 63, 67, 68, 69, 70, 73, 74, 75 }

B grade { 5, 12, 25, 26, 32, 33, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 71, 72, 76, 77, 78 }

C grade { 6, 7, 11, 13, 14, 15, 16, 17, 18, 19, 36 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 54, 55, 56 }

B grade { 5, 6, 7, 12, 13, 14, 40, 59 }

C grade { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F normal fail { }

F(-1) timedout fail { 24, 25, 31, 32, 44, 45, 46, 50, 51, 52, 53, 57, 58, 60 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 34, 36, 37, 38, 39 }

B grade { 35 }

C grade { }

F normal fail { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 12, 13, 14, 24, 25, 26, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

Giac

A grade { 8, 9, 10, 11, 15, 16, 25, 27, 28, 29, 30, 32, 34, 35, 36, 37, 47, 48, 49, 54, 55, 56 }

B grade { 1, 2, 3, 4, 17, 18, 19, 20, 21, 22, 23, 26, 33, 38, 39, 41, 42, 43, 45, 46, 51, 52, 53, 58, 59, 60 }

C grade { }

F normal fail { 40, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 12, 13, 14, 24, 31, 44, 50, 57 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 49, 55, 56 }

C grade { }

F normal fail { }

F(-1) timedout fail { 20, 21, 25, 32, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { 17, 18, 36, 37 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 21, 22, 23, 24, 25, 31, 32, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77 }

F(-1) timedout fail { 8, 9, 10, 11, 15, 16, 19, 26, 27, 28, 29, 30, 33, 34, 35, 38, 39, 40, 53, 59, 60, 66, 72, 78 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	415	373	508	444	406	0	1539	3993
N.S.	1	1.00	0.90	1.22	1.07	0.98	0.00	3.71	9.62
time (sec)	N/A	0.465	1.670	2.226	0.289	0.290	0.000	0.430	51.668

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	262	358	307	279	0	1059	2920
N.S.	1	1.00	0.92	1.25	1.07	0.98	0.00	3.70	10.21
time (sec)	N/A	0.372	1.274	1.633	0.283	0.275	0.000	0.412	38.594

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	170	159	231	174	170	0	631	736
N.S.	1	1.01	0.95	1.38	1.04	1.01	0.00	3.76	4.38
time (sec)	N/A	0.163	0.701	1.643	0.291	0.291	0.000	0.337	13.443

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	155	93	95	0	284	361
N.S.	1	1.00	0.94	1.63	0.98	1.00	0.00	2.99	3.80
time (sec)	N/A	0.069	0.302	1.646	0.287	0.277	0.000	0.316	7.872

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	156	289	0	493	0	0	5803
N.S.	1	1.00	1.28	2.37	0.00	4.04	0.00	0.00	47.57
time (sec)	N/A	0.224	0.610	1.676	0.000	4.253	0.000	0.000	28.855

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	197	899	0	1025	0	0	10198
N.S.	1	1.00	1.21	5.52	0.00	6.29	0.00	0.00	62.56
time (sec)	N/A	0.226	0.898	1.671	0.000	16.648	0.000	0.000	57.674

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	228	1449	0	1580	0	0	9097
N.S.	1	1.00	0.92	5.84	0.00	6.37	0.00	0.00	36.68
time (sec)	N/A	0.245	1.263	1.637	0.000	0.363	0.000	0.000	66.294

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	259	361	355	286	0	374	2606
N.S.	1	1.00	0.76	1.06	1.04	0.84	0.00	1.10	7.66
time (sec)	N/A	0.409	1.017	1.631	0.287	0.277	0.000	0.337	37.958

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	178	256	231	192	0	232	1732
N.S.	1	1.00	0.78	1.12	1.01	0.84	0.00	1.02	7.60
time (sec)	N/A	0.329	0.680	1.644	0.281	0.279	0.000	0.319	35.089

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	133	107	173	131	114	0	112	492
N.S.	1	1.02	0.82	1.33	1.01	0.88	0.00	0.86	3.78
time (sec)	N/A	0.169	0.463	1.655	0.312	0.283	0.000	0.288	14.683

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	57	67	0	60	232
N.S.	1	1.00	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.048	0.251	1.634	0.270	0.264	0.000	0.303	8.146

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	156	289	0	493	0	0	5803
N.S.	1	1.00	1.28	2.37	0.00	4.04	0.00	0.00	47.57
time (sec)	N/A	0.172	0.042	1.662	0.000	4.215	0.000	0.000	0.005

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	197	899	0	1025	0	0	10198
N.S.	1	1.00	1.21	5.52	0.00	6.29	0.00	0.00	62.56
time (sec)	N/A	0.189	0.101	1.657	0.000	16.955	0.000	0.000	0.010

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	228	1449	0	1580	0	0	9097
N.S.	1	1.00	0.92	5.84	0.00	6.37	0.00	0.00	36.68
time (sec)	N/A	0.206	0.157	1.652	0.000	0.415	0.000	0.000	0.007

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	139	87	78	0	76	244
N.S.	1	1.00	0.95	1.76	1.10	0.99	0.00	0.96	3.09
time (sec)	N/A	0.090	0.021	5.667	0.300	0.335	0.000	0.326	8.580

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	57	67	0	60	232
N.S.	1	1.00	1.02	1.86	0.90	1.06	0.00	0.95	3.68
time (sec)	N/A	0.042	0.013	1.635	0.275	0.350	0.000	0.309	8.110

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	96	57	81	245	196	122
N.S.	1	1.00	1.52	2.00	1.19	1.69	5.10	4.08	2.54
time (sec)	N/A	0.112	0.017	1.598	0.284	0.300	28.586	0.346	4.514

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	73	97	57	84	221	282	114
N.S.	1	1.00	1.52	2.02	1.19	1.75	4.60	5.88	2.38
time (sec)	N/A	0.110	0.020	1.635	0.287	0.309	28.220	0.376	4.937

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	70	108	98	65	0	407	312
N.S.	1	1.00	0.99	1.52	1.38	0.92	0.00	5.73	4.39
time (sec)	N/A	0.125	0.024	1.624	0.264	0.302	0.000	0.365	7.626

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	584	402	575	584	1001	0	2671	0
N.S.	1	0.99	0.68	0.97	0.99	1.69	0.00	4.52	0.00
time (sec)	N/A	0.966	0.999	1.691	0.289	0.345	0.000	1.023	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	450	286	401	417	703	0	1868	0
N.S.	1	1.00	0.63	0.89	0.92	1.56	0.00	4.14	0.00
time (sec)	N/A	0.628	0.686	1.673	0.288	0.335	0.000	0.849	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	297	183	253	248	441	0	1142	1765
N.S.	1	0.99	0.61	0.84	0.83	1.47	0.00	3.81	5.88
time (sec)	N/A	0.295	0.394	1.669	0.295	0.330	0.000	0.632	37.004

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	123	163	140	265	0	527	876
N.S.	1	1.00	0.56	0.74	0.63	1.20	0.00	2.38	3.96
time (sec)	N/A	0.097	0.211	1.658	0.277	0.310	0.000	0.431	19.441

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	178	299	0	0	0	0	9298
N.S.	1	1.00	0.64	1.08	0.00	0.00	0.00	0.00	33.45
time (sec)	N/A	0.325	0.430	1.719	0.000	0.000	0.000	0.000	56.987

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	229	1166	0	0	0	526	0
N.S.	1	1.00	0.71	3.62	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	0.389	0.756	1.703	0.000	0.000	0.000	0.457	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	361	252	1794	0	1355	0	1425	9344
N.S.	1	0.99	0.69	4.94	0.00	3.73	0.00	3.93	25.74
time (sec)	N/A	0.463	1.027	1.674	0.000	41.191	0.000	0.711	108.411

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	496	283	390	471	700	0	571	4167
N.S.	1	0.99	0.56	0.78	0.94	1.40	0.00	1.14	8.32
time (sec)	N/A	0.808	0.629	1.654	0.275	0.335	0.000	0.354	151.648

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	369	200	270	317	482	0	363	2799
N.S.	1	1.00	0.54	0.73	0.86	1.31	0.00	0.99	7.61
time (sec)	N/A	0.565	0.431	1.683	0.284	0.318	0.000	0.327	81.282

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	249	128	175	189	302	0	191	1011
N.S.	1	1.01	0.52	0.71	0.77	1.23	0.00	0.78	4.11
time (sec)	N/A	0.270	0.235	1.672	0.273	0.310	0.000	0.330	37.946

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	90	126	88	196	0	106	489
N.S.	1	1.00	0.51	0.71	0.50	1.11	0.00	0.60	2.76
time (sec)	N/A	0.085	0.135	1.666	0.295	0.282	0.000	0.315	20.546

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	178	299	0	0	0	0	9298
N.S.	1	1.00	0.64	1.08	0.00	0.00	0.00	0.00	33.45
time (sec)	N/A	0.305	0.161	1.700	0.000	0.000	0.000	0.000	0.008

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	229	1166	0	0	0	526	0
N.S.	1	1.00	0.71	3.62	0.00	0.00	0.00	1.63	0.00
time (sec)	N/A	0.386	0.267	1.677	0.000	0.000	0.000	0.492	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	361	252	1794	0	1355	0	1425	9344
N.S.	1	0.99	0.69	4.94	0.00	3.73	0.00	3.93	25.74
time (sec)	N/A	0.397	0.417	5.677	0.000	40.270	0.000	0.718	0.008

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	151	74	108	100	73	0	105	318
N.S.	1	1.74	0.85	1.24	1.15	0.84	0.00	1.21	3.66
time (sec)	N/A	0.101	0.060	1.644	0.201	0.280	0.000	0.309	16.031

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	52	135	63	96	90	61	0	80	312
N.S.	1	2.60	1.21	1.85	1.73	1.17	0.00	1.54	6.00
time (sec)	N/A	0.050	0.043	5.700	0.192	0.280	0.000	0.304	20.382

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	73	240	71	118
N.S.	1	2.45	1.25	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.115	0.047	1.648	0.286	0.281	29.129	0.320	5.821

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	C	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	55	135	69	95	56	82	216	83	118
N.S.	1	2.45	1.25	1.73	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.117	0.063	1.651	0.284	0.312	28.633	0.287	5.503

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	129	60	76	61	69	0	145	316
N.S.	1	1.55	0.72	0.92	0.73	0.83	0.00	1.75	3.81
time (sec)	N/A	0.124	0.047	5.713	0.287	0.300	0.000	0.316	14.593

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	171	71	89	86	90	0	197	304
N.S.	1	1.47	0.61	0.77	0.74	0.78	0.00	1.70	2.62
time (sec)	N/A	0.134	0.069	1.650	0.315	0.308	0.000	0.325	12.125

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	242	185	1095	0	1186	0	0	7235
N.S.	1	1.22	0.93	5.50	0.00	5.96	0.00	0.00	36.36
time (sec)	N/A	0.232	0.602	1.694	0.000	0.323	0.000	0.000	72.885

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1348	1345	1253	5734	0	3096	0	4656	0
N.S.	1	1.00	0.93	4.25	0.00	2.30	0.00	3.45	0.00
time (sec)	N/A	1.565	4.434	1.680	0.000	1.577	0.000	0.865	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	719	662	3025	0	1620	0	2592	0
N.S.	1	1.00	0.92	4.20	0.00	2.25	0.00	3.60	0.00
time (sec)	N/A	0.639	2.731	1.667	0.000	0.651	0.000	0.619	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	283	1207	0	840	0	1073	0
N.S.	1	1.00	0.86	3.66	0.00	2.55	0.00	3.25	0.00
time (sec)	N/A	0.230	0.874	1.657	0.000	0.321	0.000	0.432	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	453	404	3898	0	0	0	0	0
N.S.	1	1.01	0.90	8.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.906	1.544	5.725	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	521	521	358	4680	0	0	0	1507	0
N.S.	1	1.00	0.69	8.98	0.00	0.00	0.00	2.89	0.00
time (sec)	N/A	1.137	2.127	1.693	0.000	0.000	0.000	1.539	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	658	657	536	11204	0	0	0	8241	0
N.S.	1	1.00	0.81	17.03	0.00	0.00	0.00	12.52	0.00
time (sec)	N/A	1.752	4.755	1.692	0.000	0.000	0.000	4.811	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1032	1032	3220	3958	0	2176	0	1509	0
N.S.	1	1.00	3.12	3.84	0.00	2.11	0.00	1.46	0.00
time (sec)	N/A	1.124	16.915	1.678	0.000	3.084	0.000	0.455	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	474	2002	0	1114	0	733	0
N.S.	1	1.00	0.88	3.71	0.00	2.06	0.00	1.36	0.00
time (sec)	N/A	0.448	8.633	1.668	0.000	0.921	0.000	0.365	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	217	763	0	576	0	313	1832
N.S.	1	1.00	0.88	3.10	0.00	2.34	0.00	1.27	7.45
time (sec)	N/A	0.151	3.616	1.672	0.000	0.372	0.000	0.302	98.801

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	465	1822	0	0	0	0	0
N.S.	1	1.00	1.60	6.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	11.908	1.689	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	417	3670	0	0	0	1354	0
N.S.	1	1.00	1.15	10.08	0.00	0.00	0.00	3.72	0.00
time (sec)	N/A	0.730	11.283	1.701	0.000	0.000	0.000	1.402	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	523	9100	0	0	0	7922	0
N.S.	1	1.00	1.08	18.80	0.00	0.00	0.00	16.37	0.00
time (sec)	N/A	1.070	13.010	1.699	0.000	0.000	0.000	23.147	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	657	15990	0	0	0	25338	0
N.S.	1	1.00	0.96	23.34	0.00	0.00	0.00	36.99	0.00
time (sec)	N/A	1.214	14.908	1.709	0.000	0.000	0.000	60.944	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	715	632	2528	0	1436	0	946	0
N.S.	1	1.00	0.88	3.52	0.00	2.00	0.00	1.32	0.00
time (sec)	N/A	0.913	2.224	1.681	0.000	1.120	0.000	0.359	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	369	314	1199	0	720	0	441	2621
N.S.	1	0.99	0.85	3.23	0.00	1.94	0.00	1.19	7.06
time (sec)	N/A	0.320	0.958	1.681	0.000	0.486	0.000	0.326	140.157

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	141	425	0	380	0	190	833
N.S.	1	1.00	0.86	2.59	0.00	2.32	0.00	1.16	5.08
time (sec)	N/A	0.107	0.331	5.701	0.000	0.319	0.000	0.290	50.858

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	183	746	0	0	0	0	0
N.S.	1	1.00	0.97	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.500	1.692	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	249	2973	0	0	0	1319	0
N.S.	1	1.00	0.98	11.70	0.00	0.00	0.00	5.19	0.00
time (sec)	N/A	0.437	1.067	1.693	0.000	0.000	0.000	1.099	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	420	7119	0	4058	0	7939	0
N.S.	1	1.00	0.99	16.79	0.00	9.57	0.00	18.72	0.00
time (sec)	N/A	0.650	2.400	1.690	0.000	164.611	0.000	38.668	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	826	826	1036	18802	0	0	0	25632	0
N.S.	1	1.00	1.25	22.76	0.00	0.00	0.00	31.03	0.00
time (sec)	N/A	1.603	6.959	1.704	0.000	0.000	0.000	58.083	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1182	1154	1422	2077	0	1916	0	0	0
N.S.	1	0.98	1.20	1.76	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	2.767	33.212	1.736	0.000	0.167	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	774	769	917	1205	0	1393	0	0	0
N.S.	1	0.99	1.18	1.56	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	1.461	28.729	1.823	0.000	0.149	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	706	706	633	1163	0	1463	0	0	0
N.S.	1	1.00	0.90	1.65	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	1.217	25.895	2.279	0.000	0.166	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	687	815	1378	0	2588	0	0	0
N.S.	1	1.00	1.19	2.01	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	1.301	28.992	5.449	0.000	0.267	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	964	964	1444	2292	0	4721	0	0	0
N.S.	1	1.00	1.50	2.38	0.00	4.90	0.00	0.00	0.00
time (sec)	N/A	2.160	33.800	4.641	0.000	0.865	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1716	1716	2437	3900	0	9150	0	0	0
N.S.	1	1.00	1.42	2.27	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	5.036	36.937	5.376	0.000	3.015	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1235	1235	1470	2108	0	1931	0	0	0
N.S.	1	1.00	1.19	1.71	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	2.965	34.132	3.500	0.000	0.159	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	766	766	922	1205	0	1392	0	0	0
N.S.	1	1.00	1.20	1.57	0.00	1.82	0.00	0.00	0.00
time (sec)	N/A	1.470	28.613	1.846	0.000	0.178	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	562	812	0	1036	0	0	0
N.S.	1	1.00	1.07	1.54	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.617	26.835	1.950	0.000	0.135	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	540	540	551	861	0	1336	0	0	0
N.S.	1	1.00	1.02	1.59	0.00	2.47	0.00	0.00	0.00
time (sec)	N/A	0.739	25.022	3.020	0.000	0.146	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	596	724	1269	0	2429	0	0	0
N.S.	1	1.00	1.21	2.13	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.920	28.129	4.205	0.000	0.307	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1034	1034	1449	2330	0	4867	0	0	0
N.S.	1	1.00	1.40	2.25	0.00	4.71	0.00	0.00	0.00
time (sec)	N/A	2.169	33.497	5.678	0.000	0.999	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	838	831	1000	1233	0	1388	0	0	0
N.S.	1	0.99	1.19	1.47	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	1.455	29.122	2.923	0.000	0.157	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	524	615	812	0	1036	0	0	0
N.S.	1	0.99	1.16	1.54	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.681	25.664	1.884	0.000	0.139	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	384	418	615	0	807	0	0	0
N.S.	1	0.99	1.08	1.59	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.360	24.490	2.602	0.000	0.136	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	477	784	0	1240	0	0	0
N.S.	1	1.00	1.13	1.86	0.00	2.94	0.00	0.00	0.00
time (sec)	N/A	0.450	24.270	3.341	0.000	0.152	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	699	1249	0	2344	0	0	0
N.S.	1	1.00	1.09	1.95	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	1.014	27.394	4.564	0.000	0.302	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1116	1116	1258	2283	0	5108	0	0	0
N.S.	1	1.00	1.13	2.05	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	2.396	33.536	6.002	0.000	1.049	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [36] had the largest ratio of [.2500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	37	0.162
2	A	6	6	1.00	37	0.162
3	A	5	5	1.01	35	0.143
4	A	5	5	1.00	30	0.167
5	A	6	6	1.00	37	0.162
6	A	6	6	1.00	37	0.162
7	A	5	5	1.00	37	0.135
8	A	6	5	1.00	37	0.135
9	A	5	5	1.00	37	0.135
10	A	4	4	1.02	35	0.114
11	A	4	4	1.00	30	0.133
12	A	6	6	1.00	37	0.162
13	A	6	6	1.00	37	0.162
14	A	5	5	1.00	37	0.135
15	A	4	4	1.00	31	0.129
16	A	4	4	1.00	30	0.133
17	A	7	7	1.00	33	0.212
18	A	7	7	1.00	33	0.212
19	A	6	6	1.00	33	0.182
20	A	8	7	0.99	40	0.175
21	A	7	7	1.00	40	0.175
22	A	6	6	0.99	38	0.158
23	A	6	6	1.00	33	0.182
24	A	7	7	1.00	40	0.175

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	7	1.00	40	0.175
26	A	5	5	0.99	40	0.125
27	A	7	6	0.99	40	0.150
28	A	6	6	1.00	40	0.150
29	A	5	5	1.01	38	0.132
30	A	5	5	1.00	33	0.152
31	A	7	7	1.00	40	0.175
32	A	7	7	1.00	40	0.175
33	A	5	5	0.99	40	0.125
34	A	5	5	1.74	30	0.167
35	B	5	5	2.60	29	0.172
36	B	8	8	2.45	32	0.250
37	B	8	8	2.45	32	0.250
38	A	6	6	1.55	32	0.188
39	A	7	7	1.47	32	0.219
40	A	5	5	1.22	32	0.156
41	A	8	7	1.00	36	0.194
42	A	7	6	1.00	34	0.176
43	A	7	6	1.00	29	0.207
44	A	9	8	1.01	36	0.222
45	A	9	8	1.00	36	0.222
46	A	9	9	1.00	36	0.250
47	A	7	7	1.00	36	0.194
48	A	6	6	1.00	34	0.176
49	A	6	6	1.00	29	0.207
50	A	8	8	1.00	36	0.222
51	A	8	8	1.00	36	0.222
52	A	8	8	1.00	36	0.222
53	A	6	6	1.00	36	0.167
54	A	6	6	1.00	36	0.167
55	A	5	5	0.99	34	0.147
56	A	5	5	1.00	29	0.172
57	A	7	7	1.00	36	0.194
58	A	7	7	1.00	36	0.194
59	A	5	5	1.00	36	0.139

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	5	1.00	36	0.139
61	A	10	7	0.98	38	0.184
62	A	9	7	0.99	38	0.184
63	A	9	7	1.00	38	0.184
64	A	9	8	1.00	38	0.210
65	A	9	7	1.00	38	0.184
66	A	10	8	1.00	38	0.210
67	A	10	7	1.00	38	0.184
68	A	9	7	1.00	38	0.184
69	A	8	7	1.00	38	0.184
70	A	8	7	1.00	38	0.184
71	A	8	7	1.00	38	0.184
72	A	9	8	1.00	38	0.210
73	A	9	7	0.99	38	0.184
74	A	8	7	0.99	38	0.184
75	A	7	6	0.99	38	0.158
76	A	7	6	1.00	38	0.158
77	A	8	7	1.00	38	0.184
78	A	9	7	1.00	38	0.184

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 415

$$
\begin{aligned}
&\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-d^2x^2}}{16d^4} \\
&\quad - \frac{\left(7d^2f(Be+2Af)-C(3d^2e^2-8f^2)\right)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{70d^4f} \\
&\quad + \frac{(3Ce-7Bf)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} \\
&\quad + \frac{\left(8(C(3d^4e^4-30d^2e^2f^2-8f^4)-7d^2f(2Af(6d^2e^2+f^2)+B(d^2e^3+6ef^2)))+3d^2f(6Cd^2e^3-14Bd^2e^2 \ldots\right.}{840d^6f} \\
&\quad + \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)\arcsin(dx)}{16d^5}
\end{aligned}
$$

```
[Out] -1/70*(7*d^2*f*(2*A*f+B*e)-C*(3*d^2*e^2-8*f^2))*(f*x+e)^2*(-d^2*x^2+1)^(3/2
)/d^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-d^2*x^2+1)^(3/2)/d^2/f-1/7*C*(f*x+e
)^4*(-d^2*x^2+1)^(3/2)/d^2/f+1/840*(8*C*(3*d^4*e^4-30*d^2*e^2*f^2-8*f^4)-56
*d^2*f*(2*A*f*(6*d^2*e^2+f^2)+B*(d^2*e^3+6*e*f^2))+3*d^2*f*(-98*A*d^2*e*f^2
-14*B*d^2*e^2*f+6*C*d^2*e^3-35*B*f^3-41*C*e*f^2)*x)*(-d^2*x^2+1)^(3/2)/d^6/
f+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2
)*arcsin(d*x)/d^5+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3
+B*f^3+3*C*e*f^2)*x*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1668, 847, 794, 201, 222}

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\arcsin(dx)\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^5}$$
$$- \frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}$$
$$+ \frac{x\sqrt{1-d^2x^2}\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^4}$$
$$+ \frac{\left(1-d^2x^2\right)^{3/2}\left(3d^2fx\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+6Cd^2e^3-41Cef^2\right)+8(C(3d^4e^4-30d^2e^2f^2\right.}{840d^6f}$$
$$+ \frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^3(3Ce-7Bf)}{42d^2f} - \frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B
*f^3)*x*Sqrt[1 - d^2*x^2])/(16*d^4) - ((7*d^2*f*(B*e + 2*A*f) - C*(3*d^2*e^
2 - 8*f^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(70*d^4*f) + ((3*C*e - 7*B*f)*
(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(42*d^2*f) - (C*(e + f*x)^4*(1 - d^2*x^2)^
(3/2))/(7*d^2*f) + ((8*(C*(3*d^4*e^4 - 30*d^2*e^2*f^2 - 8*f^4) - 7*d^2*f*(2
*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2))) + 3*d^2*f*(6*C*d^2*e^3 - 1
4*B*d^2*e^2*f - 41*C*e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x)*(1 - d^2*x^2)^(3
/2))/(840*d^6*f) + ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2
+ 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(16*d^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int (e+fx)^3 \left(A+Bx+Cx^2\right) \sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f} - \frac{\int (e+fx)^3 \left(-\left(\left(4C+7Ad^2\right)f^2\right)+d^2f(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}\,dx}{7d^2f^2}$$

$$= \frac{(3Ce-7Bf)(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f}$$
$$+ \frac{\int (e+fx)^2 \left(3d^2f^2(5Ce+14Ad^2e+7Bf)+3d^2f\left(2\left(4C+7Ad^2\right)f^2-d^2e(3Ce-7Bf)\right)x\right)\sqrt{1}}{42d^4f^2}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2 \left(1-d^2x^2\right)^{3/2}}{70d^4f}$$
$$+ \frac{(3Ce-7Bf)(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f}$$
$$- \frac{\int (e+fx)\left(-3d^2f^2\left(19Cd^2e^2+70Ad^4e^2+49Bd^2ef+16Cf^2+28Ad^2f^2\right)+3d^4f\left(6Cd^2e^3-14\right.}{210d^6f^2}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2 \left(1-d^2x^2\right)^{3/2}}{70d^4f}$$
$$+ \frac{(3Ce-7Bf)(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f}$$
$$+ \frac{\left(8\left(C\left(3d^4e^4-30d^2e^2f^2-8f^4\right)-7d^2f\left(2Af\left(6d^2e^2+f^2\right)+B\left(d^2e^3+6ef^2\right)\right)\right)+3d^2f\left(6Cd^2e^3-\right.\right.}{840d^6f}$$
$$+ \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)\int \sqrt{1-d^2x^2}\,dx}{8d^4}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-d^2x^2}}{16d^4}$$
$$- \frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2 \left(1-d^2x^2\right)^{3/2}}{70d^4f}$$
$$+ \frac{(3Ce-7Bf)(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f}$$
$$+ \frac{\left(8\left(C\left(3d^4e^4-30d^2e^2f^2-8f^4\right)-7d^2f\left(2Af\left(6d^2e^2+f^2\right)+B\left(d^2e^3+6ef^2\right)\right)\right)+3d^2f\left(6Cd^2e^3-\right.\right.}{840d^6f}$$
$$+ \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{16d^4}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)x\sqrt{1-d^2x^2}}{16d^4}$$
$$- \frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2 \left(1-d^2x^2\right)^{3/2}}{70d^4f}$$
$$+ \frac{(3Ce-7Bf)(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4 \left(1-d^2x^2\right)^{3/2}}{7d^2f}$$
$$+ \frac{\left(8\left(C\left(3d^4e^4-30d^2e^2f^2-8f^4\right)-7d^2f\left(2Af\left(6d^2e^2+f^2\right)+B\left(d^2e^3+6ef^2\right)\right)\right)+3d^2f\left(6Cd^2e^3-\right.\right.}{840d^6f}$$
$$+ \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right)\sin^{-1}(dx)}{16d^5}$$

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.90

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{1-d^2x^2}(14Ad^2(-16f^3-d^2f(120e^2+45efx+8f^2x^2)+6d^4x(10e^3+20e^2fx+15ef^2x^2+4f^3x^3))+7}{}$$

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*x
^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3*d
^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x
^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) + C*(-12
8*f^3 - d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) - 6*d^4*x*(35*e^3 + 56*e^2
*f*x + 35*e*f^2*x^2 + 8*f^3*x^3) + 12*d^6*x^3*(35*e^3 + 84*e^2*f*x + 70*e*f
^2*x^2 + 20*f^3*x^3))) + 210*d*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f +
 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])
/(1680*d^6)
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{(240C\,f^3x^6d^6+280B\,d^6f^3x^5+840C\,d^6e\,f^2x^5+336A\,d^6f^3x^4+1008B\,d^6e\,f^2x^4+1008C\,d^6e^2f\,x^4+1260A\,d^6e\,f^2x^3+1260B\,d^6e^2f\,x^3}{}$
default	Expression too large to display

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/1680*(240*C*d^6*f^3*x^6+280*B*d^6*f^3*x^5+840*C*d^6*e*f^2*x^5+336*A*d^6*
f^3*x^4+1008*B*d^6*e*f^2*x^4+1008*C*d^6*e^2*f*x^4+1260*A*d^6*e*f^2*x^3+1260
*B*d^6*e^2*f*x^3+420*C*d^6*e^3*x^3+1680*A*d^6*e^2*f*x^2+560*B*d^6*e^3*x^2-4
8*C*d^4*f^3*x^4+840*A*d^6*e^3*x-70*B*d^4*f^3*x^3-210*C*d^4*e*f^2*x^3-112*A*
d^4*f^3*x^2-336*B*d^4*e*f^2*x^2-336*C*d^4*e^2*f*x^2-630*A*d^4*e*f^2*x-630*B
*d^4*e^2*f*x-210*C*d^4*e^3*x-1680*A*d^4*e^2*f-560*B*d^4*e^3-64*C*d^2*f^3*x^
2-105*B*d^2*f^3*x-315*C*d^2*e*f^2*x-224*A*d^2*f^3-672*B*d^2*e*f^2-672*C*d^2
*e^2*f-128*C*f^3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x
+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16/d^4*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d
^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2)/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^
2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(240\,Cd^6f^3x^6-560\,Bd^4e^3-672\,Bd^2ef^2+280\,(3\,Cd^6ef^2+Bd^6f^3)x^5+48\,(21\,Cd^6e^2f+21\,Bd^6ef^2+(7\ldots}{}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1680*((240*C*d^6*f^3*x^6 - 560*B*d^4*e^3 - 672*B*d^2*e*f^2 + 280*(3*C*d^6
*e*f^2 + B*d^6*f^3)*x^5 + 48*(21*C*d^6*e^2*f + 21*B*d^6*e*f^2 + (7*A*d^6 -
C*d^4)*f^3)*x^4 - 336*(5*A*d^4 + 2*C*d^2)*e^2*f - 32*(7*A*d^2 + 4*C)*f^3 +
70*(6*C*d^6*e^3 + 18*B*d^6*e^2*f - B*d^4*f^3 + 3*(6*A*d^6 - C*d^4)*e*f^2)*x
^3 + 16*(35*B*d^6*e^3 - 21*B*d^4*e*f^2 + 21*(5*A*d^6 - C*d^4)*e^2*f - (7*A*
d^4 + 4*C*d^2)*f^3)*x^2 - 105*(6*B*d^4*e^2*f + B*d^2*f^3 - 2*(4*A*d^6 - C*d
^4)*e^3 + 3*(2*A*d^4 + C*d^2)*e*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 210*
(6*B*d^3*e^2*f + B*d*f^3 + 2*(4*A*d^5 + C*d^3)*e^3 + 3*(2*A*d^3 + C*d)*e*f^
2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.07

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^4}{7\,d^2} + \frac{1}{2}\sqrt{-d^2x^2+1}Ae^3x + \frac{Ae^3\arcsin(dx)}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Be^3}{3\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Ae^2f}{d^2} - \frac{4\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^2}{35\,d^4} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x^3}{6\,d^2} - \frac{(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}x^2}{5\,d^2} - \frac{(Ce^3+3\,Be^2f+3\,Aef^2)(-d^2x^2+1)^{\frac{3}{2}}x}{4\,d^2} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{8\,d^2} - \frac{8\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3}{105\,d^6} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x}{8\,d^4} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin(dx)}{8\,d^3} - \frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}}{15\,d^4} + \frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{16\,d^4} + \frac{(3\,Cef^2+Bf^3)\arcsin(dx)}{16\,d^5}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^3*x +
1/2*A*e^3*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^3/d^2 - (-d^2*x^2 +
1)^(3/2)*A*e^2*f/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^2/d^4 - 1/6*(3*C*e
*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 - 1/5*(3*C*e^2*f + 3*B*e*f^2 + A
*f^3)*(-d^2*x^2 + 1)^(3/2)*x^2/d^2 - 1/4*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*(-
d^2*x^2 + 1)^(3/2)*x/d^2 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^
2 + 1)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1/8*(3*C*e*f^2 + B*f^
3)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*arcsin(
d*x)/d^3 - 2/15*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/2)/d^4 +
1/16*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 + 1/16*(3*C*e*f^2 + B*f^3
)*arcsin(d*x)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. 2(391) = 782.

Time = 0.43 (sec) , antiderivative size = 1539, normalized size of antiderivative = 3.71

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/1680*(840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*A*d^5*e^3 + 1680*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1
/2*sqrt(2)*sqrt(d*x + 1)))*A*d^5*e^3 + 280*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^4*e^3
+ 840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d
*x + 1)))*B*d^4*e^3 + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-
d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2*f + 2520*(sqrt(d*
x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^
4*e^2*f + 70*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1
)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^3 + 280*((
(2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt
(2)*sqrt(d*x + 1)))*C*d^3*e^3 + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x
+ 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x +
1)))*B*d^3*e^2*f + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x
 + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2*f + 210*(((2*(3*d*x
- 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f^2 + 840*(((2*d*x - 5)*(d*x + 1) +
 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d
^3*e*f^2 + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x +
 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x +
1)))*C*d^2*e^2*f + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sq
rt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^2*e^
2*f + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) +
 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*
B*d^2*e*f^2 + 210*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*
x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e*f^2 +
 14*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)
*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2
*f^3 + 70*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*s
qrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^3 + 21*(((2*(
(4*(5*d*x - 26)*(d*x + 1) + 321)*(d*x + 1) - 451)*(d*x + 1) + 745)*(d*x + 1
) - 405)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*C*d*e*f^2 + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(
d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d
*x + 1)))*C*d*e*f^2 + 7*(((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d*x + 1) -
451)*(d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*a
rcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^3 + 14*(((2*(3*(4*d*x - 17)*(d*x +
1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) +
90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^3 + (((2*((4*(5*(6*d*x - 37)*(d
*x + 1) + 661)*(d*x + 1) - 4551)*(d*x + 1) + 4781)*(d*x + 1) - 6335)*(d*x +
 1) + 2835)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 1050*arcsin(1/2*sqrt(2)*sqrt(d*x
 + 1)))*C*f^3 + 7*(((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d*x + 1) - 451)*(
d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(
1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^3)/d^6
```

Mupad [B] (verification not implemented)

Time = 51.67 (sec) , antiderivative size = 3993, normalized size of antiderivative = 9.62

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] int((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)

[Out] - ((((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^22
)/((d*x + 1)^(1/2) - 1)^22 - (((20480*C*f^3)/3 - 448*C*d^2*e^2*f)*((1 - d*x
)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((20480*C*f^3)/3 - 448*C*d^2*e^2
*f)*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (((458752*C*f^3)/1
5 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^
10 + (((458752*C*f^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 - (((1011712*C*f^3)/15 - (13184*C*d^2*e^2*f)/5)*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 - (((1011712*C*f^3)/15 -
(13184*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 +
 (((9293824*C*f^3)/105 - (15104*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^14)/(
(d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^3*((29*C*d^3*e^3)/2 - (41*
C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^25*((29*C*d
^3*e^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^25 - (((1 - d*x)^(1/2)
 - 1)^5*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^5 + (((1
 - d*x)^(1/2) - 1)^23*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2)
 - 1)^23 - (((1 - d*x)^(1/2) - 1)^7*(209*C*d^3*e^3 + (8755*C*d*e*f^2)/2))/(
(d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^21*(209*C*d^3*e^3 + (8755*C
*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2) - 1)^11*((1767*C
*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(
1/2) - 1)^17*((1767*C*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) -
1)^17 + (((1 - d*x)^(1/2) - 1)^13*(646*C*d^3*e^3 - 17527*C*d*e*f^2))/((d*x
+ 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^15*(646*C*d^3*e^3 - 17527*C*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9*((165*C*d^3*e^3)
/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2) - 1)
^19*((165*C*d^3*e^3)/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^19 - (
d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(4*((d*x + 1)^(1/2) - 1)
) + (d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1)^27)/(4*((d*x + 1)^(1
/2) - 1)^27) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) -
 1)^4 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24
)/(d^6 + (14*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (91*d^6
*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (364*d^6*((1 - d*x)^(1/
2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (1001*d^6*((1 - d*x)^(1/2) - 1)^8)/((d
*x + 1)^(1/2) - 1)^8 + (2002*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2)
 - 1)^10 + (3003*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (
3432*d^6*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (3003*d^6*((1
```

```
 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (2002*d^6*((1 - d*x)^(1/2
) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (1001*d^6*((1 - d*x)^(1/2) - 1)^20)/(
(d*x + 1)^(1/2) - 1)^20 + (364*d^6*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/
2) - 1)^22 + (91*d^6*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24 + (
14*d^6*((1 - d*x)^(1/2) - 1)^26)/((d*x + 1)^(1/2) - 1)^26 + (d^6*((1 - d*x)
^(1/2) - 1)^28)/((d*x + 1)^(1/2) - 1)^28) - ((((4928*A*f^3)/3 + 512*A*d^2*e
^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((1408*A*f^3)/3 -
 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 - (((14
08*A*f^3)/3 - 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1
)^6 + (((4928*A*f^3)/3 + 512*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^12)/((d*x +
 1)^(1/2) - 1)^12 - (((11008*A*f^3)/5 - 912*A*d^2*e^2*f)*((1 - d*x)^(1/2) -
 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)*(2*A*d^3*e^3 - (3
*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^19*(2*A*d^3*
e^3 - (3*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^19 - (((1 - d*x)^(1/2) - 1)^3
*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1
/2) - 1)^17*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^17 - ((
(1 - d*x)^(1/2) - 1)^5*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^(1/2) - 1
)^5 + (((1 - d*x)^(1/2) - 1)^15*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^
(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1)^7*(88*A*d^3*e^3 - 306*A*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^13*(88*A*d^3*e^3 - 306*A*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^9*(52*A*d^3*e^3 -
663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^11*(52*A*d
^3*e^3 - 663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 + (64*A*f^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*A*f^3*((1 - d*x)^(1/2) - 1)^16)/(
(d*x + 1)^(1/2) - 1)^16 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2
) - 1)^18)/(d^4 + (10*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (45*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 -
 d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1
)^8)/((d*x + 1)^(1/2) - 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1
)^(1/2) - 1)^10 + (210*d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^
12 + (120*d^4*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^4*
((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (10*d^4*((1 - d*x)^(1/
2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (d^4*((1 - d*x)^(1/2) - 1)^20)/((d*x
 + 1)^(1/2) - 1)^20) - ((((B*f^3)/4 + (3*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) -
 1)^23)/((d*x + 1)^(1/2) - 1)^23 - (((35*B*f^3)/12 - (93*B*d^2*e^2*f)/2)*((
1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (((35*B*f^3)/12 - (93*B*d^
2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^21)/((d*x + 1)^(1/2) - 1)^21 + (((757*B*f
^3)/4 - (417*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)
^5 - (((757*B*f^3)/4 - (417*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^19)/((d*x
 + 1)^(1/2) - 1)^19 - (((7339*B*f^3)/4 + (513*B*d^2*e^2*f)/2)*((1 - d*x)^(1
/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (((7339*B*f^3)/4 + (513*B*d^2*e^2*f)/
2)*((1 - d*x)^(1/2) - 1)^17)/((d*x + 1)^(1/2) - 1)^17 - (((25661*B*f^3)/2 -
 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 + (((2
5661*B*f^3)/2 - 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^13)/((d*x + 1)^(1/2)
```

```
 - 1)^13 + (((41929*B*f^3)/6 + 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^9)/((
d*x + 1)^(1/2) - 1)^9 - (((41929*B*f^3)/6 + 969*B*d^2*e^2*f)*((1 - d*x)^(1/
2) - 1)^15)/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^4*(16*B*d^3*e
^3 + 192*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(1
6*B*d^3*e^3 + 192*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2)
- 1)^6*((56*B*d^3*e^3)/3 - 1024*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^6 + (((1
- d*x)^(1/2) - 1)^18*((56*B*d^3*e^3)/3 - 1024*B*d*e*f^2))/((d*x + 1)^(1/2)
- 1)^18 + (((1 - d*x)^(1/2) - 1)^8*(192*B*d^3*e^3 + 2304*B*d*e*f^2))/((d*x
+ 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^16*(192*B*d^3*e^3 + 2304*B*d*e*f
^2))/((d*x + 1)^(1/2) - 1)^16 + (((1 - d*x)^(1/2) - 1)^10*(656*B*d^3*e^3 +
(9216*B*d*e*f^2)/5))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(
656*B*d^3*e^3 + (9216*B*d*e*f^2)/5))/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)
^(1/2) - 1)^12*((2848*B*d^3*e^3)/3 - (16768*B*d*e*f^2)/5))/((d*x + 1)^(1/2)
 - 1)^12 - (((B*f^3)/4 + (3*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1))/((d*x +
1)^(1/2) - 1) + (8*B*d^3*e^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 + (8*B*d^3*e^3*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^5
+ (12*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^5*((1 -
d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^5*((1 - d*x)^(1/2) - 1)
^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^
(1/2) - 1)^8 + (792*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10
+ (924*d^5*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^5*((
1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^5*((1 - d*x)^(1/2
) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^5*((1 - d*x)^(1/2) - 1)^18)/((
d*x + 1)^(1/2) - 1)^18 + (66*d^5*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2)
 - 1)^20 + (12*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^
5*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (B*f*atan((B*f*(f^2
 + 6*d^2*e^2)*((1 - d*x)^(1/2) - 1))/((B*f^3 + 6*B*d^2*e^2*f)*((d*x + 1)^(1
/2) - 1)))*(f^2 + 6*d^2*e^2))/(4*d^5) - (A*e*atan((A*e*((1 - d*x)^(1/2) - 1
)*(3*f^2 + 4*d^2*e^2))/((4*A*d^2*e^3 + 3*A*e*f^2)*((d*x + 1)^(1/2) - 1)))*(
3*f^2 + 4*d^2*e^2))/(2*d^3) - (C*e*atan((C*e*((1 - d*x)^(1/2) - 1)*(3*f^2 +
 2*d^2*e^2))/((2*C*d^2*e^3 + 3*C*e*f^2)*((d*x + 1)^(1/2) - 1)))*(3*f^2 + 2*
d^2*e^2))/(4*d^5)
```

3.2 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 286

$$\begin{aligned}
&\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{\left(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2))\right)x\sqrt{1-d^2x^2}}{16d^4} \\
&\quad+\frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f}-\frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} \\
&\quad+\frac{\left(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f(5(C+2Ad^2)f^2-2d^2e(Ce-2Bf))x\right)(1-d}{120d^4f} \\
&\quad+\frac{\left(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2))\right)\arcsin(dx)}{16d^5}
\end{aligned}$$

```
[Out] 1/10*(-2*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/6*C*(f*x+e)^3*(-d^2*
x^2+1)^(3/2)/d^2/f+1/120*(8*C*(d^2*e^3-4*e*f^2)-16*f*(5*A*d^2*e*f+B*(d^2*e^
2+f^2))-3*f*(5*(2*A*d^2+C)*f^2-2*d^2*e*(-2*B*f+C*e))*x)*(-d^2*x^2+1)^(3/2)/
d^4/f+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*arcsin(d*x
)/d^5+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*x*(-d^2*x^
2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1623, 1668, 847, 794, 201, 222}

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\arcsin(dx)\left(2d^2(A(4d^2e^2+f^2)+2Bef)+C(2d^2e^2+f^2)\right)}{16d^5}$$
$$+\frac{x\sqrt{1-d^2x^2}\left(2d^2(A(4d^2e^2+f^2)+2Bef)+C(2d^2e^2+f^2)\right)}{16d^4}$$
$$+\frac{\left(1-d^2x^2\right)^{3/2}\left(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3fx(5f^2(2Ad^2+C)-2d^2e(Ce-2B\right.}{120d^4f}$$
$$+\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)}{10d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*x*Sqrt[1 - d
^2*x^2])/(16*d^4) + ((C*e - 2*B*f)*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(10*d^2
*f) - (C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(6*d^2*f) + ((8*(C*(d^2*e^3 - 4*e
*f^2) - 2*f*(5*A*d^2*e*f + B*(d^2*e^2 + f^2))) - 3*f*(5*(C + 2*A*d^2)*f^2 -
 2*d^2*e*(C*e - 2*B*f))*x)*(1 - d^2*x^2)^(3/2))/(120*d^4*f) + ((C*(2*d^2*e^
2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*ArcSin[d*x])/(16*d^5)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int (e+fx)^2 \left(A+Bx+Cx^2\right) \sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{6d^2f} - \frac{\int (e+fx)^2 \left(-3\left(C+2Ad^2\right)f^2+3d^2f(Ce-2Bf)x\right)\sqrt{1-d^2x^2}\,dx}{6d^2f^2}$$

$$= \frac{(Ce-2Bf)(e+fx)^2 \left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3 \left(1-d^2x^2\right)^{3/2}}{6d^2f} + \frac{\int (e+fx)\left(3d^2f^2\left(3Ce+10Ad^2e+4Bf\right)+3d^2f\left(5\left(C+2Ad^2\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-}}{30d^4f^2}$$

$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f}$$
$$+ \frac{(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f(5(C+2Ad^2)f^2-2d^2e(Ce-2Bf))x)}{120d^4f}$$
$$+ \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\int\sqrt{1-d^2x^2}\,dx}{8d^4}$$
$$= \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\,x\sqrt{1-d^2x^2}}{16d^4}$$
$$+ \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f}$$
$$+ \frac{(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f(5(C+2Ad^2)f^2-2d^2e(Ce-2Bf))x)}{120d^4f}$$
$$+ \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\int\frac{1}{\sqrt{1-d^2x^2}}\,dx}{16d^4}$$
$$= \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\,x\sqrt{1-d^2x^2}}{16d^4}$$
$$+ \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f}$$
$$+ \frac{(8(C(d^2e^3-4ef^2)-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f(5(C+2Ad^2)f^2-2d^2e(Ce-2Bf))x)}{120d^4f}$$
$$+ \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\sin^{-1}(dx)}{16d^5}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.92

$$\int\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{d\sqrt{1-d^2x^2}(10Ad^2(12d^2e^2x+16ef(-1+d^2x^2)+3f^2x(-1+2d^2x^2))+4B(-8f^2-d^2(20e^2+15efx+4}{}$$

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*f^
2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2) +
 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^2*x
^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d^4*x
^4))) + 30*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*Ar
cTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(240*d^5)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{(40C f^2 x^5 d^4+48B d^4 f^2 x^4+96C d^4 ef x^4+60A d^4 f^2 x^3+120B d^4 ef x^3+60C d^4 e^2 x^3+160A d^4 ef x^2+80B d^4 e^2 x^2+120A d^4 e^2 x-10}{2}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-30C\sqrt{-d^2x^2+1}\ \mathrm{csgn}(d)d^3e^2x+30C\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2e^2+120A\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^4e^2+30A\arctan\left(\right.\right.}{}$

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/240/d^4*(40*C*d^4*f^2*x^5+48*B*d^4*f^2*x^4+96*C*d^4*e*f*x^4+60*A*d^4*f^2
*x^3+120*B*d^4*e*f*x^3+60*C*d^4*e^2*x^3+160*A*d^4*e*f*x^2+80*B*d^4*e^2*x^2+
120*A*d^4*e^2*x-10*C*d^2*f^2*x^3-16*B*d^2*f^2*x^2-32*C*d^2*e*f*x^2-30*A*d^2
*f^2*x-60*B*d^2*e*f*x-30*C*d^2*e^2*x-160*A*d^2*e*f-80*B*d^2*e^2-15*C*f^2*x-
32*B*f^2-64*C*e*f)*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)
*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16*(8*A*d^4*e^2+2*A*d^2*f^2+4*B*d^2*e*f+2*
C*d^2*e^2+C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*(
(-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(40\,Cd^5f^2x^5 - 80\,Bd^3e^2 + 48\,(2\,Cd^5ef + Bd^5f^2)x^4 - 32\,Bdf^2 + 10\,(6\,Cd^5e^2 + 12\,Bd^5ef + (6\,Ad^5 - Cd^3}{}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/240*((40*C*d^5*f^2*x^5 - 80*B*d^3*e^2 + 48*(2*C*d^5*e*f + B*d^5*f^2)*x^4
- 32*B*d*f^2 + 10*(6*C*d^5*e^2 + 12*B*d^5*e*f + (6*A*d^5 - C*d^3)*f^2)*x^3
- 32*(5*A*d^3 + 2*C*d)*e*f + 16*(5*B*d^5*e^2 - B*d^3*f^2 + 2*(5*A*d^5 - C*d
^3)*e*f)*x^2 - 15*(4*B*d^3*e*f - 2*(4*A*d^5 - C*d^3)*e^2 + (2*A*d^3 + C*d)*
f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*e*f + 2*(4*A*d^4 + C*d^2
)*e^2 + (2*A*d^2 + C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
[Out] Integral((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x^3}{6\,d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}Ae^2x+\frac{Ae^2\arcsin(dx)}{2\,d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be^2}{3\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Aef}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x}{8\,d^4}+\frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{8\,d^2}+\frac{\sqrt{-d^2x^2+1}Cf^2x}{16\,d^4}+\frac{(Ce^2+2\,Bef+Af^2)\arcsin(dx)}{8\,d^3}+\frac{Cf^2\arcsin(dx)}{16\,d^5}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)}{15\,d^4}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
[Out] -1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^2*x +
1/2*A*e^2*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^2/d^2 - 2/3*(-d^2*x^
2 + 1)^(3/2)*A*e*f/d^2 - 1/5*(-d^2*x^2 + 1)^(3/2)*(2*C*e*f + B*f^2)*x^2/d^2
 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e^2 + 2*B*e*f + A*f^2)*x/d^2 - 1/8*(-d^2*x^2
 + 1)^(3/2)*C*f^2*x/d^4 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f^2)*
x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(C*e^2 + 2*B*e*f + A*f^2)
*arcsin(d*x)/d^3 + 1/16*C*f^2*arcsin(d*x)/d^5 - 2/15*(-d^2*x^2 + 1)^(3/2)*(
2*C*e*f + B*f^2)/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(266) = 532$.

Time = 0.41 (sec) , antiderivative size = 1059, normalized size of antiderivative = 3.70

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/240*(120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^4*e^2 + 240*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d
*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2 + 1
20*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x
+ 1)))*B*d^3*e^2 + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x
+ 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f + 240*(sqrt(d*x + 1)*
(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f +
 10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d
*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 40*(((2*d*x - 5
)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d
*x + 1)))*C*d^2*e^2 + 20*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*
sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*
e*f + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e*f + 10*(((2*(3*d*x - 10)*(d*x + 1) +
 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^2*f^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sq
rt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^2 + 4*(((2*(3*(
4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1
)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e*f + 20*(((2*
(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e*f + 2*(((2*(3*(4*d*x - 17)*(d*
x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1
) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f^2 + 10*(((2*(3*d*x - 10)*(d
*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*
sqrt(2)*sqrt(d*x + 1)))*B*d*f^2 + (((2*((4*(5*d*x - 26)*(d*x + 1) + 321)*(d
*x + 1) - 451)*(d*x + 1) + 745)*(d*x + 1) - 405)*sqrt(d*x + 1)*sqrt(-d*x +
1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^2 + 2*(((2*(3*(4*d*x - 17)*
(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x
+ 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f^2)/d^5
```

Mupad [B] (verification not implemented)

Time = 38.59 (sec) , antiderivative size = 2920, normalized size of antiderivative = 10.21

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

```
[In] int((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^8*((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1
)^(1/2) - 1)^8 - (((1 - d*x)^(1/2) - 1)^14*((1408*B*f^2)/3 - (32*B*d^2*e^2)
/3))/((d*x + 1)^(1/2) - 1)^14 - (((1 - d*x)^(1/2) - 1)^6*((1408*B*f^2)/3 -
(32*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^12*((49
28*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^12 - (((1 - d*x)^(1
/2) - 1)^10*((11008*B*f^2)/5 - 304*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^10 + (
64*B*f^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*B*f^2*((1 -
 d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (8*B*d^2*e^2*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 + (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)
^(1/2) - 1)^3 - (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)
^5 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (442*B
*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 - (442*B*d*e*f*((1
- d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (204*B*d*e*f*((1 - d*x)^(1
/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^
15)/((d*x + 1)^(1/2) - 1)^15 - (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^17)/((d*x
+ 1)^(1/2) - 1)^17 + (B*d*e*f*((1 - d*x)^(1/2) - 1)^19)/((d*x + 1)^(1/2) -
1)^19 - (B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (10*d
^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^4*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 - d*x)^(1/2) - 1)^6)/((d
*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) -
 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (210*
d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^4*((1 - d*x
)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^4*((1 - d*x)^(1/2) - 1)^1
6)/((d*x + 1)^(1/2) - 1)^16 + (10*d^4*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^
(1/2) - 1)^18 + (d^4*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20) -
((((1 - d*x)^(1/2) - 1)^15*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)
^15 - (((1 - d*x)^(1/2) - 1)*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) -
1) + (((1 - d*x)^(1/2) - 1)^3*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x + 1)^(1/2
) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x +
1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^5*((273*A*f^2)/2 + 30*A*d^2*e^2))
/((d*x + 1)^(1/2) - 1)^5 + (((1 - d*x)^(1/2) - 1)^11*((273*A*f^2)/2 + 30*A*
d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*((715*A*f^2)/
2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*((715
*A*f^2)/2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9 + (16*A*d*e*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^
```

```
4)/((d*x + 1)^(1/2) - 1)^4 + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x
 + 1)^(1/2) - 1)^6) + (704*A*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(
1/2) - 1)^8) + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) -
 1)^10) - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 +
(16*A*d*e*f*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^3 + (8*d
^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*
x + 1)^(1/2) - 1)^6 + (70*d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1
)^8 + (56*d^3*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^3*
((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^3*((1 - d*x)^(1/2
) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^3*((1 - d*x)^(1/2) - 1)^16)/((d*x
+ 1)^(1/2) - 1)^16) - ((((1 - d*x)^(1/2) - 1)^23*((C*f^2)/4 + (C*d^2*e^2)/2
))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2) - 1)*((C*f^2)/4 + (C*d^2*e^
2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((35*C*f^2)/12 - (3
1*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((35*C
*f^2)/12 - (31*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2)
- 1)^5*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1
- d*x)^(1/2) - 1)^19*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2)
- 1)^19 - (((1 - d*x)^(1/2) - 1)^7*((7339*C*f^2)/4 + (171*C*d^2*e^2)/2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^17*((7339*C*f^2)/4 + (171*C*
d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^17 - (((1 - d*x)^(1/2) - 1)^11*((25661*C
*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)
^13*((25661*C*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*
x)^(1/2) - 1)^9*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9
- (((1 - d*x)^(1/2) - 1)^15*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(
1/2) - 1)^15 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^
4 - (2048*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2) - 1)^6) + (1
536*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6144*C*d*e*
f*((1 - d*x)^(1/2) - 1)^10)/(5*((d*x + 1)^(1/2) - 1)^10) - (33536*C*d*e*f*(
(1 - d*x)^(1/2) - 1)^12)/(15*((d*x + 1)^(1/2) - 1)^12) + (6144*C*d*e*f*((1
- d*x)^(1/2) - 1)^14)/(5*((d*x + 1)^(1/2) - 1)^14) + (1536*C*d*e*f*((1 - d*
x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 - (2048*C*d*e*f*((1 - d*x)^(1/2)
 - 1)^18)/(3*((d*x + 1)^(1/2) - 1)^18) + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)
^20)/((d*x + 1)^(1/2) - 1)^20)/(d^5 + (12*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2 + (66*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1
)^4 + (220*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^5*
((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (792*d^5*((1 - d*x)^(1/2
) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (924*d^5*((1 - d*x)^(1/2) - 1)^12)/((
d*x + 1)^(1/2) - 1)^12 + (792*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2
) - 1)^14 + (495*d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (
220*d^5*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (66*d^5*((1 -
d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (12*d^5*((1 - d*x)^(1/2) - 1
)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^5*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^
(1/2) - 1)^24) - (A*atan((A*(f^2 + 4*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(A*f^2 + 4*A*d^2*e^2)))*(f^2 + 4*d^2*e^2))/(2*d^3) - (C*at
```

```
an((C*(f^2 + 2*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(C*f^
2 + 2*C*d^2*e^2)))*(f^2 + 2*d^2*e^2))/(4*d^5) - (B*e*f*atan(((1 - d*x)^(1/2
) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.3 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 168

$$\begin{aligned}\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx &= \frac{(Ce+4Ad^2e+Bf)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}\\ &\quad-\frac{(4(5d^2f(Be+Af)-C(3d^2e^2-2f^2))-3d^2f(3Ce-5Bf)x)\left(1-d^2x^2\right)^{3/2}}{60d^4f}\\ &\quad+\frac{(Ce+4Ad^2e+Bf)\arcsin(dx)}{8d^3}\end{aligned}$$

```
[Out] -1/5*C*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2/f-1/60*(20*d^2*f*(A*f+B*e)-4*C*(3*d
^2*e^2-2*f^2)-3*d^2*f*(-5*B*f+3*C*e)*x)*(-d^2*x^2+1)^(3/2)/d^4/f+1/8*(4*A*d
^2*e+B*f+C*e)*arcsin(d*x)/d^3+1/8*(4*A*d^2*e+B*f+C*e)*x*(-d^2*x^2+1)^(1/2)/
d^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

$= \{1623, 1668, 794, 201, 222\}$

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\arcsin(dx)\left(4Ad^2e+Bf+Ce\right)}{8d^3} + \frac{x\sqrt{1-d^2x^2}(4Ad^2e+Bf+Ce)}{8d^2}$$
$$- \frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-\frac{1}{4}C(12d^2e^2-8f^2)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}$$
$$- \frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((C*e + 4*A*d^2*e + B*f)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (C*(e + f*x)^2*(1 -
 d^2*x^2)^(3/2))/(5*d^2*f) - ((4*(5*d^2*f*(B*e + A*f) - (C*(12*d^2*e^2 - 8*
f^2))/4) - 3*d^2*f*(3*C*e - 5*B*f)*x)*(1 - d^2*x^2)^(3/2))/(60*d^4*f) + ((C
*e + 4*A*d^2*e + B*f)*ArcSin[d*x])/(8*d^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int (e+fx)\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\int (e+fx)\left(-\left(\left(2C+5Ad^2\right)f^2\right)+d^2f(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}\,dx}{5d^2f^2}$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}C\left(12d^2e^2-8f^2\right)\right)-3d^2f(3Ce-5Bf)x\right)\left(1-d^2x^2\right)^{3/2}}{60d^4f} + \frac{\left(Ce+4Ad^2e+Bf\right)\int \sqrt{1-d^2x^2}\,dx}{4d^2}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}C\left(12d^2e^2-8f^2\right)\right)-3d^2f(3Ce-5Bf)x\right)\left(1-d^2x^2\right)^{3/2}}{60d^4f} + \frac{\left(Ce+4Ad^2e+Bf\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{8d^2}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}C\left(12d^2e^2-8f^2\right)\right)-3d^2f(3Ce-5Bf)x\right)\left(1-d^2x^2\right)^{3/2}}{60d^4f} + \frac{\left(Ce+4Ad^2e+Bf\right)\sin^{-1}(dx)}{8d^3}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{1-d^2x^2}(60Ad^4ex+40Ad^2f(-1+d^2x^2)+15Cd^2ex(-1+2d^2x^2)+5Bd^2(-8e-3fx+8d^2ex^2+6d^2fx^3)\cdots}{120d^4}$$

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*x
*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) + 8*
C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 30*d*(C*e + 4*A*d^2*e + B*f)*ArcTan[(d*x)
/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^4)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{\left(24fC\,x^4d^4+30B\,d^4f\,x^3+30C\,d^4e\,x^3+40A\,d^4f\,x^2+40B\,d^4e\,x^2+60A\,d^4ex-8C\,d^2f\,x^2-15B\,d^2fx-15C\,d^2ex-40A\,d^2f-40B\,d^2e-\cdots\right.}{120d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\,\operatorname{csgn}(d)d^4f\,x^4\sqrt{-d^2x^2+1}+30B\,\operatorname{csgn}(d)d^4f\,x^3\sqrt{-d^2x^2+1}+30C\,\operatorname{csgn}(d)d^4e\,x^3\sqrt{-d^2x^2+1}+40A\,\operatorname{csgn}(d)d^4f\cdots\right.}{\cdots}$

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/120*(24*C*d^4*f*x^4+30*B*d^4*f*x^3+30*C*d^4*e*x^3+40*A*d^4*f*x^2+40*B*d^
4*e*x^2+60*A*d^4*e*x-8*C*d^2*f*x^2-15*B*d^2*f*x-15*C*d^2*e*x-40*A*d^2*f-40*
B*d^2*e-16*C*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1
)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8/d^2*(4*A*d^2*e+B*f+C*e)/(d^2)^(1/2)*arc
tan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/
2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(24\,Cd^4fx^4-40\,Bd^2e+30\,(Cd^4e+Bd^4f)x^3+8\,(5\,Bd^4e+(5\,Ad^4-Cd^2)f)x^2-8\,(5\,Ad^2+2\,C)f-15}{120\,d}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/120*((24*C*d^4*f*x^4 - 40*B*d^2*e + 30*(C*d^4*e + B*d^4*f)*x^3 + 8*(5*B*d
^4*e + (5*A*d^4 - C*d^2)*f)*x^2 - 8*(5*A*d^2 + 2*C)*f - 15*(B*d^2*f - (4*A*
d^4 - C*d^2)*e)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3 + C*
d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (e+fx)\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{2}\sqrt{-d^2x^2+1}Aex-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{Ae\arcsin{(dx)}}{2\,d}$$
$$-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Af}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Ce+Bf)x}{4\,d^2}$$
$$+\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{8\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Cf}{15\,d^4}+\frac{(Ce+Bf)\arcsin{(dx)}}{8\,d^3}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*e*x - 1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*A
*e*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 - 1/3*(-d^2*x^2 + 1)^(3
/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e + B*f)*x/d^2 + 1/8*sqrt(-d^2*x^
2 + 1)*(C*e + B*f)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*f/d^4 + 1/8*(C*e + B
*f)*arcsin(d*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(151) = 302$.

Time = 0.34 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.76

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{60\left(\sqrt{dx+1}(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^3e+120\left(\sqrt{dx+1}\sqrt{-dx+1}+2\arcsin\left(\frac{1}{2}\right.\right.}{}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/120*(60*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1)))*A*d^3*e + 120*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sq
rt(2)*sqrt(d*x + 1)))*A*d^3*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x +
1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e + 60*(sqrt
(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B
*d^2*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*a
rcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 60*(sqrt(d*x + 1)*(d*x - 2)*sqr
t(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 5*(((2*(3*d*x
- 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arc
sin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e + 5*
(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x +
 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f + 20*(((2*d*x - 5)*(d*x +
 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))
)*B*d*f + (((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1)
+ 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))
*C*f + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sq
rt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f)/d^4
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.38

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\frac{Bf(\sqrt{1-dx}-1)}{2(\sqrt{dx+1}-1)} - \frac{35Bf(\sqrt{1-dx}-1)^3}{2(\sqrt{dx+1}-1)^3} + \frac{273Bf(\sqrt{1-dx}-1)^5}{2(\sqrt{dx+1}-1)^5} - \frac{715Bf(\sqrt{1-dx}-1)^7}{2(\sqrt{dx+1}-1)^7} + \frac{715Bf(\sqrt{1-dx}-1)^9}{2(\sqrt{dx+1}-1)^9} - \frac{273Bf(\sqrt{1-dx}-1)^{11}}{2(\sqrt{dx+1}-1)^{11}} \cdots}{d^3\left(\frac{(\sqrt{1-dx}-1)^2}{(\sqrt{dx+1}-1)^2}+1\right)^8}$$
$$- \sqrt{1-dx}\left(\frac{2Cf\sqrt{dx+1}}{15d^4} - \frac{Cfx^4\sqrt{dx+1}}{5} + \frac{Cfx^2\sqrt{dx+1}}{15d^2}\right)$$
$$+ \frac{\frac{Ce(\sqrt{1-dx}-1)}{2(\sqrt{dx+1}-1)} - \frac{35Ce(\sqrt{1-dx}-1)^3}{2(\sqrt{dx+1}-1)^3} + \frac{273Ce(\sqrt{1-dx}-1)^5}{2(\sqrt{dx+1}-1)^5} - \frac{715Ce(\sqrt{1-dx}-1)^7}{2(\sqrt{dx+1}-1)^7} + \frac{715Ce(\sqrt{1-dx}-1)^9}{2(\sqrt{dx+1}-1)^9} - \frac{273Ce(\sqrt{1-d \cdots}}{2(\sqrt{dx+1}- \cdots} \cdots}{d^3\left(\frac{(\sqrt{1-dx}-1)^2}{(\sqrt{dx+1}-1)^2}+1\right)^8}$$
$$- \frac{Bf\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3} - \frac{Ce\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3} + \frac{Aex\sqrt{1-dx}\sqrt{dx+1}}{2}$$
$$- \frac{A\sqrt{d}\,e\ln\left(\sqrt{-d}\sqrt{1-dx}\sqrt{dx+1}-d^{3/2}x\right)}{2(-d)^{3/2}}$$
$$+ \frac{Af(d^2x^2-1)\sqrt{1-dx}\sqrt{dx+1}}{3d^2} + \frac{Be(d^2x^2-1)\sqrt{1-dx}\sqrt{dx+1}}{3d^2}$$

```
[In] int((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*f*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*B*f*((1 - d*x)
^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*B*f*((1 - d*x)^(1/2) - 1)
^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*B*f*((1 - d*x)^(1/2) - 1)^7)/(2*((d*
x + 1)^(1/2) - 1)^7) + (715*B*f*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2
) - 1)^9) - (273*B*f*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11)
 + (35*B*f*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) - (B*f*((
1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((1 - d*x)^(1/2
) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (1 - d*x)^(1/2)*((2*C*f*(d*x + 1
)^(1/2))/(15*d^4) - (C*f*x^4*(d*x + 1)^(1/2))/5 + (C*f*x^2*(d*x + 1)^(1/2))
/(15*d^2)) + ((C*e*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*C
*e*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*C*e*((1 - d*
x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*C*e*((1 - d*x)^(1/2) -
1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (715*C*e*((1 - d*x)^(1/2) - 1)^9)/(2*((
d*x + 1)^(1/2) - 1)^9) - (273*C*e*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(
1/2) - 1)^11) + (35*C*e*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^
13) - (C*e*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((
1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (B*f*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (C*e*atan(((1 - d*x)^(1/2) -
```

```
 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) + (A*e*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2
))/2 - (A*d^(1/2)*e*log((-d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2
)*x))/(2*(-d)^(3/2)) + (A*f*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/
(3*d^2) + (B*e*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

3.4 $\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{(C+4Ad^2)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{B(1-d^2x^2)^{3/2}}{3d^2}-\frac{Cx(1-d^2x^2)^{3/2}}{4d^2}+\frac{(C+4Ad^2)\arcsin(dx)}{8d^3}$$

```
[Out] -1/3*B*(-d^2*x^2+1)^(3/2)/d^2-1/4*C*x*(-d^2*x^2+1)^(3/2)/d^2+1/8*(4*A*d^2+C
)*arcsin(d*x)/d^3+1/8*(4*A*d^2+C)*x*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {913, 1829, 655, 201, 222}

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{(4Ad^2+C)\arcsin(dx)}{8d^3}+\frac{x\sqrt{1-d^2x^2}(4Ad^2+C)}{8d^2}-\frac{B(1-d^2x^2)^{3/2}}{3d^2}-\frac{Cx(1-d^2x^2)^{3/2}}{4d^2}$$

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((C + 4*A*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (B*(1 - d^2*x^2)^(3/2))/(3*d^
2) - (C*x*(1 - d^2*x^2)^(3/2))/(4*d^2) + ((C + 4*A*d^2)*ArcSin[d*x])/(8*d^3
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\text{integral} = \int \left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{Cx(1-d^2x^2)^{3/2}}{4d^2} - \frac{\int \left(-C-4Ad^2-4Bd^2x\right)\sqrt{1-d^2x^2}\,dx}{4d^2}$$

$$= -\frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3/2}}{4d^2} - \frac{\left(-C-4Ad^2\right)\int \sqrt{1-d^2x^2}\,dx}{4d^2}$$

$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3/2}}{4d^2} + \frac{\left(C+4Ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{8d^2}$$

$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B(1-d^2x^2)^{3/2}}{3d^2} - \frac{Cx(1-d^2x^2)^{3/2}}{4d^2} + \frac{\left(C+4Ad^2\right)\sin^{-1}(dx)}{8d^3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$$
$$=\frac{d\sqrt{1-d^2x^2}(-8B-3Cx+12Ad^2x+8Bd^2x^2+6Cd^2x^3)+6(C+4Ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{24d^3}$$

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^3
) + 6*(C + 4*A*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^3)
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(6C\,d^2x^3+8B\,d^2x^2+12A\,d^2x-3Cx-8B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{24d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(4A\,d^2+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx-}}{8d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6C\ \mathrm{csgn}(d)d^3x^3\sqrt{-d^2x^2+1}+8B\ \mathrm{csgn}(d)d^3x^2\sqrt{-d^2x^2+1}+12A\ \mathrm{csgn}(d)d^3\sqrt{-d^2x^2+1}\,x-3C\ \mathrm{csgn}(d)d\sqrt{-d^2x^2+}\right.}{24\sqrt{-d^2x^2+1}\,d^3}$

```
[In] int((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(6*C*d^2*x^3+8*B*d^2*x^2+12*A*d^2*x-3*C*x-8*B)*(d*x+1)^(1/2)*(d*x-1)/
d^2/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*(4
*A*d^2+C)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1
)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$$
$$=\frac{(6\,Cd^3x^3+8\,Bd^3x^2-8\,Bd+3\,(4\,Ad^3-Cd)x)\sqrt{dx+1}\sqrt{-dx+1}-6\,(4\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}}{dx}\right.}{24\,d^3}$$

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x +
1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/d^3
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=&\ \frac{1}{2}\sqrt{-d^2x^2+1}Ax-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cx}{4\,d^2}\\&+\frac{A\arcsin\left(dx\right)}{2\,d}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}B}{3\,d^2}\\&+\frac{\sqrt{-d^2x^2+1}Cx}{8\,d^2}+\frac{C\arcsin\left(dx\right)}{8\,d^3}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcsi
n(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*C*x/d^2
+ 1/8*C*arcsin(d*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.99

$$\begin{aligned}&\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx\\&=\frac{12\left(\sqrt{dx+1}(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^2+24\left(\sqrt{dx+1}\sqrt{-dx+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\right.\right.}{}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(12*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqr
t(d*x + 1)))*A*d^2 + 24*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1)))*A*d^2 + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt
(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + 12*(sqrt(d*x + 1)*(
d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + (((2*(
3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sq
rt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C)/d^3
```

Mupad [B] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.80

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx = \frac{A\,x\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{2} - \frac{\frac{35\,C\,{\left(\sqrt{1-d\,x}-1\right)}^3}{2\,{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{273\,C\,{\left(\sqrt{1-d\,x}-1\right)}^5}{2\,{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{715\,C\,{\left(\sqrt{1-d\,x}-1\right)}^7}{2\,{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{715\,C\,{\left(\sqrt{1-d\,x}-1\right)}^9}{2\,{\left(\sqrt{d\,x+1}-1\right)}^9}+\frac{273\,C\,{\left(\sqrt{1-d\,x}-1\right)}^{11}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{11}}-\frac{35\,C\,{\left(\sqrt{1-d\,x}-1\right)}^{13}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{13}}+\frac{C\,{\left(\sqrt{1-d\,x}-1\right)}^{15}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{15}}-\frac{C\,\left(\sqrt{1-d\,x}-1\right)}{2\,\left(\sqrt{d\,x+1}-1\right)}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^8} - \frac{C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3} - \frac{A\,\sqrt{d}\,\ln\left(\sqrt{-d}\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}-d^{3/2}\,x\right)}{2\,{\left(-d\right)}^{3/2}} + \frac{B\,\left(d^2\,x^2-1\right)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2}$$

```
[In] int((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] (A*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/2 - ((35*C*((1 - d*x)^(1/2) - 1)^3)/(
2*((d*x + 1)^(1/2) - 1)^3) - (273*C*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^
(1/2) - 1)^5) + (715*C*((1 - d*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7)
 - (715*C*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) + (273*C*((1
 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11) - (35*C*((1 - d*x)^(1/2
) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) + (C*((1 - d*x)^(1/2) - 1)^15)/(2*(
(d*x + 1)^(1/2) - 1)^15) - (C*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) -
1)))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (C*ata
n(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (A*d^(1/2)*log((-
d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2)*x))/(2*(-d)^(3/2)) + (B*
(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

3.5 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 122

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\arcsin(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

```
[Out] -(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*e
^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^(1
/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1623, 1668, 858, 222, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \frac{(Af^2-Bef+Ce^2)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}} - \frac{\arcsin(dx)(Ce-Bf)}{df^2} - \frac{C\sqrt{1-d^2x^2}}{d^2f}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{(Ce^2-Bef+Af^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{(Ce^2-Bef+Af^2)\,\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$

$$= \frac{-\frac{Cf\sqrt{1-d^2x^2}}{d^2} + \frac{2(-Ce+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - \frac{2\sqrt{d^2e^2-f^2}(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(de-f)(de+f)}}{f^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + (2*(-(C*e) + B*f)*ArcTan[(d*x)/(-1 + Sqrt
[1 - d^2*x^2])])/d - (2*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*ArcT
an[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*(d*
e + f)))/f^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(114) = 228.

Time = 1.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.37

method	result
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{f\sqrt{d^2}}-\frac{\left(Af^2-Bef+Ce^2\right)\ln\left(\frac{-\frac{2\left(d^2e^2-f^2\right)}{f^2}+\frac{2d^2e\left(x+\frac{e}{f}\right)}{f}+2\sqrt{-\frac{d^2}{}}}{}\right.}{f^2\sqrt{-\frac{d^2e^2-f}{f^2}}}\right.}{f\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2ef-C\,\mathrm{csgn}\right.}{}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] C/f/d^2*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(
1/2)/(-d*x+1)^(1/2)+1/f*((B*f-C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2
*x^2+1)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2*(d
^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x+e/f
)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d*x+1)
)^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(114) = 228.

Time = 4.25 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$
$$= \left[-\frac{(Cd^2e^2-Bd^2ef+Ad^2f^2)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}(d^2ex+f)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+(d^2e^2-f^2)\right.}{fx+e}\right.}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\, dx = \int \frac{A + Bx + Cx^2}{(e + fx)\sqrt{-dx + 1}\sqrt{dx + 1}}\, dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 28.86 (sec) , antiderivative size = 5803, normalized size of antiderivative = 47.57

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)} \, dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
- (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
 + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
```

+ 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
+ (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
x + 1)^(1/2) - 1)) + (4096((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
f^4((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
- 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
e^2((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))
/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/
2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) +
(C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d
x)^(1/2) - 1)^2(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/
2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) -
1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384
*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e
^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((409
6*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2)
- 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096
*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*
x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^
4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d
x + 1)^(1/2) - 1)) + (4096((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6
*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)
^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f -

d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*e^2*((4096*(32*C^
3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2) - 1)^2*(32*
C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (458752*
C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C*e^2*((4096*
(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1
)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((
1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 + 9*C^2*d^4*e^
7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d^2*e^3*f^7 -
30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*C*e^2*f^6 -
22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^2*e^3*f^7 -
90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)
- (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (16384*((1 - d*x
)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(1/2) - 1)) +
(4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4*((d
x + 1)^(1/2) - 1)^2)))/(f^2(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d
e)^(1/2)(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(
f + d*e)^(1/2)*(f - d*e)^(1/2)) + (917504*C^4*e^7*((1 - d*x)^(1/2) - 1)^2)/
(d*f^4*((d*x + 1)^(1/2) - 1)^2)))*2i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))
+ (B*e*atan(((B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1
- d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2)
- 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131
072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 -
d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(
d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f
^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((
d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x
)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(7*d^4*e^3*f^4 -
9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e
^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*
f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f
- d*e)^(1/2)))*1i)/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (B*e*((4096*(24*B^
3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*
e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((4096*(16*B^2*e*f^4 + 9*B^
2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*
e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^
5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (B*
e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 3604
48*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*
B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1
/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^
(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) +
(4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x +

```
 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2
)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f*(f + d*e)^
(1/2)*(f - d*e)^(1/2)))/((131072*B^4*e^3)/d + (917504*B^4*e^3*((1 - d*x)^(1
/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B^3*d^2*e^4 + 32*
B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^
2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) -
1))/((d*x + 1)^(1/2) - 1) + (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d +
 (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x +
 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f
^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*
d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^
3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 -
 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (
B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(819
20*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*
x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^
2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/
2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1
/2)) - (B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^
(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)
 + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((
4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2
*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(
1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x
 + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d
+ ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2)
 - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e
^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3)
)/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9
*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(
1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^
(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))*2i)/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))
```

3.6 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} dx$

Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} dx = \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\arcsin(dx)}{df^2} - \frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2(d^2e^2-f^2)^{3/2}}$$

```
[Out] C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*
x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f^2
-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1623, 1665, 858, 222, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} dx = -\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2)}{f^2(d^2e^2-f^2)^{3/2}} + \frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{f(d^2e^2-f^2)(e+fx)} + \frac{C\arcsin(dx)}{df^2}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx,x,\frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{(d^2e^2-f^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(de-f)(de+f)(e+fx)} + \frac{2C\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} + \frac{2\sqrt{d^2e^2-f^2}(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(-de+f)^2(de+f)^2}}{f^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((d*e - f)*(d*e + f)*(e +
 f*x)) + (2*C*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d + (2*Sqrt[d^2*e^2 -
 f^2]*(C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*e^2 -
f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-(d*e) + f)^2*(d*e + f)^2))/f^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 899, normalized size of antiderivative = 5.52

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e\,f^3x+C\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e^3fx-A\,\mathrm{c}}{}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(-d*x+1)
^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)/(f*x+e)/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(155) = 310$.

Time = 16.65 (sec) , antiderivative size = 1025, normalized size of antiderivative = 6.29

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$$

$$= \left[\frac{Cd^3e^5f - Bd^3e^4f^2 + Bde^2f^4 - Adef^5 + (Ad^3 - Cd)e^3f^3 - (Cd^3e^5 + Bde^2f^3 - (Ad^3 + 2\,Cd)e^3f^2 + (C}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 57.67 (sec) , antiderivative size = 10198, normalized size of antiderivative = 62.56

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
```

^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
e^3((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
(A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
- 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
f((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2

) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3*e^4*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 - (2*d*e^2*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (d*e^2*f^2*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 + (4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)) - (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (B*f^4*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) + (B*f^4*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 - B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (4*B*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (4*B*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (B*d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 - (B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*(

(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 - (B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 + (8*B*d*e*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (B*d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4)/(d^3*e^3*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (4*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - d*e*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) - (4*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (2*d^3*e^3*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3*e^3*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 - (4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (2*d*e*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (d*e*f^2*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4) - ((4*C*d*e*((1 - d*x)^(1/2) - 1))/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)) - (4*C*d*e*((1 - d*x)^(1/2) - 1)^3)/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^3) + (8*C*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^2))/(d^2*e + (4*d*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (4*d*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (2*d^2*e*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e*((1 - d*x)^(1/2) - 1)^4)/(

(d*x + 1)^(1/2) - 1)^4) + (4*C*atan((((((1 - d*x)^(1/2) - 1)*((2097152*(288
*e^3*f^11 - 6*d^10*e^13*f - 912*d^2*e^5*f^9 + 1048*d^4*e^7*f^7 - 532*d^6*e^
9*f^5 + 112*d^8*e^11*f^3))/(d*f^2*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9
- 4*d^7*e^6*f^7 + d^9*e^8*f^5)) - (33554432*(20*d^2*e*f^21 - 103*d^4*e^3*f^
19 + 215*d^6*e^5*f^17 - 230*d^8*e^7*f^15 + 130*d^10*e^9*f^13 - 35*d^12*e^11
*f^11 + 3*d^14*e^13*f^9))/(d^5*f^10*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^
9 - 4*d^7*e^6*f^7 + d^9*e^8*f^5)) + (8388608*(72*e*f^17 - 452*d^2*e^3*f^15
+ 1024*d^4*e^5*f^13 - 1106*d^6*e^7*f^11 + 597*d^8*e^9*f^9 - 144*d^10*e^11*f
^7 + 9*d^12*e^13*f^5))/(d^3*f^6*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9 -
4*d^7*e^6*f^7 + d^9*e^8*f^5))))/((d*x + 1)^(1/2) - 1) - (33554432*(7*d^2*e^
2*f^19 - 35*d^4*e^4*f^17 + 70*d^6*e^6*f^15 - 70*d^8*e^8*f^13 + 35*d^10*e^10
*f^11 - 7*d^12*e^12*f^9))/(d^5*f^10*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (2097152*(112*e^4*f^9 + 28*d^8*e^12*f - 3
36*d^2*e^6*f^7 + 364*d^4*e^8*f^5 - 168*d^6*e^10*f^3))/(d*f^2*(f^12 - 4*d^2*
e^2*f^10 + 6*d^4*e^4*f^8 - 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (8388608*(28*e^2
*f^15 - 168*d^2*e^4*f^13 + 364*d^4*e^6*f^11 - 371*d^6*e^8*f^9 + 182*d^8*e^1
0*f^7 - 35*d^10*e^12*f^5))/(d^3*f^6*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)))*(d^4*f^14 - 4*d^6*e^2*f^12 + 6*d^8*e^4*f^1
0 - 4*d^10*e^6*f^8 + d^12*e^8*f^6))/(67108864*e*f^12 + 37748736*d^12*e^13 -
268435456*d^2*e^3*f^10 + 536870912*d^4*e^5*f^8 - 637534208*d^6*e^7*f^6 + 4
69762048*d^8*e^9*f^4 - 201326592*d^10*e^11*f^2)))/(d*f^2) + (log(16*f^15 -
9*d^14*e^14*f - (16*f^15*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 -
92*d^2*e^2*f^13 + 236*d^4*e^4*f^11 - 352*d^6*e^6*f^9 + 329*d^8*e^8*f^7 - 1
91*d^10*e^10*f^5 + 63*d^12*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2
) + 12*d^6*e^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/
2)*(f - d*e)^(3/2) - (6*d^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1) + (16*d*e*f^14*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (92*d^2*e
^2*f^13*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (236*d^4*e^4*f^1
1*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (352*d^6*e^6*f^9*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (329*d^8*e^8*f^7*((1 - d*x)^(
1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (191*d^10*e^10*f^5*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)
*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 120*d^4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^
6*e^6*f^6*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(
f - d*e)^(9/2) + 207*d^8*e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e
^4*f^2*(f + d*e)^(9/2)*(f - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(
f - d*e)^(3/2) - (88*d^3*e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1) + (216*d^5*e^5*f^10*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (308
*d^7*e^7*f^8*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (274*d^9*e^9*f^
6*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (150*d^11*e^11*f^4*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (46*d^13*e^13*f^2*((1 - d*x)^(1/2)
- 1))/((d*x + 1)^(1/2) - 1) + (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f

- d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3
f^9((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(
3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d
e)^(3/2)(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d
x)^(1/2) - 1)(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (26
4*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
+ 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d
e)^(9/2)(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
+ (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2
+ (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1
- d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^
2 + (56*d^5*e^5*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((
d*x + 1)^(1/2) - 1) + (44*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)
*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1))*(C*d^2*e^3 - 2*C*e*f^2))/(f^2*(f +
d*e)^(3/2)*(f - d*e)^(3/2)) + (C*e*log(9*d^14*e^14*f - 16*f^15 + (16*f^15*
((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + 92*d^2*e^2*f^13 - 236*d^
4*e^4*f^11 + 352*d^6*e^6*f^9 - 329*d^8*e^8*f^7 + 191*d^10*e^10*f^5 - 63*d^1
2*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 12*d^6*e^6*(f + d*e)^
(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (6*d
^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (16*d*e*f^14*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (92*d^2*e^2*f^13*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (236*d^4*e^4*f^11*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (352*d^6*e^6*f^9*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2 + (329*d^8*e^8*f^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(
1/2) - 1)^2 - (191*d^10*e^10*f^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 + (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
- (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x +
1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 120*d^
4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^6*e^6*f^6*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 207*d^8*
e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e^4*f^2*(f + d*e)^(9/2)*(f
- d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (88*d^3*
e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (216*d^5*e^5*f^10*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (308*d^7*e^7*f^8*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) - (274*d^9*e^9*f^6*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1) + (150*d^11*e^11*f^4*((1 - d*x)^(1/2) - 1))/((d*x + 1

)^(1/2) - 1) - (46*d^13*e^13*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3*f^9*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (264*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (56*d^5*e^5*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) + (44*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1))*(2*f^2 - d^2*e^2))/(f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))

3.7 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} dx$

Optimal result

Integrand size = 37, antiderivative size = 248

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} \, dx
&= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} \\
&\quad - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} \\
&\quad + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}
\end{aligned}
$$

```
[Out] 1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)/
(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*e*
f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^2+B
*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)^
2/(f*x+e)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used

$= \{1623, 1665, 821, 739, 210\}$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$$
$$= \frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))}{2\left(d^2e^2-f^2\right)^{5/2}}$$
$$+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$
$$-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} + \frac{\int \frac{2(Ce+Ad^2e-Bf)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2(d^2e^2-f^2)}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{2(d^2e^2-f^2)^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} - \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\,\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = -\frac{\frac{(de-f)(de+f)\sqrt{1-d^2x^2}(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(e+fx)^2}+2\sqrt{d^2e^2-f^2}(C(d^2e^2+}{2(de-f)^3(de+f)^3}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] -1/2*(((d*e - f)*(d*e + f)*Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x)
 + B*f^2*(e + 2*f*x) - A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x -
4*f^2*x)))/(e + f*x)^2 + 2*Sqrt[d^2*e^2 - f^2]*(C*(d^2*e^2 + 2*f^2) + d^2*(
-3*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x -
e*Sqrt[1 - d^2*x^2])])/((d*e - f)^3*(d*e + f)^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1449

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/
(f*x+e))*d^4*e^2*f^2*x^2-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C
*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*
x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^4-3*C*e
^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*f^4*x*(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(
1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^2*e*x+(-d
^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+B*e*f^3*(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1
/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*f^4*x^2+A*ln(2*(d^2*e*x+(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*l
n(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^
2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*e*f^3*x+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+4*A*
```

```
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*
f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f
^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x^2+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x-6*B*ln(2*(d^2*e
*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*
x+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x
+e))*d^2*e^3*f*x-4*A*d^2*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)+2*B*d^2*e^3*f*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-4*C*e*f^3*x
*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)^2*(-d*x+1)^(1/2)*(d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/(d*e+f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(232) = 464$.

Time = 0.36 (sec) , antiderivative size = 1580, normalized size of antiderivative = 6.37

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
```

```
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume?`
for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 66.29 (sec) , antiderivative size = 9097, normalized size of antiderivative = 36.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
```

7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2) - 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5)

)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))/((8*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8*((1 - d*x)^(1/2) - 1)^2*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e*f^4))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(2*f^2 + d^2*e^2)*1i)/((f + d*e)^(5/2)*(f - d*e)^(5/2)) + (A*d^2*atan(((A*d^2*(f^2 + 2*d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1

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1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)) - (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^
9*e^7*f - 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4
 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7
*f - 12*A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2
*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))/((8*(4*A^2*d^9*
e^5 + 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 +
6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8*((1 - d*x)^(1/2) - 1)^2*(4*A^2*d^9*e^5
+ 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e
^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*
e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*
e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e
^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5
*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (A
*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 +
8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^1
1 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*
d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*
e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)
^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f -
 d*e)^(5/2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f
- 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6
*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*
A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11
- 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f
^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*
((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72
*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8
 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*
f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*
e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(f^2 + 2*d^2*e^2)*1i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2)) - (B*d^2*e*f*atan(((B*d^2*e*f*((4*((1 - d*x)
^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((
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d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^
6*e^6*f^2)) - (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/
(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*
e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*
d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8
- 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x +
 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6
*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f
 + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (
B*d^2*e*f*((4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x
)^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/((
(d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d
^6*e^6*f^2)) + (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)))/((72*B^2*d^5*e^3*f^2)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6
+ 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) -
 (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (3*B*d^2*e*f*((4*
(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^8 -
4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) +
 (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7
*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^
3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10)
)/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/
```

```
2)) + (72*B^2*d^5*e^3*f^2*((1 - d*x)^(1/2) - 1)^2)/((((d*x + 1)^(1/2) - 1)^2
*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))))*3i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2))
```

3.8 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 340

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f}$$
$$+\frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f}-\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f}$$
$$+\frac{(4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2)))+d^2f(6Cd^2e^3-30Bd^2}{120d^6f}$$
$$+\frac{\left(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3\right)\arcsin(dx)}{8d^5}$$

```
[Out] 1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C*d^2*e^3+3*B*f^3+9*C*e*f^
2)*arcsin(d*x)/d^5-1/60*(4*(5*A*d^2+4*C)*f^2-3*d^2*e*(-5*B*f+C*e))*(f*x+e)^
2*(-d^2*x^2+1)^(1/2)/d^4/f+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-d^2*x^2+1)^(1/2)/d
^2/f-1/5*C*(f*x+e)^4*(-d^2*x^2+1)^(1/2)/d^2/f+1/120*(4*C*(3*d^4*e^4-52*d^2*
e^2*f^2-16*f^4)-20*d^2*f*(4*A*f*(4*d^2*e^2+f^2)+3*B*(d^2*e^3+4*e*f^2))+d^2*
f*(-100*A*d^2*e*f^2-30*B*d^2*e^2*f+6*C*d^2*e^3-45*B*f^3-71*C*e*f^2)*x)*(-d^
2*x^2+1)^(1/2)/d^6/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1623, 1668, 847, 794, 222}

$$\int \frac{(e+fx)^3 \left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}} \, dx$$
$$= \frac{\arcsin(dx) \left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)}{8d^5}$$
$$- \frac{\sqrt{1-d^2x^2}(e+fx)^2 \left(5f(4Af+3Be)-C\left(3e^2-\frac{16f^2}{d^2}\right)\right)}{60d^2f}$$
$$+ \frac{\sqrt{1-d^2x^2}(d^2fx(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2)+4(C(3d^4e^4-52d^2e^2f^2-16}{120d^6f}$$
$$+ \frac{\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{20d^2f} - \frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/60*((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2)/d^2))*(e + f*x)^2*Sqrt[1
- d^2*x^2])/(d^2*f) + ((C*e - 5*B*f)*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(20*d^2
*f) - (C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(5*d^2*f) + ((4*(C*(3*d^4*e^4 - 52*
d^2*e^2*f^2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4
*e*f^2))) + d^2*f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 - 100*A*d^2*e*
f^2 - 45*B*f^3)*x)*Sqrt[1 - d^2*x^2])/(120*d^6*f) + ((4*C*d^2*e^3 + 8*A*d^4
*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSin[d*x])/
(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
```

```
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} - \frac{\int \frac{(e+fx)^3\left(-\left(\left(4C+5Ad^2\right)f^2\right)+d^2f(Ce-5Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{5d^2f^2} \\
&= \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} \\
&\quad + \frac{\int \frac{(e+fx)^2\left(d^2f^2\left(13Ce+20Ad^2e+15Bf\right)+d^2f\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}\,dx}{20d^4f^2} \\
&= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} \\
&\quad + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} \\
&\quad - \frac{\int \frac{(e+fx)\left(-d^2f^2\left(33Cd^2e^2+60Ad^4e^2+75Bd^2ef+32Cf^2+40Ad^2f^2\right)+d^4f\left(6Cd^2e^3-30Bd^2e^2f-71Cef^2-100Ad^2ef^2-45Bf^3\right)x\right)}{\sqrt{1-d^2x^2}}}{60d^6f^2}
\end{aligned}
$$

$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} + \frac{(4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2)))+d^2f(6Cd^2e^3-}{120d^6f} + \frac{(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3)\int\frac{1}{\sqrt{1-d^2x^2}}dx}{8d^4}$$

$$= -\frac{(4(4C+5Ad^2)f^2-3d^2e(Ce-5Bf))(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} + \frac{(4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2)))+d^2f(6Cd^2e^3-}{120d^6f} + \frac{(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3)\sin^{-1}(dx)}{8d^5}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.76

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}dx$$

$$= \frac{-\sqrt{1-d^2x^2}(20Ad^2f(4f^2+d^2(18e^2+9efx+2f^2x^2))+15B(d^2f^2(16e+3fx)+2d^4(4e^3+6e^2fx+4ef^2}{}$$

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^2
)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2
+ f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d^4*
x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*d*(4*C*d^2*e^3 +
 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcTa
n[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^6)
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06

method	result
risch	$\frac{(24C\,d^4f^3x^4+30B\,d^4f^3x^3+90C\,d^4e\,f^2x^3+40A\,d^4f^3x^2+120B\,d^4e\,f^2x^2+120C\,d^4e^2f\,x^2+180A\,d^4e\,f^2x+180B\,d^4e^2fx+60C\,d^4e^3x}{120d^6\sqrt{}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^4+30B\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^3+90C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4e\,f^2x^3+40A\sqrt{-}\right.}{}$

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/120*(24*C*d^4*f^3*x^4+30*B*d^4*f^3*x^3+90*C*d^4*e*f^2*x^3+40*A*d^4*f^3*x^
2+120*B*d^4*e*f^2*x^2+120*C*d^4*e^2*f*x^2+180*A*d^4*e*f^2*x+180*B*d^4*e^2*f
*x+60*C*d^4*e^3*x+360*A*d^4*e^2*f+120*B*d^4*e^3+32*C*d^2*f^3*x^2+45*B*d^2*f
^3*x+135*C*d^2*e*f^2*x+80*A*d^2*f^3+240*B*d^2*e*f^2+240*C*d^2*e^2*f+64*C*f^
3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1
/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C*d^2*e
^3+3*B*f^3+9*C*e*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/
2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{(24\,Cd^4f^3x^4+120\,Bd^4e^3+240\,Bd^2ef^2+120\,(3\,Ad^4+2\,Cd^2)e^2f+16\,(5\,Ad^2+4\,C)f^3+30\,(3\,Cd^4ef}{}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/120*((24*C*d^4*f^3*x^4 + 120*B*d^4*e^3 + 240*B*d^2*e*f^2 + 120*(3*A*d^4
+ 2*C*d^2)*e^2*f + 16*(5*A*d^2 + 4*C)*f^3 + 30*(3*C*d^4*e*f^2 + B*d^4*f^3)*
x^3 + 8*(15*C*d^4*e^2*f + 15*B*d^4*e*f^2 + (5*A*d^4 + 4*C*d^2)*f^3)*x^2 + 1
5*(4*C*d^4*e^3 + 12*B*d^4*e^2*f + 3*B*d^2*f^3 + 3*(4*A*d^4 + 3*C*d^2)*e*f^2
)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*e^2*f + 3*B*d*f^3 + 4*(2*A
*d^5 + C*d^3)*e^3 + 3*(4*A*d^3 + 3*C*d)*e*f^2)*arctan((sqrt(d*x + 1)*sqrt(-
d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = &-\frac{\sqrt{-d^2x^2+1}Cf^3x^4}{5\,d^2}+\frac{Ae^3\arcsin(dx)}{d}\\&-\frac{\sqrt{-d^2x^2+1}Be^3}{d^2}-\frac{3\sqrt{-d^2x^2+1}Ae^2f}{d^2}\\&-\frac{4\sqrt{-d^2x^2+1}Cf^3x^2}{15\,d^4}-\frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x^3}{4\,d^2}\\&-\frac{(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}x^2}{3\,d^2}\\&-\frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{2\,d^2}\\&+\frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin(dx)}{2\,d^3}\\&-\frac{8\sqrt{-d^2x^2+1}Cf^3}{15\,d^6}-\frac{3\,(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{8\,d^4}\\&-\frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}}{3\,d^4}\\&+\frac{3\,(3\,Cef^2+Bf^3)\arcsin(dx)}{8\,d^5}\end{aligned}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
[Out] -1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 + A*e^3*arcsin(d*x)/d - sqrt(-d^2*x^2
 + 1)*B*e^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*A*e^2*f/d^2 - 4/15*sqrt(-d^2*x^2 + 1
)*C*f^3*x^2/d^4 - 1/4*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x^3/d^2 - 1/3*
(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)*x^2/d^2 - 1/2*(C*e^3 + 3
*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 + 1/2*(C*e^3 + 3*B*e^2*f + 3
*A*e*f^2)*arcsin(d*x)/d^3 - 8/15*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(3*C*e*
f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 - 2/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)
*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(3*C*e*f^2 + B*f^3)*arcsin(d*x)/d^5
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.10

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(120\,Bd^4e^3+360\,Ad^4e^2f-60\,Cd^3e^3-180\,Bd^3e^2f-180\,Ad^3ef^2+360\,Cd^2e^2f+360\,Bd^2ef^2+120\,A}{}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/120*((120*B*d^4*e^3 + 360*A*d^4*e^2*f - 60*C*d^3*e^3 - 180*B*d^3*e^2*f -
 180*A*d^3*e*f^2 + 360*C*d^2*e^2*f + 360*B*d^2*e*f^2 + 120*A*d^2*f^3 - 225*
C*d*e*f^2 - 75*B*d*f^3 + 120*C*f^3 + (60*C*d^3*e^3 + 180*B*d^3*e^2*f + 180*
A*d^3*e*f^2 - 240*C*d^2*e^2*f - 240*B*d^2*e*f^2 - 80*A*d^2*f^3 + 405*C*d*e*
f^2 + 135*B*d*f^3 - 160*C*f^3 + 2*(60*C*d^2*e^2*f + 60*B*d^2*e*f^2 + 20*A*d
^2*f^3 - 135*C*d*e*f^2 - 45*B*d*f^3 + 88*C*f^3 + 3*(15*C*d*e*f^2 + 4*(d*x +
 1)*C*f^3 + 5*B*d*f^3 - 16*C*f^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*sqrt(d*x
 + 1)*sqrt(-d*x + 1) - 30*(8*A*d^5*e^3 + 4*C*d^3*e^3 + 12*B*d^3*e^2*f + 12*
A*d^3*e*f^2 + 9*C*d*e*f^2 + 3*B*d*f^3)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d
^6
```

Mupad [B] (verification not implemented)

Time = 37.96 (sec) , antiderivative size = 2606, normalized size of antiderivative = 7.66

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14
)/((d*x + 1)^(1/2) - 1)^14 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (((12288*C*f^3)/5 +
768*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1
- d*x)^(1/2) - 1)^3*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)
^3 - (((1 - d*x)^(1/2) - 1)^17*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)
^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^7*(88*C*d^3*e^3 - 42*C*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^13*(88*C*d^3*e^3 - 42*C*d*e*
f^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*(40*C*d^3*e^3 + 4
26*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)^15*(40*C*d^
```

3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9
*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2
) - 1)^11*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 - (d*(4*
C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) + (
d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1)^19)/(2*((d*x + 1)^(1/2) -
1)^19) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4
+ (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16)/(d^
6 + (10*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^6*((1
- d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^6*((1 - d*x)^(1/2) -
1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^6*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1
)^(1/2) - 1)^8 + (252*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^1
0 + (210*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^6*
((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^6*((1 - d*x)^(1/
2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (10*d^6*((1 - d*x)^(1/2) - 1)^18)/((
d*x + 1)^(1/2) - 1)^18 + (d^6*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) -
1)^20) - (((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^
(1/2) - 1)^4 + ((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x
+ 1)^(1/2) - 1)^8 - (((128*A*f^3)/3 - 144*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1
)^6)/((d*x + 1)^(1/2) - 1)^6 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)
^(1/2) - 1)^10 - (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)
+ (30*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (36*A*d*
e*f^2*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (36*A*d*e*f^2*((1
- d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (30*A*d*e*f^2*((1 - d*x)^(1/
2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 + (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^11)
/((d*x + 1)^(1/2) - 1)^11)/(d^4 + (6*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 +
(20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (15*d^4*((1 - d
*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6*d^4*((1 - d*x)^(1/2) - 1)^10
)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2
) - 1)^12) - ((((3*B*f^3)/2 + 6*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^15)/((d*
x + 1)^(1/2) - 1)^15 - (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)^(1/2) -
1)^3)/((d*x + 1)^(1/2) - 1)^3 + (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (((333*B*f^3)/2 + 90*B*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (((333*B*f^3)/2 + 90*B
*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (((671*B*f
^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 +
(((671*B*f^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2)
- 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(48*B*d^3*e^3 + 192*B*d*e*f^2))/((d*x +
1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(48*B*d^3*e^3 + 192*B*d*e*f^2))
/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8*(160*B*d^3*e^3 + 128*B
*d*e*f^2))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^6*(120*B*d^3*e^
3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^10*(12
0*B*d^3*e^3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^10 - (((3*B*f^3)/2 + 6*
B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (8*B*d^3*e^3*((

```
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^3*e^3*((1 - d*x)^(1
/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*((1 - d*x)^(1/2) - 1)^
2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1
/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (70
*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (56*d^5*((1 - d*x)^
(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1 - d*x)^(1/2) - 1)^12)
/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/
2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16) - (3*
B*f*atan((B*f*(f^2 + 4*d^2*e^2)*((1 - d*x)^(1/2) - 1))/((B*f^3 + 4*B*d^2*e^
2*f)*((d*x + 1)^(1/2) - 1)))*(f^2 + 4*d^2*e^2))/(2*d^5) - (2*A*e*atan((A*e*
((1 - d*x)^(1/2) - 1)*(3*f^2 + 2*d^2*e^2))/((2*A*d^2*e^3 + 3*A*e*f^2)*((d*x
 + 1)^(1/2) - 1)))*(3*f^2 + 2*d^2*e^2))/d^3 - (C*e*atan((C*e*((1 - d*x)^(1/
2) - 1)*(9*f^2 + 4*d^2*e^2))/((4*C*d^2*e^3 + 9*C*e*f^2)*((d*x + 1)^(1/2) -
1)))*(9*f^2 + 4*d^2*e^2))/(2*d^5)
```

3.9 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 228

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$=\frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f}-\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}$$
$$+\frac{\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-f\left(3\left(3C+4Ad^2\right)f^2-2d^2e(Ce-4Bf)\right)x\right)\sqrt{1-d^2x^2}}{24d^4f}$$
$$+\frac{\left(C\left(4d^2e^2+3f^2\right)+4d^2\left(2Bef+A\left(2d^2e^2+f^2\right)\right)\right)\arcsin(dx)}{8d^5}$$

```
[Out] 1/8*(C*(4*d^2*e^2+3*f^2)+4*d^2*(2*B*e*f+A*(2*d^2*e^2+f^2)))*arcsin(d*x)/d^5
+1/12*(-4*B*f+C*e)*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^2/f-1/4*C*(f*x+e)^3*(-d^2
*x^2+1)^(1/2)/d^2/f+1/24*(4*C*(d^2*e^3-8*e*f^2)-16*f*(3*A*d^2*e*f+B*(d^2*e^
2+f^2))-f*(3*(4*A*d^2+3*C)*f^2-2*d^2*e*(-4*B*f+C*e))*x)*(-d^2*x^2+1)^(1/2)/
d^4/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used

= {1623, 1668, 847, 794, 222}

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{\arcsin(dx)\,(4d^2(A(2d^2e^2+f^2)+2Bef)+C(4d^2e^2+3f^2))}{8d^5}$$
$$+ \frac{\sqrt{1-d^2x^2}(4(C(d^2e^3-8ef^2)-4f(3Ad^2ef+B(d^2e^2+f^2)))-fx(3f^2(4Ad^2+3C)-2d^2e(Ce-4Bf}{24d^4f}$$
$$+ \frac{\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{12d^2f} - \frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(12*d^2*f) - (C*(e + f*x)^3*S
qrt[1 - d^2*x^2])/(4*d^2*f) + ((4*(C*(d^2*e^3 - 8*e*f^2) - 4*f*(3*A*d^2*e*f
 + B*(d^2*e^2 + f^2))) - f*(3*(3*C + 4*A*d^2)*f^2 - 2*d^2*e*(C*e - 4*B*f))*
x)*Sqrt[1 - d^2*x^2])/(24*d^4*f) + ((C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f
 + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(8*d^5)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
```

```
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} - \frac{\int \frac{(e+fx)^2\left(-\left(\left(3C+4Ad^2\right)f^2\right)+d^2f(Ce-4Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{4d^2f^2}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\int \frac{(e+fx)\left(d^2f^2\left(7Ce+12Ad^2e+8Bf\right)+d^2f\left(3\left(3C+4Ad^2\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{\sqrt{1-d^2x^2}}\,dx}{12d^4f^2}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-f\left(3\left(3C+4Ad^2\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{24d^4f} + \frac{\left(C\left(4d^2e^2+3f^2\right)+4d^2\left(2Bef+A\left(2d^2e^2+f^2\right)\right)\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{8d^4}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} + \frac{\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-f\left(3\left(3C+4Ad^2\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{24d^4f} + \frac{\left(C\left(4d^2e^2+3f^2\right)+4d^2\left(2Bef+A\left(2d^2e^2+f^2\right)\right)\right)\sin^{-1}(dx)}{8d^5}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-d\sqrt{1-d^2x^2}(12Ad^2f(4e+fx)+C(12d^2e^2x+16ef(2+d^2x^2)+3f^2x(3+2d^2x^2))+8B(2f^2+d^2(3e^2 \cdots}{24d^5}$$

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f*(
2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f*x
 + f^2*x^2)))) + 6*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^2 +
 f^2)))*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^5)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12

method	result
risch	$\frac{(6C d^2 f^2 x^3+8B d^2 f^2 x^2+16C d^2 ef x^2+12A d^2 f^2 x+24B d^2 efx+12C d^2 e^2 x+48A d^2 ef+24B d^2 e^2+9C f^2 x+16B f^2+32Cef)\sqrt{dx+\cdots}}{24d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(6C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3 f^2 x^3+8B\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3 f^2 x^2+16C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3 ef x^2+12A\sqrt{-d^2x^2 \cdots}\right.}{\cdots}$

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/24*(6*C*d^2*f^2*x^3+8*B*d^2*f^2*x^2+16*C*d^2*e*f*x^2+12*A*d^2*f^2*x+24*B*
d^2*e*f*x+12*C*d^2*e^2*x+48*A*d^2*e*f+24*B*d^2*e^2+9*C*f^2*x+16*B*f^2+32*C*
e*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^
(1/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^2+4*A*d^2*f^2+8*B*d^2*e*f+4*C*d^2*e^2+3
*C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*
(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,Cd^3f^2x^3+24\,Bd^3e^2+16\,Bdf^2+16\,(3\,Ad^3+2\,Cd)ef+8\,(2\,Cd^3ef+Bd^3f^2)x^2+3\,(4\,Cd^3e^2+8\,Bd}{}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/24*((6*C*d^3*f^2*x^3 + 24*B*d^3*e^2 + 16*B*d*f^2 + 16*(3*A*d^3 + 2*C*d)*
e*f + 8*(2*C*d^3*e*f + B*d^3*f^2)*x^2 + 3*(4*C*d^3*e^2 + 8*B*d^3*e*f + (4*A
*d^3 + 3*C*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*e*f + 4*(2*
A*d^4 + C*d^2)*e^2 + (4*A*d^2 + 3*C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x +
 1) - 1)/(d*x)))/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cf^2x^3}{4\,d^2} + \frac{Ae^2 \arcsin{(dx)}}{d} - \frac{\sqrt{-d^2x^2+1}Be^2}{d^2} - \frac{2\sqrt{-d^2x^2+1}Aef}{d^2} - \frac{\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)x^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{2\,d^2} - \frac{3\sqrt{-d^2x^2+1}Cf^2x}{8\,d^4} + \frac{(Ce^2+2\,Bef+Af^2)\arcsin{(dx)}}{2\,d^3} + \frac{3\,Cf^2\arcsin{(dx)}}{8\,d^5} - \frac{2\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)}{3\,d^4}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
[Out] -1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*e^2*arcsin(d*x)/d - sqrt(-d^2*x^2
 + 1)*B*e^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*e*f/d^2 - 1/3*sqrt(-d^2*x^2 + 1)*(
2*C*e*f + B*f^2)*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f^2)
*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/2*(C*e^2 + 2*B*e*f + A*f^2)
*arcsin(d*x)/d^3 + 3/8*C*f^2*arcsin(d*x)/d^5 - 2/3*sqrt(-d^2*x^2 + 1)*(2*C*
e*f + B*f^2)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(24\,Bd^3e^2+48\,Ad^3ef-12\,Cd^2e^2-24\,Bd^2ef-12\,Ad^2f^2+48\,Cdef+24\,Bdf^2-15\,Cf^2+(12\,Cd^2e^2}{}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
[Out] -1/24*((24*B*d^3*e^2 + 48*A*d^3*e*f - 12*C*d^2*e^2 - 24*B*d^2*e*f - 12*A*d^
2*f^2 + 48*C*d*e*f + 24*B*d*f^2 - 15*C*f^2 + (12*C*d^2*e^2 + 24*B*d^2*e*f +
 12*A*d^2*f^2 - 32*C*d*e*f - 16*B*d*f^2 + 27*C*f^2 + 2*(8*C*d*e*f + 3*(d*x
+ 1)*C*f^2 + 4*B*d*f^2 - 9*C*f^2)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*sqrt(
-d*x + 1) - 6*(8*A*d^4*e^2 + 4*C*d^2*e^2 + 8*B*d^2*e*f + 4*A*d^2*f^2 + 3*C*
f^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

Mupad [B] (verification not implemented)

Time = 35.09 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.60

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*A*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (2*A*f^2*((
1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*A*f^2*((1 - d*x)^(1/2) - 1
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*A*f^2*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)
^(1/2) - 1)^7 + (16*A*d*e*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^
2 + (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*A*d*
e*f*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6)/(d^3 + (4*d^3*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((1 - d*x)^(1/2) - 1)^4)
/((d*x + 1)^(1/2) - 1)^4 + (4*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2)
 - 1)^6 + (d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8) - ((((1 -
d*x)^(1/2) - 1)^4*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (((1
 - d*x)^(1/2) - 1)^8*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^8 - (
((1 - d*x)^(1/2) - 1)^6*((128*B*f^2)/3 - 48*B*d^2*e^2))/((d*x + 1)^(1/2) -
1)^6 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B
*d^2*e^2*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (20*B*d*e*f*(
(1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (24*B*d*e*f*((1 - d*x)^(1
/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (24*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/
((d*x + 1)^(1/2) - 1)^7 - (20*B*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(
1/2) - 1)^9 + (4*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11
 - (4*B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (6*d^4*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (15*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (6*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*
x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12) - ((((1 - d*x)^(1/2) - 1)^15*((
3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1
)^3*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/
2) - 1)*((3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1
/2) - 1)^13*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 -
 d*x)^(1/2) - 1)^5*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^5
- (((1 - d*x)^(1/2) - 1)^11*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2
) - 1)^11 - (((1 - d*x)^(1/2) - 1)^7*((671*C*f^2)/2 - 22*C*d^2*e^2))/((d*x
+ 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^9*((671*C*f^2)/2 - 22*C*d^2*e^2)
)/((d*x + 1)^(1/2) - 1)^9 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1
)^(1/2) - 1)^4 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2)
- 1)^6) + (256*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(1/2) - 1)^8)
 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) - 1)^10) + (1
```

```
28*C*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12)/(d^5 + (8*d^
5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/
2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x
 + 1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)
^8 + (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2)
 - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x +
 1)^(1/2) - 1)^16) - (C*atan((C*((1 - d*x)^(1/2) - 1)*(3*f^2 + 4*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(3*C*f^2 + 4*C*d^2*e^2)))*(3*f^2 + 4*d^2*e^2))/(2*d^
5) - (2*A*atan((A*(f^2 + 2*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(A*f^2 + 2*A*d^2*e^2)))*(f^2 + 2*d^2*e^2))/d^3 - (4*B*e*f*atan(((1 -
 d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.10 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{(2(3d^2f(Be+Af)-C(d^2e^2-2f^2))-d^2f(Ce-3Bf)x)\sqrt{1-d^2x^2}}{6d^4f} + \frac{(Ce+2Ad^2e+Bf)\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*(2*A*d^2*e+B*f+C*e)*arcsin(d*x)/d^3-1/3*C*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/
d^2/f-1/6*(6*d^2*f*(A*f+B*e)-2*C*(d^2*e^2-2*f^2)-d^2*f*(-3*B*f+C*e)*x)*(-d^
2*x^2+1)^(1/2)/d^4/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1623, 1668, 794, 222}

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\arcsin(dx)\left(2Ad^2e+Bf+Ce\right)}{2d^3} - \frac{\sqrt{1-d^2x^2}\left(2\left(3d^2f(Af+Be)-\frac{1}{2}C(2d^2e^2-4f^2)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f} - \frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(d^2*f) - ((2*(3*d^2*f*(B*e + A*f) -
 (C*(2*d^2*e^2 - 4*f^2))/2) - d^2*f*(C*e - 3*B*f)*x)*Sqrt[1 - d^2*x^2])/(6*
d^4*f) + ((C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\int \frac{(e+fx)\left(-\left(\left(2C+3Ad^2\right)f^2\right)+d^2f(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2f^2}$$

$$
\begin{aligned}
&= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} \\
&\quad -\frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C(2d^2e^2-4f^2)\right)-d^2f(Ce-3Bf)x\right)\sqrt{1-d^2x^2}}{6d^4f} \\
&\quad +\frac{(Ce+2Ad^2e+Bf)\int\frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} \\
&\quad -\frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C(2d^2e^2-4f^2)\right)-d^2f(Ce-3Bf)x\right)\sqrt{1-d^2x^2}}{6d^4f} \\
&\quad +\frac{(Ce+2Ad^2e+Bf)\sin^{-1}(dx)}{2d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$
\begin{aligned}
&\int\frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx \\
&= \frac{\sqrt{1-d^2x^2}(-6Bd^2e-4Cf-6Ad^2f-3Cd^2ex-3Bd^2fx-2Cd^2fx^2)}{6d^4} \\
&\quad +\frac{(Ce+2Ad^2e+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}
\end{aligned}
$$

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(-6*B*d^2*e - 4*C*f - 6*A*d^2*f - 3*C*d^2*e*x - 3*B*d^2*
f*x - 2*C*d^2*f*x^2))/(6*d^4) + ((C*e + 2*A*d^2*e + B*f)*ArcTan[(d*x)/(-1 +
 Sqrt[1 - d^2*x^2])])/d^3
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

method	result
risch	$\frac{(2C\,d^2f\,x^2+3B\,d^2fx+3C\,d^2ex+6A\,d^2f+6B\,d^2e+4fC)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{6d^4\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2A\,d^2e+Bf+Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right.}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{d}}$
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(2C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2f\,x^2+3B\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2fx+3C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2ex+6A\sqrt{-d^2x^2+1}\,\operatorname{csgn}(\right.}{}$

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/6*(2*C*d^2*f*x^2+3*B*d^2*f*x+3*C*d^2*e*x+6*A*d^2*f+6*B*d^2*e+4*C*f)*(d*x+
1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*
x+1)^(1/2)+1/2*(2*A*d^2*e+B*f+C*e)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d
^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\left(2\,Cd^2fx^2+6\,Bd^2e+2\left(3\,Ad^2+2\,C\right)f+3\left(Cd^2e+Bd^2f\right)x\right)\sqrt{dx+1}\sqrt{-dx+1}+6\left(Bdf+\left(2\,Ad^3+\right.\right.}{6\,d^4}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*((2*C*d^2*f*x^2 + 6*B*d^2*e + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*e + B*d^2
*f)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arctan(
(sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = &-\frac{\sqrt{-d^2x^2+1}Cfx^2}{3\,d^2}+\frac{Ae\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^2x^2+1}Be}{d^2}\\&-\frac{\sqrt{-d^2x^2+1}Af}{d^2}-\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{2\,d^2}\\&+\frac{(Ce+Bf)\arcsin\left(dx\right)}{2\,d^3}-\frac{2\sqrt{-d^2x^2+1}Cf}{3\,d^4}\end{aligned}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*e*arcsin(d*x)/d - sqrt(-d^2*x^2 + 1
)*B*e/d^2 - sqrt(-d^2*x^2 + 1)*A*f/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e + B*f)
*x/d^2 + 1/2*(C*e + B*f)*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,Bd^2e+6\,Ad^2f-3\,Cde-3\,Bdf+(3\,Cde+2\,(dx+1)Cf+3\,Bdf-4\,Cf)(dx+1)+6\,Cf)\sqrt{dx+1}}{6\,d^4}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] -1/6*((6*B*d^2*e + 6*A*d^2*f - 3*C*d*e - 3*B*d*f + (3*C*d*e + 2*(d*x + 1)*C
*f + 3*B*d*f - 4*C*f)*(d*x + 1) + 6*C*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(
2*A*d^3*e + C*d*e + B*d*f)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^4
```

Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.78

$$\begin{aligned}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \frac{\frac{2\,B\,f\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,B\,f\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}\\ &\quad-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,C\,f}{3\,d^4}+\frac{2\,C\,f\,x}{3\,d^3}+\frac{C\,f\,x^3}{3\,d}+\frac{C\,f\,x^2}{3\,d^2}\right)}{\sqrt{d\,x+1}}\\ &\quad+\frac{\frac{2\,C\,e\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}-\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}-\frac{2\,C\,e\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}\\ &\quad-\frac{\left(\frac{A\,f}{d^2}+\frac{A\,f\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}}-\frac{\left(\frac{B\,e}{d^2}+\frac{B\,e\,x}{d}\right)\,\sqrt{1-d\,x}}{\sqrt{d\,x+1}}\\ &\quad-\frac{4\,A\,e\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,B\,f\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{2\,C\,e\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}\end{aligned}$$

```
[In] int(((e + f*x)*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)

[Out] ((2*B*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*B*f*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*B*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (2*B*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1
)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((1 -
d*x)^(1/2)*((2*C*f)/(3*d^4) + (2*C*f*x)/(3*d^3) + (C*f*x^3)/(3*d) + (C*f*x^
2)/(3*d^2)))/(d*x + 1)^(1/2) + ((2*C*e*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) - (14*C*e*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*C
*e*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (2*C*e*((1 - d*x)^(1/
2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1
)^(1/2) - 1)^2 + 1)^4) - (((A*f)/d^2 + (A*f*x)/d)*(1 - d*x)^(1/2))/(d*x + 1
)^(1/2) - (((B*e)/d^2 + (B*e*x)/d)*(1 - d*x)^(1/2))/(d*x + 1)^(1/2) - (4*A*
e*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2
)^(1/2) - (2*B*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - (
2*C*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(C+2Ad^2)\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*(2*A*d^2+C)*arcsin(d*x)/d^3-B*(-d^2*x^2+1)^(1/2)/d^2-1/2*C*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1829, 655, 222}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(2Ad^2+C)\arcsin(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((B*Sqrt[1 - d^2*x^2])/d^2) - (C*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-C-2Ad^2-2Bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{(-C-2Ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(C+2Ad^2)\sin^{-1}(dx)}{2d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$
\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(-2B-Cx)\sqrt{1-d^2x^2}}{2d^2} + \frac{(C+2Ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] ((-2*B - C*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*d^2)*ArcTan[(d*x)/(-1
+ Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2A\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2-C\operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,x-2B\sqrt{-d^2x^2+1}\operatorname{csgn}(d)d+C\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)\right)\operatorname{csgn}(d)}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(Cx+2B)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2A\,d^2+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^2\sqrt{d^2}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

```
[In] int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(
1/2))*d^2-C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x-2*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d
+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(Cdx+2\,Bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{A\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}Cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}B}{d^2} + \frac{C\arcsin(dx)}{2\,d^3}$$

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt(-d^2*x^2 + 1)*B/d^2
 + 1/2*C*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)C+2\,Bd-C)\sqrt{dx+1}\sqrt{-dx+1} - 2\,(2\,Ad^2+C)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*C + 2*B*d - C)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*A*d^2 +
 C)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4} - \frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\sqrt{1-d\,x}\,\left(\frac{B}{d^2}+\frac{B\,x}{d}\right)}{\sqrt{d\,x+1}}$$

```
[In] int((A + B*x + C*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*C*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*C*((1 - d*x
)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*C*((1 - d*x)^(1/2) - 1)^7)/((d
*x + 1)^(1/2) - 1)^7 - (2*C*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(
d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (4*A*atan((d
*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) -
 (2*C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((1 - d*x)^(
1/2)*(B/d^2 + (B*x)/d))/(d*x + 1)^(1/2)
```

3.12 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 122

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\arcsin(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

```
[Out] -(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)/(d^2*e
^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(1/2)-C*(-d^2*x^2+1)^(1
/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1668, 858, 222, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \frac{(Af^2-Bef+Ce^2)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}} - \frac{\arcsin(dx)(Ce-Bf)}{df^2} - \frac{C\sqrt{1-d^2x^2}}{d^2f}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{(Ce^2-Bef+Af^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{(Ce^2-Bef+Af^2)\,\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{(Ce^2-Bef+Af^2)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \frac{-\frac{Cf\sqrt{1-d^2x^2}}{d^2} + \frac{2(-Ce+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - \frac{2\sqrt{d^2e^2-f^2}(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(de-f)(de+f)}}{f^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + (2*(-(C*e) + B*f)*ArcTan[(d*x)/(-1 + Sqrt
[1 - d^2*x^2])])/d - (2*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*ArcT
an[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*(d*
e + f)))/f^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(114) = 228$.

Time = 1.66 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.37

method	result
risch	$\frac{C\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{f\,d^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{f\sqrt{d^2}}-\frac{\left(A\,f^2-Bef+C\,e^2\right)\ln\left(\frac{-\frac{2\left(d^2e^2-f^2\right)}{f^2}+\frac{2d^2e\left(x+\frac{e}{f}\right)}{f}+2\sqrt{-\frac{d^2e^2}{f^2}}\cdots\right)}{f^2\sqrt{-\frac{d^2e^2-f^2}{f^2}}}\right)\cdots}{f\sqrt{-dx+1}\,\sqrt{dx+1}}$
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2f^2+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^2ef-C\,\mathrm{csgn}(d\cdots\right.}{\cdots}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] C/f/d^2*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(
1/2)/(-d*x+1)^(1/2)+1/f*((B*f-C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2
*x^2+1)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2*(d
^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x+e/f
)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d*x+1)
)^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.

Time = 4.22 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$
$$=\left[-\frac{(Cd^2e^2-Bd^2ef+Ad^2f^2)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}\,(d^2ex+f)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+(d^2e^2-f^2)\sqrt{-\cdots}\right.}{fx+e}\right.}{\cdots}\right.$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 5803, normalized size of antiderivative = 47.57

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
- (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
 + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
```

```
 + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
 + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
 - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))
/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/
2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) +
 (C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d
*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/
2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) -
 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384
*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e
^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((409
6*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2)
- 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096
*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*
x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^
4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6
*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)
^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f -
```

d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*e^2*((4096*(32*C^
3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2) - 1)^2*(32*
C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (458752*
C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C*e^2*((4096*
(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1
)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((
1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 + 9*C^2*d^4*e^
7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d^2*e^3*f^7 -
30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*C*e^2*f^6 -
22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^2*e^3*f^7 -
90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)
- (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (16384*((1 - d*x
)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(1/2) - 1)) +
(4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4*((d
x + 1)^(1/2) - 1)^2)))/(f^2(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d
e)^(1/2)(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(
f + d*e)^(1/2)*(f - d*e)^(1/2)) + (917504*C^4*e^7*((1 - d*x)^(1/2) - 1)^2)/
(d*f^4*((d*x + 1)^(1/2) - 1)^2)))*2i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))
+ (B*e*atan(((B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1
- d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2)
- 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131
072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 -
d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(
d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f
^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((
d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x
)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(7*d^4*e^3*f^4 -
9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e
^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*
f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f
- d*e)^(1/2)))*1i)/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (B*e*((4096*(24*B^
3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*
e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((4096*(16*B^2*e*f^4 + 9*B^
2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*
e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^
5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (B*
e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 3604
48*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*
B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1
/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^
(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) +
(4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x +

```
1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2
)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f*(f + d*e)^
(1/2)*(f - d*e)^(1/2)))/((131072*B^4*e^3)/d + (917504*B^4*e^3*((1 - d*x)^(1
/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B^3*d^2*e^4 + 32*
B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^
2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) -
1))/((d*x + 1)^(1/2) - 1) + (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d +
 (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x +
 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f
^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*
d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^
3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 -
 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (
B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(819
20*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*
x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^
2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/
2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1
/2)) - (B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^
(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)
 + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((
4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2
*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(
1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x
 + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d
+ ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2)
 - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e
^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3)
)/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9
*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(
1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^
(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))*2i)/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} \, dx = \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)}+\frac{C\arcsin(dx)}{df^2} - \frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2(d^2e^2-f^2)^{3/2}}$$

```
[Out] C*arcsin(d*x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*
x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)+(A*f^2
-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1623, 1665, 858, 222, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2} \, dx = -\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2)}{f^2(d^2e^2-f^2)^{3/2}} + \frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{f(d^2e^2-f^2)(e+fx)}+\frac{C\arcsin(dx)}{df^2}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{d^2e^2-f^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{(d^2e^2-f^2)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$$
$$= \frac{\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(de-f)(de+f)(e+fx)} + \frac{2C\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} + \frac{2\sqrt{d^2e^2-f^2}(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(-de+f)^2(de+f)^2}}{f^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((d*e - f)*(d*e + f)*(e +
 f*x)) + (2*C*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d + (2*Sqrt[d^2*e^2 -
 f^2]*(C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*e^2 -
f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-(d*e) + f)^2*(d*e + f)^2))/f^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.66 (sec) , antiderivative size = 899, normalized size of antiderivative = 5.52

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e\,f^3x+C\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e^3fx-A}{}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)
+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)
/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)
^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e
^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^3*f*(-(d^
2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2
-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(csgn(d
)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(csgn(d)
*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*(-d*x+1)
^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/d/(d*e+f)/(f*x+e)/(-(d^2*e^
2-f^2)/f^2)^(1/2)/f^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(155) = 310$.

Time = 16.95 (sec) , antiderivative size = 1025, normalized size of antiderivative = 6.29

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$$

$$= \left[\frac{Cd^3e^5f - Bd^3e^4f^2 + Bde^2f^4 - Adef^5 + (Ad^3 - Cd)e^3f^3 - (Cd^3e^5 + Bde^2f^3 - (Ad^3 + 2\,Cd)e^3f^2 + (\right.$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10198, normalized size of antiderivative = 62.56

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
```

```
^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
 - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
 d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
 (A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
*f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
```

) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3
e^4((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1
/2) - 1)^4 - (2*d*e^2*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)
^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)^3*(f +
 d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (d*e^2*f^2*((1 - d*x
)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 + (
4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
1)^(1/2) - 1)) - (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - ((
(1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2)
 - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f
^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (B*f^4*atan(((f + d*e)^(3/2)*(f -
 d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1
i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e
^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) -
1)*8i)/((d*x + 1)^(1/2) - 1) + (B*f^4*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)
*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x +
1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((
d*x + 1)^(1/2) - 1)^3 - B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i
- (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(
1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d
*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (4*B*f*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (4*B*f*((1 - d
x)^(1/2) - 1)(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (B*
d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^
3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 - (B
*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^
2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e
^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*(

```
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) -
 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 - (B*d*
e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 + (8*B*d*e
*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2)
- 1)^2 + (B*d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d
*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^
2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1
 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1
)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) -
 1) + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f
^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2
*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*
x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/
((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)
^2 + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1
/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^
3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*
d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x
)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/(
(d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^
4)/(d^3*e^3*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (4*f^3*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - d*e*f^2*(f + d
*e)^(3/2)*(f - d*e)^(3/2) - (4*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
 - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (2*d^3*e^3*((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3*e^3*((1 - d*
x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 -
(4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
 + 1)^(1/2) - 1)^3 + (4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
- d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (2*d*e*f^2*((1 - d*x)^(1/2) - 1)^2*(f
 + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (d*e*f^2*((1 - d*x
)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4) -
((4*C*d*e*((1 - d*x)^(1/2) - 1))/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)) -
(4*C*d*e*((1 - d*x)^(1/2) - 1)^3)/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^3)
 + (8*C*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2
) - 1)^2))/(d^2*e + (4*d*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (
4*d*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (2*d^2*e*((1 - d*x
)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e*((1 - d*x)^(1/2) - 1)^4)/(
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(d*x + 1)^(1/2) - 1)^4) + (4*C*atan((((((1 - d*x)^(1/2) - 1)*((2097152*(288
*e^3*f^11 - 6*d^10*e^13*f - 912*d^2*e^5*f^9 + 1048*d^4*e^7*f^7 - 532*d^6*e^
9*f^5 + 112*d^8*e^11*f^3))/(d*f^2*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9
- 4*d^7*e^6*f^7 + d^9*e^8*f^5)) - (33554432*(20*d^2*e*f^21 - 103*d^4*e^3*f^
19 + 215*d^6*e^5*f^17 - 230*d^8*e^7*f^15 + 130*d^10*e^9*f^13 - 35*d^12*e^11
*f^11 + 3*d^14*e^13*f^9))/(d^5*f^10*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^
9 - 4*d^7*e^6*f^7 + d^9*e^8*f^5)) + (8388608*(72*e*f^17 - 452*d^2*e^3*f^15
+ 1024*d^4*e^5*f^13 - 1106*d^6*e^7*f^11 + 597*d^8*e^9*f^9 - 144*d^10*e^11*f
^7 + 9*d^12*e^13*f^5))/(d^3*f^6*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9 -
4*d^7*e^6*f^7 + d^9*e^8*f^5))))/((d*x + 1)^(1/2) - 1) - (33554432*(7*d^2*e^
2*f^19 - 35*d^4*e^4*f^17 + 70*d^6*e^6*f^15 - 70*d^8*e^8*f^13 + 35*d^10*e^10
*f^11 - 7*d^12*e^12*f^9))/(d^5*f^10*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (2097152*(112*e^4*f^9 + 28*d^8*e^12*f - 3
36*d^2*e^6*f^7 + 364*d^4*e^8*f^5 - 168*d^6*e^10*f^3))/(d*f^2*(f^12 - 4*d^2*
e^2*f^10 + 6*d^4*e^4*f^8 - 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (8388608*(28*e^2
*f^15 - 168*d^2*e^4*f^13 + 364*d^4*e^6*f^11 - 371*d^6*e^8*f^9 + 182*d^8*e^1
0*f^7 - 35*d^10*e^12*f^5))/(d^3*f^6*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)))*(d^4*f^14 - 4*d^6*e^2*f^12 + 6*d^8*e^4*f^1
0 - 4*d^10*e^6*f^8 + d^12*e^8*f^6))/(67108864*e*f^12 + 37748736*d^12*e^13 -
 268435456*d^2*e^3*f^10 + 536870912*d^4*e^5*f^8 - 637534208*d^6*e^7*f^6 + 4
69762048*d^8*e^9*f^4 - 201326592*d^10*e^11*f^2)))/(d*f^2) + (log(16*f^15 -
9*d^14*e^14*f - (16*f^15*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 -
 92*d^2*e^2*f^13 + 236*d^4*e^4*f^11 - 352*d^6*e^6*f^9 + 329*d^8*e^8*f^7 - 1
91*d^10*e^10*f^5 + 63*d^12*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2
) + 12*d^6*e^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/
2)*(f - d*e)^(3/2) - (6*d^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (16*d*e*f^14*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (92*d^2*e
^2*f^13*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (236*d^4*e^4*f^1
1*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (352*d^6*e^6*f^9*((1 -
 d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (329*d^8*e^8*f^7*((1 - d*x)^(
1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (191*d^10*e^10*f^5*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)
*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 120*d^4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^
6*e^6*f^6*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(
f - d*e)^(9/2) + 207*d^8*e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e
^4*f^2*(f + d*e)^(9/2)*(f - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(
f - d*e)^(3/2) - (88*d^3*e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (216*d^5*e^5*f^10*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (308
*d^7*e^7*f^8*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (274*d^9*e^9*f^
6*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (150*d^11*e^11*f^4*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (46*d^13*e^13*f^2*((1 - d*x)^(1/2)
- 1))/((d*x + 1)^(1/2) - 1) + (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f
```

- d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3
f^9((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(
3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d
e)^(3/2)(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d
x)^(1/2) - 1)(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (26
4*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
+ 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d
e)^(9/2)(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
+ (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2
+ (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1
- d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^
2 + (56*d^5*e^5*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((
d*x + 1)^(1/2) - 1) + (44*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)
*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1))*(C*d^2*e^3 - 2*C*e*f^2))/(f^2*(f +
d*e)^(3/2)*(f - d*e)^(3/2)) + (C*e*log(9*d^14*e^14*f - 16*f^15 + (16*f^15*
((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + 92*d^2*e^2*f^13 - 236*d^
4*e^4*f^11 + 352*d^6*e^6*f^9 - 329*d^8*e^8*f^7 + 191*d^10*e^10*f^5 - 63*d^1
2*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 12*d^6*e^6*(f + d*e)^
(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (6*d
^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (16*d*e*f^14*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (92*d^2*e^2*f^13*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (236*d^4*e^4*f^11*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (352*d^6*e^6*f^9*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2 + (329*d^8*e^8*f^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(
1/2) - 1)^2 - (191*d^10*e^10*f^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 + (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
- (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x +
1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 120*d^
4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^6*e^6*f^6*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 207*d^8*
e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e^4*f^2*(f + d*e)^(9/2)*(f
- d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (88*d^3*
e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (216*d^5*e^5*f^10*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (308*d^7*e^7*f^8*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) - (274*d^9*e^9*f^6*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1) + (150*d^11*e^11*f^4*((1 - d*x)^(1/2) - 1))/((d*x + 1

```
)^(1/2) - 1) - (46*d^13*e^13*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1) - (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (48*
d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)
^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f -
d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3*f^9*((1 - d*x)^(1/2) - 1
)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (688*d^5*e^5*f^7
*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) -
1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2)
)/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^
(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (264*d^9*e^9*f^3*((1 - d*x)^
(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (80*d*e
*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/
2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2)
)/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (560*d^4*e^4*f^8*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
- (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (733*d^8*e^8*f^4*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
- (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))
/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1 - d*x)^(1/2) - 1)^2*(f +
d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (56*d^5*e^5*f*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) + (44
*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
 1)^(1/2) - 1))*(2*f^2 - d^2*e^2))/(f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
```

3.14 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 248

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx &= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} \\ &\quad -\frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} \\ &\quad +\frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}\end{aligned}$$

```
[Out] 1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^2+f^2)))*arctan((d^2*e*x+f)/
(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e^2-f^2)^(5/2)+1/2*(A*f^2-B*e*
f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(-3*A*d^2*e*f^2+B
*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)^
2/(f*x+e)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used

= {1623, 1665, 821, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$$
$$= \frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))}{2\left(d^2e^2-f^2\right)^{5/2}}$$
$$+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$
$$-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} + \frac{\int \frac{2(Ce+Ad^2e-Bf)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2(d^2e^2-f^2)}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{2(d^2e^2-f^2)^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} - \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\,\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-x^2}\,dx, x, \frac{f+d^2ex}{\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^2}$$

$$= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = -\frac{\frac{(de-f)(de+f)\sqrt{1-d^2x^2}(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(e+fx)^2}+2\sqrt{d^2e^2-f^2}(C(d^2e^2}{2(de-f)^3(de+f)^3}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] -1/2*((((d*e - f)*(d*e + f)*Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x)
 + B*f^2*(e + 2*f*x) - A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x -
4*f^2*x)))/(e + f*x)^2 + 2*Sqrt[d^2*e^2 - f^2]*(C*(d^2*e^2 + 2*f^2) + d^2*(
-3*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x -
e*Sqrt[1 - d^2*x^2])])/((d*e - f)^3*(d*e + f)^3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1449

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/
(f*x+e))*d^4*e^2*f^2*x^2-3*A*d^2*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)+B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C
*d^2*e^3*f*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*A*ln(2*(d^2*e*
x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^4-3*C*e
^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*f^4*x*(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^4+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(
1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*f^4*x^2+2*C*ln(2*(d^2*e*x+(-d
^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+B*e*f^3*(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1
/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*f^4*x^2+A*ln(2*(d^2*e*x+(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*l
n(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^
2*e^3*f+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*e*f^3*x+A*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+4*A*
```

```
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^4*e^3*f*x-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*
f+f)/(f*x+e))*d^2*e*f^3*x^2+C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f
^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x^2+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)
^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x-6*B*ln(2*(d^2*e
*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*
x+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x
+e))*d^2*e^3*f*x-4*A*d^2*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)+2*B*d^2*e^3*f*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-4*C*e*f^3*x
*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)^2*(-d*x+1)^(1/2)*(d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e-f)/(d*e+f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(232) = 464$.

Time = 0.42 (sec) , antiderivative size = 1580, normalized size of antiderivative = 6.37

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
```

```
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume?`
for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9097, normalized size of antiderivative = 36.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3} dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
```

7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2) - 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5)

)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1
- d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*
e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*
d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(
4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9
*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*
e)^(5/2)*(f - d*e)^(5/2)))/((8*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e
*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8
*((1 - d*x)^(1/2) - 1)^2*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e*f^4))
/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e
*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d
^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7
+ 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*
e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x
)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*
f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*
f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4
*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 -
4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4
*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9
*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^
5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/
2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))
+ (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)
^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(2*f^2 + d^
2*e^2)*1i)/((f + d*e)^(5/2)*(f - d*e)^(5/2)) + (A*d^2*atan(((A*d^2*(f^2 + 2
*d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A
*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*
d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^
7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)
+ (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1

1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)) - (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^
9*e^7*f - 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4
 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7
*f - 12*A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2
*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))/((8*(4*A^2*d^9*
e^5 + 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 +
6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8*((1 - d*x)^(1/2) - 1)^2*(4*A^2*d^9*e^5
+ 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e
^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*
e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*
e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e
^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5
*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (A
d^2(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 +
8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^1
1 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*
d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*
e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)
^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f -
 d*e)^(5/2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f
- 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6
*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*
A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11
- 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f
^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*
((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72
*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8
 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*
f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*
e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(f^2 + 2*d^2*e^2)*1i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2)) - (B*d^2*e*f*atan(((B*d^2*e*f*((4*((1 - d*x)
^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((

d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^
6*e^6*f^2)) - (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/
(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*
e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*
d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8
- 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x +
 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6
*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f
 + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (
B*d^2*e*f*((4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x
)^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/((
(d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d
^6*e^6*f^2)) + (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)))/((72*B^2*d^5*e^3*f^2)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6
+ 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) -
 (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (3*B*d^2*e*f*((4*
(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^8 -
4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) +
 (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7
*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^
3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10)
)/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/

```
2)) + (72*B^2*d^5*e^3*f^2*((1 - d*x)^(1/2) - 1)^2)/((((d*x + 1)^(1/2) - 1)^2
*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))))*3i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2))
```

3.15 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 79

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*d
^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1623, 1823, 794, 222}

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d^2x^2}(2(3ad^2+2c)+3bd^2x)}{6d^4} + \frac{b\arcsin(dx)}{2d^3} - \frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2} \\
&= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-4c-6ad^2-3bd^2x-2cd^2x^2)}{6d^4} + \frac{b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (Sqrt[1 - d^2*x^2]*(-4*c - 6*a*d^2 - 3*b*d^2*x - 2*c*d^2*x^2))/(6*d^4) + (b
*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(2\,\mathrm{csgn}(d) c\, d^{2} x^{2} \sqrt{-d^{2} x^{2}+1}+3 \sqrt{-d^{2} x^{2}+1}\,\mathrm{csgn}(d) b\, d^{2} x+6\,\mathrm{csgn}(d) \sqrt{-d^{2} x^{2}+1}\, a\, d^{2}+4\,\mathrm{csgn}(d) \sqrt{-d^{2} x^{2}+1}\, c-3\, \mathrm{a}\right.}{6 d^{4} \sqrt{-d^{2} x^{2}+1}}$
risch	$\frac{\left(2 c\, d^{2} x^{2}+3 b\, d^{2} x+6 a\, d^{2}+4 c\right) \sqrt{dx+1}\,\left(dx-1\right) \sqrt{\left(-dx+1\right) \left(dx+1\right)}}{6 d^{4} \sqrt{-\left(dx+1\right) \left(dx-1\right)}\, \sqrt{-dx+1}}+\frac{b \arctan\left(\frac{\sqrt{d^{2}}\, x}{\sqrt{-d^{2} x^{2}+1}}\right) \sqrt{\left(-dx+1\right) \left(dx+1\right)}}{2 d^{2} \sqrt{d^{2}}\, \sqrt{-dx+1}\, \sqrt{dx+1}}$

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*b*d^2*x+6*csgn(d)*(-d^2*x^2+1)^(1/2)*a*d^2+4*cs
gn(d)*(-d^2*x^2+1)^(1/2)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}bx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}a}{d^2} + \frac{b\arcsin(dx)}{2\,d^3} - \frac{2\sqrt{-d^2x^2+1}c}{3\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{6\,bd\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right) - (6\,ad^2 + (2\,(dx+1)c + 3\,bd - 4\,c)(dx+1) - 3\,bd + 6\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.09

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,b\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4} - \frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)

[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

3.16 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {913, 1829, 655, 222}

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(2ad^2+c)\arcsin(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 913

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}-\frac{(-c-2ad^2)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}\\ &= -\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\sin^{-1}(dx)}{2d^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx=\frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2}+\frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] ((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
+ Sqrt[1 - d^2*x^2])])/d^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(\sqrt{-d^{2}x^{2}+1}\ \operatorname{csgn}(d) d c x-2 \arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) a\,d^{2}+2\ \operatorname{csgn}(d) d \sqrt{-d^{2}x^{2}+1}\, b-\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^{2}x^{2}+1}}\right) c\right) \operatorname{csgn}(d}{2 d^{3}\sqrt{-d^{2}x^{2}+1}}$
risch	$\frac{(cx+2b)\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{(2a\,d^{2}+c)\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-dx+1)(dx+1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{-dx+1}\,\sqrt{dx+1}}$

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*a
rctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2)*
b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2 +
 c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

3.17 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-a\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1623, 1823, 858, 222, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx \\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2} - \frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2} \end{aligned}$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}$$

$$= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{2b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - a\log(x) + a\log\left(-1+\sqrt{1-d^2x^2}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (2*b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])
/d - a*Log[x] + a*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-\operatorname{csgn}(d)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2-\operatorname{csgn}(d)\sqrt{-d^2x^2+1}\,c+\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-(dx+1)(dx-1)}}\right)bd\right)\operatorname{csgn}(d)}{d^2\sqrt{-d^2x^2+1}}$	96

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*
d^2-csgn(d)*(-d^2*x^2+1)^(1/2)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2
))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.59 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.10

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(44) = 88.

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.08

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)-\left(\pi+2\arcta\right.}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(
sqrt(2) - sqrt(-d*x + 1)) + 2)) - a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))
/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) - 2)) - (pi + 2*a
rctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(
2) - sqrt(-d*x + 1))))*b*d + sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

3.18 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\,\mathrm{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1623, 1821, 858, 222, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\,\mathrm{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx
\end{aligned}
$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}$$

$$= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - b\log(x) + b\log\left(-1+\sqrt{1-d^2x^2}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (2*c*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d
 - b*Log[x] + b*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\operatorname{csgn}(d)dbx-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)da+\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)cx\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}\,xd}$	97
risch	$\frac{a\sqrt{dx+1}\,(dx-1)\sqrt{(-dx+1)(dx+1)}}{x\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}}+\frac{\left(\frac{c\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}-b\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{(-dx+1)(dx+1)}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$	129

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-arctanh(1/(-d^2*x^2+1)^(1/2))*csgn(d)*d*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d*
a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*
csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.60

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(44) = 88.

Time = 0.38 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.88

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{\frac{4\,ad^2\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)^2-4}+bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx-}}{\sqrt{2}-\sqrt{-}}\right.\right.}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] -(4*a*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2
) - sqrt(-d*x + 1)))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x
```

```
+ 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4) + b*d*log(abs(-(sqrt(2) - sqrt(-d*x
 + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - b*d
*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2)
 - sqrt(-d*x + 1)) - 2)) - (pi + 2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqr
t(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(2) - sqrt(-d*x + 1))))*c)/d
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

3.19 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}} dx$

Optimal result

Integrand size = 33, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}(2c+ad^2)\,\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

```
[Out] -1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b
*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1623, 1821, 821, 272, 65, 214}

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\left(ad^2+2c\right)\text{arctanh}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[1 - d^2*x^2])/x^2 - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1623

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)$$

$$= -\frac{a\sqrt{1-d^2x^2}}{2x^2} - \frac{b\sqrt{1-d^2x^2}}{x} - \frac{1}{2}(2c+ad^2)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{1}{2}\left(-\frac{(a+2bx)\sqrt{1-d^2x^2}}{x^2} - (2c+ad^2)\log(x) + (2c+ad^2)\log\left(-1+\sqrt{1-d^2x^2}\right)\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2) - (2*c + a*d^2)*Log[x] + (2*c + a*d
^2)*Log[-1 + Sqrt[1 - d^2*x^2]])/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^2\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2x^2+2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)c\,x^2+2\sqrt{-d^2x^2+1}\,bx+\sqrt{-d^2x^2+1}\,a\right)}{2\sqrt{-d^2x^2+1}\,x^2}$	108
risch	$\frac{\sqrt{dx+1}\,(dx-1)(2bx+a)\sqrt{(-dx+1)(dx+1)}}{2x^2\sqrt{-(dx+1)(dx-1)}\,\sqrt{-dx+1}} - \frac{\left(c+\frac{a\,d^2}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-dx+1)(dx+1)}}{\sqrt{-dx+1}\,\sqrt{dx+1}}$	113

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(ad^2+2\,c)x^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\sqrt{-d^2x^2+1}b}{x} - \frac{\sqrt{-d^2x^2+1}a}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(61) = 122$.

Time = 0.37 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)}{}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/2*((a*d^3 + 2*c*d)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + s
qrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - (a*d^3 + 2*c*d)*log(abs(-(s
qrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x
 + 1)) - 2)) - 4*(a*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 - 2*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/
sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 + 4*a*d^3*((sqr
t(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x +
 1))) + 8*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(
sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sq
rt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.39

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} + \frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2} - \frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2} - \frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x} + \frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log((((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log((((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log((((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log((((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.20 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 591

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(A(8b^4e^3+6a^2b^2ef^2)+a^2(a^2f^2(3Ce+Bf)+2b^2e^2(Ce+3Bf)))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$- \frac{(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af)))\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f}$$
$$+ \frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f}$$
$$- \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f}$$
$$- \frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4e^2(3Ce^2-7f(Be+12Af)))+3b}{840b^6f}$$
$$+ \frac{a^2\sqrt{c}(A(8b^4e^3+6a^2b^2ef^2)+a^2(a^2f^2(3Ce+Bf)+2b^2e^2(Ce+3Bf)))\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b}{\sqrt{a^2c}}\right.}{16b^5\sqrt{a^2c-b^2cx^2}}$$

```
[Out] 1/16*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^2*(B*f+3*C*e)+2*b^2*e^2*(3*B
*f+C*e)))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4-1/70*(8*a^2*C*f^2-b^2*(3*C
*e^2-7*f*(2*A*f+B*e)))*(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^
(1/2)/b^4/f+1/42*(-7*B*f+3*C*e)*(f*x+e)^3*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*
c*x+a*c)^(1/2)/b^2/f-1/7*C*(f*x+e)^4*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a
*c)^(1/2)/b^2/f-1/840*(64*a^4*C*f^4+16*a^2*b^2*f^2*(15*C*e^2+7*f*(A*f+3*B*e
))-8*b^4*e^2*(3*C*e^2-7*f*(12*A*f+B*e))+3*b^2*f*(a^2*f^2*(35*B*f+41*C*e)-2*
b^2*e*(3*C*e^2-7*f*(7*A*f+B*e)))*x)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*
c)^(1/2)/b^6/f+1/16*a^2*(A*(6*a^2*b^2*e*f^2+8*b^4*e^3)+a^2*(a^2*f^2*(B*f+3*
C*e)+2*b^2*e^2*(3*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c
^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/(-b^2*c*x^2+a^2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1624, 1668, 847, 794, 201, 223, 209}

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\sqrt{a+bx}(a^2-b^2x^2)\,(e+fx)^2\sqrt{ac-bcx}\left(-\frac{8a^2Cf^2}{b^2}-7f(2Af+Be)+3Ce^2\right)}{70b^2f}$$

$$+\frac{\sqrt{a+bx}(a^2-b^2x^2)\,(e+fx)^3\sqrt{ac-bcx}(3Ce-7Bf)}{42b^2f}$$

$$-\frac{C\sqrt{a+bx}(a^2-b^2x^2)\,(e+fx)^4\sqrt{ac-bcx}}{7b^2f}$$

$$+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)(a^4f^2(Bf+3Ce)+A(6a^2b^2ef^2+8b^4e^3)+2a^2b^2e^2(3Bf}{16b^5\sqrt{a^2c-b^2cx^2}}$$

$$+\frac{x\sqrt{a+bx}\sqrt{ac-bcx}(a^4f^2(Bf+3Ce)+A(6a^2b^2ef^2+8b^4e^3)+2a^2b^2e^2(3Bf+Ce))}{16b^4}$$

$$-\frac{\sqrt{a+bx}(a^2-b^2x^2)\,\sqrt{ac-bcx}(3b^2fx(a^2f^2(35Bf+41Ce)-b^2(6Ce^3-14ef(7Af+Be)))+8(8a^4Cf}{840b^6f}$$

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^
2*b^2*e*f^2))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((3*C*e^2 - (8*
a^2*C*f^2)/b^2 - 7*f*(B*e + 2*A*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*
x)^2*(a^2 - b^2*x^2))/(70*b^2*f) + ((3*C*e - 7*B*f)*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(42*b^2*f) - (C*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]*(e + f*x)^4*(a^2 - b^2*x^2))/(7*b^2*f) - (Sqrt[a + b*x]*Sqrt[a*c
 - b*c*x]*(8*(8*a^4*C*f^4 + 2*a^2*b^2*f^2*(15*C*e^2 + 7*f*(3*B*e + A*f)) -
b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f))) + 3*b^2*f*(a^2*f^2*(41*C*e + 35*B*f
) - b^2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x)*(a^2 - b^2*x^2))/(840*b^6*f) +
 (a^2*Sqrt[c]*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b
^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c
]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)^3\sqrt{a^2c-b^2cx^2}(A+Bx+Cx^2)\,dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)^3\left(-c(7Ab^2+4a^2C)f^2+b^2cf(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}\,dx}{7b^2cf^2\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} + \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)^2\left(3b^2c^2f^2(14Ab^2e+a^2(5Ce+7Bf))+3b^2c^2f(8a^2Cf^2-b^2(3C\right.}{42b^4c^2f^2\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f} + \frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(e+fx)\left(-3b^2c^3f^2(16a^4Cf^2+a^2b^2e(19Ce+49Bf)+14A(5b^4e^2+2a\right.}{210b^6c^3f^2\sqrt{a^2}}$$

$$= -\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f} + \frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4(3Ce^4-7e^2f(Be+12A\right.}{840b^6f} + \frac{\left((a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2))\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\sqrt{a^2c-}}{8b^4\sqrt{a^2c-b^2cx^2}}$$

$$=\frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2)\right)x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$-\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f}$$
$$+\frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4(3Ce^4-7e^2f(Be+12A}{840b^6f}$$
$$+\frac{\left(a^2c(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2))\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\frac{}{\sqrt{a^2c-}}}{16b^4\sqrt{a^2c-b^2cx^2}}$$

$$=\frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2)\right)x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$-\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f}$$
$$+\frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4(3Ce^4-7e^2f(Be+12A}{840b^6f}$$
$$+\frac{\left(a^2c(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2))\sqrt{a+bx}\sqrt{ac-bcx}\right)\text{Subst}(}{16b^4\sqrt{a^2c-b^2cx^2}}$$

$$=\frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2)\right)x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$-\frac{\left(8a^2Cf^2-b^2(3Ce^2-7f(Be+2Af))\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{70b^4f}$$
$$+\frac{(3Ce-7Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(8a^4Cf^4+2a^2b^2f^2(15Ce^2+7f(3Be+Af))-b^4(3Ce^4-7e^2f(Be+12A}{840b^6f}$$
$$+\frac{a^2\sqrt{c}(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2))\sqrt{a+bx}\sqrt{ac-bcx}\tan^{-1}(}{16b^5\sqrt{a^2c-b^2cx^2}}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.68

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\left(-\sqrt{a-bx}\sqrt{a+bx}(128a^6Cf^3+a^4b^2f(7f(96Be+32Af+15Bfx)+C(672e^2+315efx+\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(-(Sqrt[a - b*x]*Sqrt[a + b*x]*(128*a^6*C*f^3 + a^4*b^2*
f*(7*f*(96*B*e + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2))
 + 2*a^2*b^4*(7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*e^2
*f*x + 24*e*f^2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^2*x^
2 + 8*f^3*x^3)) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3
*x^3) + x*(7*B*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*C*x*(3
5*e^3 + 84*e^2*f*x + 70*e*f^2*x^2 + 20*f^3*x^3))))) + 210*a^2*b*(a^4*f^2*(3
*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2)
)*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(1680*b^6*Sqrt[a - b*x])
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(-240C\,f^3x^6b^6-280B\,b^6f^3x^5-840C\,b^6e\,f^2x^5-336A\,b^6f^3x^4-1008B\,b^6e\,f^2x^4+48C\,a^2b^4f^3x^4-1008C\,b^6e^2f\,x^4-1260A\,b^6e\,f^2x^3}{}$
default	Expression too large to display

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/1680*(-240*C*b^6*f^3*x^6-280*B*b^6*f^3*x^5-840*C*b^6*e*f^2*x^5-336*A*b^6
*f^3*x^4-1008*B*b^6*e*f^2*x^4+48*C*a^2*b^4*f^3*x^4-1008*C*b^6*e^2*f*x^4-126
0*A*b^6*e*f^2*x^3+70*B*a^2*b^4*f^3*x^3-1260*B*b^6*e^2*f*x^3+210*C*a^2*b^4*e
*f^2*x^3-420*C*b^6*e^3*x^3+112*A*a^2*b^4*f^3*x^2-1680*A*b^6*e^2*f*x^2+336*B
*a^2*b^4*e*f^2*x^2-560*B*b^6*e^3*x^2+64*C*a^4*b^2*f^3*x^2+336*C*a^2*b^4*e^2
*f*x^2+630*A*a^2*b^4*e*f^2*x-840*A*b^6*e^3*x+105*B*a^4*b^2*f^3*x+630*B*a^2*
b^4*e^2*f*x+315*C*a^4*b^2*e*f^2*x+210*C*a^2*b^4*e^3*x+224*A*a^4*b^2*f^3+168
0*A*a^2*b^4*e^2*f+672*B*a^4*b^2*e*f^2+560*B*a^2*b^4*e^3+128*C*a^6*f^3+672*C
*a^4*b^2*e^2*f)*(b*x+a)^(1/2)/b^6*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/16*a^2/b^
4*(6*A*a^2*b^2*e*f^2+8*A*b^4*e^3+B*a^4*f^3+6*B*a^2*b^2*e^2*f+3*C*a^4*e*f^2+
```

```
2*C*a^2*b^2*e^3)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1
/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$= \left[\frac{105\,(6\,Ba^4b^3e^2f+Ba^6bf^3+2\,(Ca^4b^3+4\,Aa^2b^5)e^3+3\,(Ca^6b+2\,Aa^4b^3)ef^2)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bc\cdots}\right.}{\cdots}\right.$$

$$-\frac{105\,(6\,Ba^4b^3e^2f+Ba^6bf^3+2\,(Ca^4b^3+4\,Aa^2b^5)e^3+3\,(Ca^6b+2\,Aa^4b^3)ef^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\cdots}{b^2cx^2-a^2c}\right.}{\cdots}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/3360*(105*(6*B*a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)
*e^3 + 3*(C*a^6*b + 2*A*a^4*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6 - 5
60*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^
5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 -
336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70
*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*
f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)
*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*
a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f
^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(6*B*a^4*b^3*e^2
*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3 + 3*(C*a^6*b + 2*A*a^4*b
^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2
*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 - 560*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e
*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*
e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 - 336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^
2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B
*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B
*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^
4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b
^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b^6]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{-c(-a+bx)}\sqrt{a+bx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)

[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**3*(A + B*x + C*x**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^3x^4}{7\,b^2c}+\frac{Aa^2\sqrt{c}e^3\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^3x$$
$$-\frac{4\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^3x^2}{35\,b^4c}+\frac{(3\,Cef^2+Bf^3)a^6\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{16\,b^5}$$
$$+\frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^3}{3\,b^2c}$$
$$-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ae^2f}{b^2c}-\frac{8\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^4f^3}{105\,b^6c}$$
$$+\frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^4x}{16\,b^4}+\frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)a^2x}{8\,b^2}$$
$$-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)x^3}{6\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{5\,b^2c}$$
$$-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^3+3\,Be^2f+3\,Aef^2)x}{4\,b^2c}$$
$$-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{15\,b^4c}$$

[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")

[Out] -1/7*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^3*x^4/(b^2*c) + 1/2*A*a^2*sqrt(c)*e^3*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^3*x - 4/35*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^2*f^3*x^2/(b^4*c) + 1/16*(3*C*e*f^2 + B*f^3)*a^6*sqrt(c)*a

```
rcsin(b*x/a)/b^5 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^4*sqrt(c)*arcsin(b
*x/a)/b^3 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^3/(b^2*c) - (-b^2*c*x^2 + a^
2*c)^(3/2)*A*e^2*f/(b^2*c) - 8/105*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^4*f^3/(b^
6*c) + 1/16*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^4*x/b^4 + 1/8*sq
rt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*x/b^2 - 1/6*(-b^
2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/5*(-b^2*c*x^2 +
a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) - 1/8*(-b^2*c*x^2
+ a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*c) - 1/4*(-b^2*c*x^2 + a^2*c)
^(3/2)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c
)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2671 vs. $2(550) = 1100$.

Time = 1.02 (sec) , antiderivative size = 2671, normalized size of antiderivative = 4.52

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/1680*(1680*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^5*e^3 -
 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c))
)/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^4*
e^3 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b
^5*e^3 - 2520*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
+ 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))
*A*a*b^4*e^2*f + 280*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x +
 a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^3*e^3 + 280*(6*a^3*c*log(abs(-sqrt(b*x
+ a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x
 + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^4*e^3 + 840*(6
*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(
-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x
 + a))*B*a*b^3*e^2*f + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-
(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^4*e^2*f + 840*(6*a^3*c*log(abs(-sqr
t(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a
)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^3*e*f^
2 - 70*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*
sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b^3*e^3 - 210*(18*a^4*c*log(abs
```

```
(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3
- (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*
c)*sqrt(b*x + a))*C*a*b^2*e^2*f - 210*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*
(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*
b^3*e^2*f - 210*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b^2*e*f^2 - 210*(18*
a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-
c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x
+ a)*c + 2*a*c)*sqrt(b*x + a))*A*b^3*e*f^2 - 70*(18*a^4*c*log(abs(-sqrt(b*x
 + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*
x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*
x + a))*A*a*b^2*f^3 + 42*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-
(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)
*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt
(b*x + a))*C*b^2*e^2*f + 42*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqr
t(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17
*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a))*C*a*b*e*f^2 + 42*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) +
sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x -
 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c
)*sqrt(b*x + a))*B*b^2*e*f^2 + 14*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
 + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*
x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*
a*c)*sqrt(b*x + a))*B*a*b*f^3 + 14*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c
) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b
*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2
*a*c)*sqrt(b*x + a))*A*b^2*f^3 - 21*(150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(
-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*
a^3 - (4*(5*b*x - 26*a)*(b*x + a) + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a
))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b*e*f^2 - 7*(150*a^6*c*log(a
bs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (405*a
^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a) + 321*a^2)*(b*x +
a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*f^3
 - 7*(150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x +
a) + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a))*B*b*f^3 + (1050*a^7*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (2835*a^6 - (6335*a^5 - 2*(4781*a^4 - (4
551*a^3 - 4*(5*(6*b*x - 37*a)*(b*x + a) + 661*a^2)*(b*x + a))*(b*x + a))*(b
*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*f^3)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

```
[In] int((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.21 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 451

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(2A(4b^4e^2+a^2b^2f^2)+a^2(a^2Cf^2+2b^2e(Ce+2Bf)))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$+\frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(8(2a^2f^2(2Ce+Bf)-b^2e(Ce^2-2f(Be+5Af)))+3f(5a^2Cf^2-b^2(2Ce^2-2f(2}{120b^4f}$$
$$+\frac{a^2\sqrt{c}(2A(4b^4e^2+a^2b^2f^2)+a^2(a^2Cf^2+2b^2e(Ce+2Bf)))\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{16b^5\sqrt{a^2c-b^2cx^2}}$$

```
[Out] 1/16*(2*A*(a^2*b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*x*(b
*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4+1/10*(-2*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+a^
2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/6*C*(f*x+e)^3*(-b^2*x^2+a^2)*(b
*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/120*(16*a^2*f^2*(B*f+2*C*e)-8*b^2*e*
(C*e^2-2*f*(5*A*f+B*e))+3*f*(5*a^2*C*f^2-b^2*(2*C*e^2-2*f*(5*A*f+2*B*e)))*x
)*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/16*a^2*(2*A*(a^2*
b^2*f^2+4*b^4*e^2)+a^2*(a^2*C*f^2+2*b^2*e*(2*B*f+C*e)))*arctan(b*x*c^(1/2)/
(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/(-b^
2*c*x^2+a^2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1624, 1668, 847, 794, 201, 223, 209}

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx =$$
$$-\frac{\sqrt{a+bx}(a^2-b^2x^2)\sqrt{ac-bcx}\left(8\left(2a^2f^2(Bf+2Ce)-\frac{1}{8}b^2(8Ce^3-16ef(5Af+Be))\right)+3fx(5a^2Cf^2-\right.}{120b^4f}$$
$$+\frac{\sqrt{a+bx}(a^2-b^2x^2)(e+fx)^2\sqrt{ac-bcx}(Ce-2Bf)}{10b^2f}$$
$$-\frac{C\sqrt{a+bx}(a^2-b^2x^2)(e+fx)^3\sqrt{ac-bcx}}{6b^2f}$$
$$+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)(a^4Cf^2+2A(a^2b^2f^2+4b^4e^2)+2a^2b^2e(2Bf+Ce))}{16b^5\sqrt{a^2c-b^2cx^2}}$$
$$+\frac{x\sqrt{a+bx}\sqrt{ac-bcx}(a^4Cf^2+2A(a^2b^2f^2+4b^4e^2)+2a^2b^2e(2Bf+Ce))}{16b^4}$$

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*x*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((C*e - 2*B*f)*Sqrt[a + b*x]*Sq
rt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(10*b^2*f) - (C*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(6*b^2*f) - (Sqrt[a + b*x]*S
qrt[a*c - b*c*x]*(8*(2*a^2*f^2*(2*C*e + B*f) - (b^2*(8*C*e^3 - 16*e*f*(B*e
+ 5*A*f)))/8) + 3*f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2*f*(2*B*e + 5*A*f)))*x)*
(a^2 - b^2*x^2))/(120*b^4*f) + (a^2*Sqrt[c]*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e +
 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*Ar
cTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2
])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] && !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int (e+fx)^2\sqrt{a^2c-b^2cx^2}(A+Bx+Cx^2)\,dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int (e+fx)^2\left(-3c(2Ab^2+a^2C)f^2+3b^2cf(Ce-2Bf)x\right)\sqrt{a^2c-b^2cx^2}\,dx}{6b^2cf^2\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} + \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int (e+fx)\left(3b^2c^2f^2(10Ab^2e+a^2(3Ce+4Bf))+3b^2c^2f(5(2Ab^2+a^2C)f^2\right.}{30b^4c^2f^2\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(8\left(2a^2f^2(2Ce+Bf)-\frac{1}{8}b^2(8Ce^3-16ef(Be+5Af))\right)+3f(5a^2Cf^2-b^2(2C\right.}{120b^4f} + \frac{\left((a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\sqrt{a+bx}\sqrt{ac-bcx}\right)\int \sqrt{a^2c-b^2cx^2}\,dx}{8b^4\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4} + \frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(8\left(2a^2f^2(2Ce+Bf)-\frac{1}{8}b^2(8Ce^3-16ef(Be+5Af))\right)+3f(5a^2Cf^2-b^2(2C\right.}{120b^4f} + \frac{\left(a^2c(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\sqrt{a+bx}\sqrt{ac-bcx}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{16b^4\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$+\frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\,(a^2-b^2x^2)}{10b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\,(a^2-b^2x^2)}{6b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}\big(8\big(2a^2f^2(2Ce+Bf)-\frac{1}{8}b^2(8Ce^3-16ef(Be+5Af))\big)+3f(5a^2Cf^2-b^2(}{120b^4f}$$
$$+\frac{\big(a^2c(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\,\sqrt{a+bx}\sqrt{ac-bcx}\big)\,\mathrm{Subst}\Big(\int\frac{1}{1+b^2cx^2}\,d}{16b^4\sqrt{a^2c-b^2cx^2}}$$
$$= \frac{(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4}$$
$$+\frac{(Ce-2Bf)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\,(a^2-b^2x^2)}{10b^2f}$$
$$-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\,(a^2-b^2x^2)}{6b^2f}$$
$$-\frac{\sqrt{a+bx}\sqrt{ac-bcx}\big(8\big(2a^2f^2(2Ce+Bf)-\frac{1}{8}b^2(8Ce^3-16ef(Be+5Af))\big)+3f(5a^2Cf^2-b^2(}{120b^4f}$$
$$+\frac{a^2\sqrt{c}(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2))\,\sqrt{a+bx}\sqrt{ac-bcx}\,\tan^{-1}\Big(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}}{16b^5\sqrt{a^2c-b^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.63

$$\int\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\,(A+Bx+Cx^2)\;dx$$
$$=\frac{\sqrt{c(a-bx)}\Big(b\sqrt{a-bx}\sqrt{a+bx}(-a^4f(64Ce+32Bf+15Cfx)-2a^2b^2(5Af(16e+3fx)+Cx(15e^2+16}{}$$

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^4*f*(64*C*e + 32*B*f
 + 15*C*f*x)) - 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x +
5*f^2*x^2) + B*(40*e^2 + 30*e*f*x + 8*f^2*x^2)) + 4*b^4*x*(5*A*(6*e^2 + 8*e
*f*x + 3*f^2*x^2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^2 +
24*e*f*x + 10*f^2*x^2)))) + 30*a^2*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) +
 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(240*
b^5*Sqrt[a - b*x])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\left(-40 C f^{2} x^{5} b^{4}-48 B b^{4} f^{2} x^{4}-96 C b^{4} e f x^{4}-60 A b^{4} f^{2} x^{3}-120 B b^{4} e f x^{3}+10 C a^{2} b^{2} f^{2} x^{3}-60 C b^{4} e^{2} x^{3}-160 A b^{4} e f x^{2}+16 B a^{2} b^{2} f^{2} x^{2}\right.}{}$
default	$\frac{\sqrt{b x+a}\, \sqrt{c(-b x+a)}\, \left(60 C b^{4} e^{2} x^{3} \sqrt{b^{2} c}\, \sqrt{c(-b^{2} x^{2}+a^{2})}+80 B b^{4} e^{2} x^{2} \sqrt{b^{2} c}\, \sqrt{c(-b^{2} x^{2}+a^{2})}-80 B a^{2} b^{2} e^{2} \sqrt{b^{2} c}\, \sqrt{c(-b^{2} x^{2}+a^{2})}-64 C a^{4}\right.}{}$

[In] int((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETU RNVERBOSE)

[Out] -1/240/b^4*(-40*C*b^4*f^2*x^5-48*B*b^4*f^2*x^4-96*C*b^4*e*f*x^4-60*A*b^4*f^ 2*x^3-120*B*b^4*e*f*x^3+10*C*a^2*b^2*f^2*x^3-60*C*b^4*e^2*x^3-160*A*b^4*e*f *x^2+16*B*a^2*b^2*f^2*x^2-80*B*b^4*e^2*x^2+32*C*a^2*b^2*e*f*x^2+30*A*a^2*b^ 2*f^2*x-120*A*b^4*e^2*x+60*B*a^2*b^2*e*f*x+15*C*a^4*f^2*x+30*C*a^2*b^2*e^2* x+160*A*a^2*b^2*e*f+32*B*a^4*f^2+80*B*a^2*b^2*e^2+64*C*a^4*e*f)*(b*x+a)^(1/ 2)*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/16*a^2*(2*A*a^2*b^2*f^2+8*A*b^4*e^2+4*B* a^2*b^2*e*f+C*a^4*f^2+2*C*a^2*b^2*e^2)/b^4/(b^2*c)^(1/2)*arctan((b^2*c)^(1/ 2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c *(b*x-a))^(1/2)*c

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.56

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\sqrt{-bcx+ac}\sqrt{bx+a}b\right.}{}\right.$$
$$-\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(40\,Cb^5f\right.}{}$$

[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor ithm="fricas")

[Out] [1/480*(15*(4*B*a^4*b^2*e*f + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2* A*a^4*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^

```
4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f
 - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(
5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^
2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*s
qrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*e*f + 2*(C*a^
4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*A*a^4*b^2)*f^2)*sqrt(c)*arctan(sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40*C*b^5*f^2
*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4
 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*
C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3
 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 +
(C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2*(A + B*x + C*x**2),
 x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{Aa^2\sqrt{c}e^2\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{Ca^6\sqrt{c}f^2\arcsin\left(\frac{bx}{a}\right)}{16\,b^5} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^2x \\
&+ \frac{\sqrt{-b^2cx^2+a^2c}Ca^4f^2x}{16\,b^4} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^2x^3}{6\,b^2c} \\
&+ \frac{(Ce^2+2\,Bef+Af^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce^2+2\,Bef+Af^2)a^2x}{8\,b^2} \\
&- \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^2x}{8\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^2}{3\,b^2c} \\
&- \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Aef}{3\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,b^2c} \\
&- \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,b^2c} - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)a^2}{15\,b^4c}
\end{aligned}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*A*a^2*sqrt(c)*e^2*arcsin(b*x/a)/b + 1/16*C*a^6*sqrt(c)*f^2*arcsin(b*x/a
)/b^5 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*x + 1/16*sqrt(-b^2*c*x^2 + a^2*c
)*C*a^4*f^2*x/b^4 - 1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^2*x^3/(b^2*c) + 1/8*
(C*e^2 + 2*B*e*f + A*f^2)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x
^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*a^2*x/b^2 - 1/8*(-b^2*c*x^2 + a^2*c)^
(3/2)*C*a^2*f^2*x/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^2/(b^2*c) -
2/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*e*f/(b^2*c) - 1/5*(-b^2*c*x^2 + a^2*c)^(3/
2)*(2*C*e*f + B*f^2)*x^2/(b^2*c) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(C*e^2 +
2*B*e*f + A*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*(2*C*e*f + B*f
^2)*a^2/(b^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. $2(412) = 824$.

Time = 0.85 (sec) , antiderivative size = 1868, normalized size of antiderivative = 4.14

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^4*e^2 - 1
20*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^3*e^
2 - 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^4
*e^2 - 240*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*
a*b^3*e*f + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
 + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c
+ 2*a*c)*sqrt(b*x + a))*C*a*b^2*e^2 + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sq
rt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) +
 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^3*e^2 + 80*(6*a^3*c*l
og(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((
2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B
*a*b^2*e*f + 80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*
c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c
 + 2*a*c)*sqrt(b*x + a))*A*b^3*e*f + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqr
t(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) +
9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^2*f^2 - 10*(18*a^4*c
```

```
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
(39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*C*b^2*e^2 - 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*
a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*C*a*b*e*f - 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^2*e*f - 10*(18*a^4*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
(39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*
c + 2*a*c)*sqrt(b*x + a))*B*a*b*f^2 - 10*(18*a^4*c*log(abs(-sqrt(b*x + a)*s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*
a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*A*b^2*f^2 + 4*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*
c + 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a)
 + 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))
*C*b*e*f + 2*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
+ 2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) +
 133*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C
*a*f^2 + 2*(90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 1
33*a^2)*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b
*f^2 - (150*a^6*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - (405*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x
+ a) + 321*a^2)*(b*x + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)
*sqrt(b*x + a))*C*f^2)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

```
[In] int((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.22 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 300

$$\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{(4Ab^2e+a^2(Ce+Bf))\,x\sqrt{a+bx}\sqrt{ac-bcx}}{8b^2}-\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\,(a^2-b^2x^2)}{5b^2f} \\
&\quad-\frac{\sqrt{a+bx}\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Be+Af)))-3b^2f(3Ce-5Bf)x)\,(a^2-b^2x^2)}{60b^4f} \\
&\quad+\frac{a^2\sqrt{c}(4Ab^2e+a^2(Ce+Bf))\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}
\end{aligned}$$

```
[Out] 1/8*(4*A*b^2*e+a^2*(B*f+C*e))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/5*C*
(f*x+e)^2*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/f-1/60*(8*a^2
*C*f^2-4*b^2*(3*C*e^2-5*f*(A*f+B*e))-3*b^2*f*(-5*B*f+3*C*e)*x)*(-b^2*x^2+a^
2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/f+1/8*a^2*(4*A*b^2*e+a^2*(B*f+C*e))
*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x
+a*c)^(1/2)/b^3/(-b^2*c*x^2+a^2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{1624, 1668, 794, 201, 223, 209\}$

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)(a^2(Bf+Ce)+4Ab^2e)}{8b^3\sqrt{a^2c-b^2cx^2}} + \frac{1}{8}x\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{a^2(Bf+Ce)}{b^2}+4Ae\right) - \frac{\sqrt{a+bx}(a^2-b^2x^2)\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Af+Be)))-3b^2fx(3Ce-5Bf))}{60b^4f} - \frac{C\sqrt{a+bx}(a^2-b^2x^2)(e+fx)^2\sqrt{ac-bcx}}{5b^2f}$$

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A*e + (a^2*(C*e + B*f))/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (C*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(5*b^2*f) - (S
qrt[a + b*x]*Sqrt[a*c - b*c*x]*(4*(2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e +
A*f))) - 3*b^2*f*(3*C*e - 5*B*f)*x)*(a^2 - b^2*x^2))/(60*b^4*f) + (a^2*Sqrt
[c]*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
```

```
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int (e+fx)\sqrt{a^2c-b^2cx^2}\left(A+Bx+Cx^2\right)\,dx}{\sqrt{a^2c-b^2cx^2}} \\
&= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} \\
&\quad -\frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int (e+fx)\left(-c\left(5Ab^2+2a^2C\right)f^2+b^2cf(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}\,dx}{5b^2cf^2\sqrt{a^2c-b^2cx^2}} \\
&= -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} \\
&\quad -\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(4\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Be+Af)\right)\right)-3b^2f(3Ce-5Bf)x\right)\left(a^2-b^2x^2\right)}{60b^4f} \\
&\quad +\frac{\left(\left(4Ab^2e+a^2(Ce+Bf)\right)\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\sqrt{a^2c-b^2cx^2}\,dx}{4b^2\sqrt{a^2c-b^2cx^2}}
\end{aligned}
$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Be+Af)))-3b^2f(3Ce-5Bf)x)\left(a^2-b^2x^2\right)}{60b^4f} + \frac{\left(a^2c(4Ab^2e+a^2(Ce+Bf))\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{8b^2\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Be+Af)))-3b^2f(3Ce-5Bf)x)\left(a^2-b^2x^2\right)}{60b^4f} + \frac{\left(a^2c(4Ab^2e+a^2(Ce+Bf))\sqrt{a+bx}\sqrt{ac-bcx}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^2\sqrt{a^2c-b^2cx^2}}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{C\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\sqrt{a+bx}\sqrt{ac-bcx}(4(2a^2Cf^2-b^2(3Ce^2-5f(Be+Af)))-3b^2f(3Ce-5Bf)x)\left(a^2-b^2x^2\right)}{60b^4f} + \frac{a^2\sqrt{c}(4Ab^2e+a^2(Ce+Bf))\sqrt{a+bx}\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.61

$$\int\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt{c(a-bx)}\left(\sqrt{a-bx}\sqrt{a+bx}(-16a^4Cf-a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))+2b^4x(10A(3\right.}{120b^4\sqrt{a-}}$$

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
[Out] (Sqrt[c*(a - b*x)]*(Sqrt[a - b*x]*Sqrt[a + b*x]*(-16*a^4*C*f - a^2*b^2*(40*
A*f + 5*B*(8*e + 3*f*x) + C*x*(15*e + 8*f*x)) + 2*b^4*x*(10*A*(3*e + 2*f*x)
 + x*(5*B*(4*e + 3*f*x) + 3*C*x*(5*e + 4*f*x)))) + 30*a^2*b*(4*A*b^2*e + a^
2*(C*e + B*f))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(120*b^4*Sqrt[a - b*x]
)
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{(-24fC x^4 b^4 - 30B b^4 f x^3 - 30C b^4 e x^3 - 40A b^4 f x^2 - 40B b^4 e x^2 + 8C a^2 b^2 f x^2 - 60A b^4 e x + 15B a^2 b^2 f x + 15C a^2 b^2 e x + 40A a^2 f b^2 +}{120 b^4 \sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(24C b^4 f x^4 \sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+30B b^4 f x^3 \sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+30C b^4 e x^3 \sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+60A \operatorname{arct}\right.}{}$

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/120*(-24*C*b^4*f*x^4-30*B*b^4*f*x^3-30*C*b^4*e*x^3-40*A*b^4*f*x^2-40*B*b
^4*e*x^2+8*C*a^2*b^2*f*x^2-60*A*b^4*e*x+15*B*a^2*b^2*f*x+15*C*a^2*b^2*e*x+4
0*A*a^2*b^2*f+40*B*a^2*b^2*e+16*C*a^4*f)*(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*(b*
x-a))^(1/2)*c+1/8*a^2/b^2*(4*A*b^2*e+B*a^2*f+C*a^2*e)/(b^2*c)^(1/2)*arctan(
(b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a
)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.47

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$= \left[\frac{15\left(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(24\,Cb^4fx^4\right.}{}\right.$$

$$-\frac{15\left(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(24\,Cb^4fx^4-40\,Ba^2b^2e+30\,(Cb^4e+\right.}{}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2
 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f
*x^4 - 40*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*
b^2 - 5*A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*
a^2*b^2 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120*(15*
(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*s
```

```
qrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 - 40*B*a^2*
b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*b^2 - 5*A*b^4)*f
)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*a^2*b^2 - 4*A*b^
4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{-c\,(-a+bx)}\sqrt{a+bx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.83

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{Aa^2\sqrt{c}e\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}Aex+\frac{(Ce+Bf)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}$$
$$+\frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)a^2x}{8\,b^2}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cfx^2}{5\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be}{3\,b^2c}$$
$$-\frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f}{15\,b^4c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Af}{3\,b^2c}-\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce+Bf)x}{4\,b^2c}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*A*a^2*sqrt(c)*e*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e*x +
1/8*(C*e + B*f)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x^2 + a^2*c
)*(C*e + B*f)*a^2*x/b^2 - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f*x^2/(b^2*c) -
1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2
)*C*a^2*f/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*f/(b^2*c) - 1/4*(-b^2*
c*x^2 + a^2*c)^(3/2)*(C*e + B*f)*x/(b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(267) = 534$.

Time = 0.63 (sec) , antiderivative size = 1142, normalized size of antiderivative = 3.81

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/120*(120*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^3*e - 60*
(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqr
t(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^2*e - 6
0*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^3*e - 6
0*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/s
qrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a*b^2*f +
 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sq
rt(b*x + a))*C*a*b*e + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-
(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(
b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^2*e + 20*(6*a^3*c*log(abs(-sqrt(b*x
+ a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x
 + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a*b*f + 20*(6*a^
3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
 - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x +
a))*A*b^2*f - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b
*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*b*e - 5*(18*a^4*c*log(
abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a
^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2
*a*c)*sqrt(b*x + a))*C*a*f - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) +
sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x +
a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b*f + (
90*a^5*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqr
t(-c) - (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x
 + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*f)/b^4
```

Mupad [B] (verification not implemented)

Time = 37.00 (sec) , antiderivative size = 1765, normalized size of antiderivative = 5.88

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] int((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*a^4*c^8*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^
(1/2))) - (B*a^4*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^
(1/2) - a^(1/2))^15) - (35*B*a^4*c^7*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*B*a^4*c^6*f*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*B*a^4*c^5*f*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (7
15*B*a^4*c^4*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) -
 a^(1/2))^9) - (273*B*a^4*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*
((a + b*x)^(1/2) - a^(1/2))^11) + (35*B*a^4*c^2*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28
*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))
^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) -
a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^
(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((
a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) - (a*c - b*c*x)^(1/2)*((2*C*
a^4*f*(a + b*x)^(1/2))/(15*b^4) - (C*f*x^4*(a + b*x)^(1/2))/5 + (C*a^2*f*x^
2*(a + b*x)^(1/2))/(15*b^2)) + ((C*a^4*c^8*e*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(2*((a + b*x)^(1/2) - a^(1/2))) - (C*a^4*c*e*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^15)/(2*((a + b*x)^(1/2) - a^(1/2))^15) - (35*C*a^4*c^7*e*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*
C*a^4*c^6*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^
(1/2))^5) - (715*C*a^4*c^5*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a
+ b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c^4*e*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9) - (273*C*a^4*c^3*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/2) - a^(1/2))^11) + (35*C*a^4*
c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2)
)^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(
1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a
+ b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c
```

```
*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*
b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^1
4) + (A*e*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (A*f*(a^2 - b^2*x^2)*(
a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*e*(a^2 - b^2*x^2)*(a*c - b
*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*a^4*c^(1/2)*f*atan(((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(2*b^3) - (C*
a^4*c^(1/2)*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*e*log((-b*c)^(1/2)*(c*(a -
 b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.23 $\int \sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} \\
&\quad -\frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\
&\quad +\frac{a^2\sqrt{c}(4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}
\end{aligned}$$

```
[Out] 1/8*(4*A+a^2*C/b^2)*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)-1/3*B*(-b^2*x^2+a^2)
*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2-1/4*C*x*(-b^2*x^2+a^2)*(b*x+a)^(1/2)*
(-b*c*x+a*c)^(1/2)/b^2+1/8*a^2*(4*A*b^2+C*a^2)*arctan(b*x*c^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*c^(1/2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/(-b^2*c*x^2+a
^2*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

= {915, 1829, 655, 201, 223, 209}

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{a^2\sqrt{c}\sqrt{a+bx}(a^2C+4Ab^2)\sqrt{ac-bcx}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}$$
$$+\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}$$
$$-\frac{B\sqrt{a+bx}(a^2-b^2x^2)\sqrt{ac-bcx}}{3b^2}-\frac{Cx\sqrt{a+bx}(a^2-b^2x^2)\sqrt{ac-bcx}}{4b^2}$$

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A + (a^2*C)/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (B*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(a^2 - b^2*x^2))/(3*b^2) - (C*x*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(a^2 - b^2*x^2))/(4*b^2) + (a^2*Sqrt[c]*(4*A*b^2 + a^2*C)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b
^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\sqrt{a^2c-b^2cx^2}(A+Bx+Cx^2)\,dx}{\sqrt{a^2c-b^2cx^2}} \\
&= -\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\
&\quad -\frac{\left(\sqrt{a+bx}\sqrt{ac-bcx}\right)\int(-c(4Ab^2+a^2C)-4b^2Bcx)\sqrt{a^2c-b^2cx^2}\,dx}{4b^2c\sqrt{a^2c-b^2cx^2}} \\
&= -\frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\
&\quad +\frac{\left((4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\sqrt{a^2c-b^2cx^2}\,dx}{4b^2\sqrt{a^2c-b^2cx^2}} \\
&= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2} \\
&\quad -\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\
&\quad +\frac{\left(a^2c(4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{8b^2\sqrt{a^2c-b^2cx^2}} \\
&= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2} \\
&\quad -\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} \\
&\quad +\frac{\left(a^2c(4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^2\sqrt{a^2c-b^2cx^2}}
\end{aligned}$$

$$= \frac{1}{8}\left(4A + \frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{3b^2} - \frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{4b^2} + \frac{a^2\sqrt{c}(4Ab^2+a^2C)\sqrt{a+bx}\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\Big(b\sqrt{a-bx}\sqrt{a+bx}(-a^2(8B+3Cx)+2b^2x(6A+x(4B+3Cx)))+6a^2(4Ab^2+a^2C)\arctan\Big(}{24b^3\sqrt{a-bx}}$$

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^2*(8*B + 3*C*x)) + 2
*b^2*x*(6*A + x*(4*B + 3*C*x))) + 6*a^2*(4*A*b^2 + a^2*C)*ArcTan[Sqrt[a + b
*x]/Sqrt[a - b*x]]))/(24*b^3*Sqrt[a - b*x])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\left(6C\,b^2x^3+8b^2B\,x^2+12A\,b^2x-3C\,a^2x-8a^2B\right)\sqrt{bx+a}\,(-bx+a)c}{24b^2\sqrt{-c(bx-a)}}+\frac{a^2\left(4b^2A+C\,a^2\right)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}\,c}{8b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6C\,b^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2c+8B\,b^2x^2\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+3C\arctan\right.}{24\sqrt{c(-b^2x^2+a^2)}\,b^2\sqrt{}}$

```
[In] int((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*C*b^2*x^3+8*B*b^2*x^2+12*A*b^2*x-3*C*a^2*x-8*B*a^2)*(b*x+a)^(1/2)/b
^2*(-b*x+a)/(-c*(b*x-a))^(1/2)*c+1/8*a^2*(4*A*b^2+C*a^2)/b^2/(b^2*c)^(1/2)*
arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)
/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,\right.}{48\,b^3}\right.$$
$$-\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+ab}\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\,Ab^3\right)x\right.}{24\,b^3}$$

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a
*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*
B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3, -
1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B*a^2*
b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx=\frac{Ca^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{Aa^2\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ax+\frac{\sqrt{-b^2cx^2+a^2c}Ca^2x}{8\,b^2}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cx}{4\,b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}B}{3\,b^2c}$$

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/8*C*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)/b + 1
/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x + 1/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*x/b^2
- 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*C*x/(b^2*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/
2)*B/(b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(191) = 382$.

Time = 0.43 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.38

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)\ dx =$$

$$-\frac{24\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\right)Aab^2-12\left(\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\cdots\right.\right.}{\sqrt{-c}}\right.}{\cdots}$$

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^2 - 12*(2*a
^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c
) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b - 12*(2*a^2
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
+ sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^2 + 4*(6*a^3*c*
log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (
(2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*
C*a + 4*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c
)*sqrt(b*x + a))*B*b - (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b
*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*
a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C)/b^3
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 876, normalized size of antiderivative = 3.96

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\frac{C\,a^4\,c^8\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}-\frac{C\,a^4\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{15}}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{15}}-\frac{35\,C\,a^4\,c^7\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}+\frac{273\,C\,a^4\,c^6\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}-\frac{715\,C\,a^4}{2}}{b^3\,c^8+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{16}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{16}}+\frac{8\,b^3\,c^7\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{28\,b^3\,c^6\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{56\,b^3\,c^5\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}+\frac{7}{}}$$
$$+\frac{A\,x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{B\,\left(a^2-b^2\,x^2\right)\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{3\,b^2}$$
$$-\frac{C\,a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{2\,b^3}-\frac{A\,a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((C*a^4*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^(1
/2))) - (C*a^4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^(1/2
) - a^(1/2))^15) - (35*C*a^4*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*
((a + b*x)^(1/2) - a^(1/2))^3) + (273*C*a^4*c^6*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*C*a^4*c^5*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c
^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9)
 - (273*C*a^4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/
2) - a^(1/2))^11) + (35*C*a^4*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(
2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70
*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))
^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) -
 a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*
x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(
(a + b*x)^(1/2) - a^(1/2))^14) + (A*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/
2 - (B*(a^2 - b^2*x^2)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (C*a^
4*c^(1/2)*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2
) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*log((-b*c)^(1/2)*(c*(a - b*x))
^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.24 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} dx$

Optimal result

Integrand size = 40, antiderivative size = 278

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] -C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan(
b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2
)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a^2*
f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a
^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used

= {1624, 1668, 858, 223, 209, 739, 210}

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)}\,dx$$
$$= \frac{\sqrt{a^2c - b^2cx^2}(Af^2 - Bef + Ce^2)\arctan\left(\frac{\sqrt{c}(a^2f + b^2ex)}{\sqrt{a^2c - b^2cx^2}\sqrt{b^2e^2 - a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a + bx}\sqrt{ac - bcx}\sqrt{b^2e^2 - a^2f^2}}$$
$$- \frac{\sqrt{a^2c - b^2cx^2}(Ce - Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c - b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a + bx}\sqrt{ac - bcx}} - \frac{C(a^2 - b^2x^2)}{b^2f\sqrt{a + bx}\sqrt{ac - bcx}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2c-b^2cx^2}\int\frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\sqrt{a^2c-b^2cx^2}\int\frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad+\frac{\left((Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad-\frac{\left((Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx,x,\frac{a^2cf+b^2cex}{\sqrt{a^2c-b^2cx^2}}\right)}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}$$

$$= -\frac{C(a^2 - b^2x^2)}{b^2 f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2} - \frac{2(Ce-Bf)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} + \frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] ((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[S
qrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x
]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/
(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^2\sqrt{-c(bx-a)}} + \frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{f\sqrt{b^2c}} - \frac{\left(A\,f^2-Bef+C\,e^2\right)\ln\left(\frac{\frac{2c\left(a^2f^2-b^2e^2\right)}{f^2}+\frac{2b^2ce\left(x+\frac{e}{f}\right)}{f}+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}{x+}\right.}{f^2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}}\right.}{f\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\,\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2c\,f^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\,\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)\right.}{}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -C*(b*x+a)^(1/2)*(-b*x+a)/f/b^2/(-c*(b*x-a))^(1/2)+1/f*((B*f-C*e)/f/(b^2*c)
^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-(A*f^2-B*e*f+C*e^2)
/f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*
e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-b^2*c*(x+e/f)^2+2*b^2*c*e/f
*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a))^(1/
2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c(-a+bx)}\sqrt{a+bx}\,(e+fx)}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 56.99 (sec) , antiderivative size = 9298, normalized size of antiderivative = 33.45

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
```

) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) + (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(

1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b
^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8
e^6((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2))
 + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
+ b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a*e*(
(4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*
(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2)
- 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*B^2*
a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(24*B*
a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3
/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1
/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^
6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2
) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f
^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x
)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 9
0*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) -
a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2
)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (
4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144*B^2
*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a^4*c
^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/
2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (917504*B^4*a^4*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*((a + b*x)^(1/2) - a^(1/2))^2)))*2i
)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) - (C*e^2*atan(((C*e^2*((4096*(32*C^
3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2
)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e
^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2
) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((409
6*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/
2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))
+ (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b
^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*
f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f
^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^
(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^
4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*
((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (409

6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) + (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)))/((131072*C^4*a^4*c^5)/(b^8*f^4) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)

^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^
2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*
a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(
b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4
*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*
c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32
*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(
3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))
)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e
^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^
4) - (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e
^4*f^4) + (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2
)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^
7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*
f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6)
)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2
)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096
*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)
^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 +
 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)
+ (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*
c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x
)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C
^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1
/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f
^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (91
7504*C^4*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))*2i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4*B*atan(
(67108864*B^5*a^16*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)
^(1/2) - a^(1/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c
^(15/2)*e^4 - 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) + (37748736*B^5*a^1
2*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1
/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 -
 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) - (100663296*B^5*a^14*b^2*c^7*e^
2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67
108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 - 1006632

```
96*B^5*a^14*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f) - (A*a*atan((a*c*(a*c -
b*c*x)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - (a*c)^(3/2)*(a^4*c*f^2
- a^2*b^2*c*e^2)^(1/2)*1i + a*c*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/
2)*1i + b*c*x*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - a^(1/2)*c*
(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*(a + b*x)^(1/2)*2i)/(2*a^(5/2
)*b*c^2*e - 2*a^3*c^2*f*(a + b*x)^(1/2) - 2*a^2*b*c^2*e*(a + b*x)^(1/2) + 2
*a^(5/2)*b*c^2*f*x + 2*a^(5/2)*c*f*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) - 2*a^(3
/2)*b*c*e*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) + 2*a*b*c*e*(a*c - b*c*x)^(1/2)*(
a*c)^(1/2)*(a + b*x)^(1/2)))*2i)/(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2) + (4*C*e
*atan((67108864*C^5*a^8*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a +
 b*x)^(1/2) - a^(1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^
4*c^(15/2)*e^4 - 100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) + (37748736*C^5*a
^4*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 -
100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) - (100663296*C^5*a^6*b^2*c^7*e^2*f
^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67108
864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 - 100663296*C^
5*a^6*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f^2) - (8*C*a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*f*((a + b*x)^(1/2) - a^(1/2))^2*
(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2
+ (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
))
```

3.25 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 322

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx
&= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad + \frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

```
[Out] f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(1
/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2
*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*f^2
*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^
2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^2/(-a
^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

$= \{1624, 1665, 858, 223, 209, 739, 210\}$

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$
$$= \frac{\sqrt{a^2c-b^2cx^2}(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{3/2}}$$
$$+ \frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)} + \frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{c(Ab^2e+a^2(Ce-Bf))+cC\left(\frac{b^2e^2}{f}-a^2f\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{c\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+ \frac{\left(\left(-cCe\left(\frac{b^2e^2}{f}-a^2f\right)+cf(Ab^2e+a^2(Ce-Bf))\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{cf\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}$$
$$+ \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx, x, \frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{f\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$- \frac{\left(\left(-cCe\left(\frac{b^2e^2}{f}-a^2f\right)+cf(Ab^2e+a^2(Ce-Bf))\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx,\right.}{cf\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$=\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.71

$$\int\frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$
$$=\frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)}+\frac{C\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}-\frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] (2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a*
f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a -
b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*x]*
ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((
b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(290) = 580.

Time = 1.70 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1166

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*
f^4*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*
c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x
^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
```

```
)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^
2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^3*f*x*(b
^2*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^
(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x
/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*
ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4
*(b^2*c)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))
^(1/2)*(b^2*c)^(1/2)-C*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2
+a^2))^(1/2)*(b^2*c)^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2
+a^2))^(1/2)/(a*f+b*e)/(b^2*c)^(1/2)/(a*f-b*e)/(f*x+e)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx=\text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-c\,(-a+bx)}\sqrt{a+bx}\,(e+fx)^2}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^2} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.63

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^2} \, dx$$
$$= \frac{\frac{2\left(Cb^3\sqrt{-c}e^3 - 2\,Ca^2b\sqrt{-c}ef^2 - Ab^3\sqrt{-c}ef^2 + Ba^2b\sqrt{-c}f^3\right)\arctan\left(-\frac{2\,bce - \left(\sqrt{bx+a}\sqrt{-c} - \sqrt{-(bx+a)c+2\,ac}\right)^2 f}{2\sqrt{-b^2e^2+a^2f^2c}}\right)}{(b^2e^2f^2 - a^2f^4)\sqrt{-b^2e^2+a^2f^2c}} - \frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c} - \sqrt{-(bx}\right.\right.}{\sqrt{-c}f^2}}{}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] (2*(C*b^3*sqrt(-c)*e^3 - 2*C*a^2*b*sqrt(-c)*e*f^2 - A*b^3*sqrt(-c)*e*f^2 +
B*a^2*b*sqrt(-c)*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^2 -
a^2*f^4)*sqrt(-b^2*e^2 + a^2*f^2)*c) - C*log((sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(C*b^3*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3 - B*b^3*(sqrt(b*x + a)*sqrt(
-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^2*f - 2*C*a^2*b^2*sqrt(-c)*c
*e^2*f + A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt
(-c)*e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c*f^3)/((b
^2*e^2*f^2 - a^2*f^4)*(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^2*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f - 4
*a^2*c^2*f)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.26 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}$$
$$+\frac{\left(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af))\right)(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}$$
$$+\frac{\left(A(2b^4e^2+a^2b^2f^2)+a^2(2a^2Cf^2+b^2e(Ce-3Bf))\right)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}\left(b^2e^2-a^2f^2\right)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] 1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*x
+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(-3
*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(-3*B
*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^(1/2)/
(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {1624, 1665, 821, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)}$$
$$+\frac{(a^2-b^2x^2)\,(2a^2f^2(2Ce-Bf)-b^2(ef(Be-3Af)+Ce^3))}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^2}$$
$$+\frac{\sqrt{a^2c-b^2cx^2}(2a^4Cf^2+A(a^2b^2f^2+2b^4e^2)+a^2b^2e(Ce-3Bf))\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{5/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2c-b^2cx^2}\int\frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int\frac{2c(Ab^2e+a^2(Ce-Bf))-c\left(2a^2Cf-b^2\left(Be+\frac{Ce^2}{f}-Af\right)\right)x}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{2c\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af)))\,(a^2-b^2x^2)}{2f\,(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left((2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{2\,(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af)))\,(a^2-b^2x^2)}{2f\,(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} - \frac{\left((2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx,x\right.}{2\,(b^2e^2-a^2f^2)^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$
\begin{aligned}
&= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} \\
&\quad + \frac{(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af)))(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} \\
&\quad + \frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}\left(b^2e^2-a^2f^2\right)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.69

$$
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx
= \frac{\frac{(-a+bx)\sqrt{a+bx}(b^2e(Ce^2x+Be(2e+fx)-Af(4e+3fx))+a^2f(-Ce(3e+4fx)+f(Af+B(e+2fx))))}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+}{(be-af)^{5/}}}{\sqrt{c(a-bx)}}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] (((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e + 3
*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e -
 a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f
) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt
[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e + a*f)
^(5/2)))/Sqrt[c*(a - b*x)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. 2(333) = 666.

Time = 1.67 (sec) , antiderivative size = 1794, normalized size of antiderivative = 4.94

method	result	size
default	Expression too large to display	1794

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^2
*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b
```

```
^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*e*
f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c
*e^3*f+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2
*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*f^4*x^2+2*B*a^2*f^4*x*(c*(-b^2*x^2+a^2))
^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*A*b^2*e^2*f^2*(c*(-b^2*x^2+a^2))^(
1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*a^2*f^4*(c*(-b^2*x^2+a^2))^(1/2)*(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^
2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^4+2*C*ln(2*(b^2*
c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(
f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^4+B*a^2*e*f^3*(c*(-b^
2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^2*e^3*f*(c*(-b^2*x^
2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^2*e^2*f^2*(c*(-b^2*x^2+
a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*A*b^2*e*f^3*x*(c*(-b^2*x^2+a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*b^2*e^2*f^2*x*(c*(-b^2*x^2+a^2)
)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*C*a^2*e*f^3*x*(c*(-b^2*x^2+a^2))^
(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+C*b^2*e^3*f*x*(c*(-b^2*x^2+a^2))^(1/2
)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b
^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x^2
+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(
a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*
e*f^3*x-6*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^
2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2*x+2*C*ln(2*(b^2*c*e*x+a^2*c
*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2
*b^2*c*e^3*f*x)/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2+a^2))^(1/2)
/(a*f+b*e)/(a*f-b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 41.19 (sec) , antiderivative size = 1355, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2
)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 +
```

```
A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f
 - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*a^2*
b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f^2)*x
^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2*e^3*f^
2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*
A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4
*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*
sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*
b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^
4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3
*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2
 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a
^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2
*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sq
rt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f^2)*(b^2*e*x + a^2
*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^4*c*f^2 - (b^4*c*e^
2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*e^3*f^2 - B*a^4*e*f^4 -
 A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*A*a^2*b^2)*e^2*f^
3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4*f^5 - (5*C*a^2*b
^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*b^2*c*e^4*f^4 - a
^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^4*b^2*c*e^2*f^6 -
 a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^
5 - a^6*c*e*f^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(335) = 670$.

Time = 0.71 (sec) , antiderivative size = 1425, normalized size of antiderivative = 3.93

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -((C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2 - 3*B*a^2*b^3*sqrt(-c)*e*f
 + 2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2)*arctan(-1/2*(2*b*c*e -
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 +
 a^2*f^2)*c))/((b^4*e^4 - 2*a^2*b^2*e^2*f^2 + a^4*f^4)*sqrt(-b^2*e^2 + a^2*
f^2)*c) + 2*(4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^
4*sqrt(-c)*c*e^5 - 2*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^6*sqrt(-c)*e^4*f + 4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*
c + 2*a*c))^4*sqrt(-c)*c*e^4*f - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^4*f - 14*C*a^2*b^4*(sqrt(b*x + a)*
sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 12*A*b^6*(sqr
t(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 16
*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)
*c^2*e^3*f^2 + 8*C*a^4*b^4*sqrt(-c)*c^3*e^3*f^2 + 5*C*a^2*b^3*(sqrt(b*x + a
)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 + 2*A*b^5*(sqrt
(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 + 10*B*
a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*
e^2*f^3 + 44*C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*c^2*e^2*f^3 + 40*A*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 8*B*a^4*b^4*sqrt(-c)*c^3*e^2*f^
```

```
3 - 3*B*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqr
t(-c)*e*f^4 - 8*C*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^4*sqrt(-c)*c*e*f^4 - 6*A*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^4*sqrt(-c)*c*e*f^4 - 20*B*a^4*b^3*(sqrt(b*x + a)*sqrt(-c)
- sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e*f^4 - 32*C*a^6*b^2*sqrt(-c)*
c^3*e*f^4 - 24*A*a^4*b^4*sqrt(-c)*c^3*e*f^4 + A*a^2*b^3*(sqrt(b*x + a)*sqrt
(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^5 + 4*B*a^4*b^2*(sqrt(b*x +
 a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^5 - 4*A*a^4*b^3*(
sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^5 + 1
6*B*a^6*b^2*sqrt(-c)*c^3*f^5)/((b^4*e^4*f^2 - 2*a^2*b^2*e^2*f^4 + a^4*f^6)*
(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c*e - (sqrt(b*
x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f - 4*a^2*c^2*f)^2))/b
```

Mupad [B] (verification not implemented)

Time = 108.41 (sec) , antiderivative size = 9344, normalized size of antiderivative = 25.74

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
```

c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c *f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^ (1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2* b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4 - 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a ^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^ 3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5* f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^ 2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a ^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)) ^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4 *c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c *x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2 *b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6* f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^ (1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c *f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3 *f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1 /2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1 /2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a* c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2 *f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2 *b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^ (1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1 /2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^ 4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2* a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^ 4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4* c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/ 2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) - (6*B*a^2*b*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(a^4*f^4 +

b^4*e^4 - 2*a^2*b^2*e^2*f^2)) + (6*B*a^2*b*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(a^4*f^4 + b^4*e^4 - 2*a^2*b^2*e^2*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (C*a^2*(2*a^2*f^2 + b^2*e^2)*(2*atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*atan((((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 + 4*C^2*a^6*b^2*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (C*a^(3/2)*(2*a^2*f^2 + b^2*e^2)*(8*C*a^(17/2)*f^7*(a*c)^(1/2) - 12*C*a^(13/2)*b^2*e^2*f^5*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*e^6*f*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2)/(b*e*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)) - (C*a^(3/2)*(2*a^2*f^2 + b^2*e^2)*(8*C*a^(17/2)*c*f^7*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*c*e^6*f*(a*c)^(1/2) - 12*C*a^(13/2)*b^2*c*e^2*f^5*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))))/((a + b*x)^(1/2) - a^(1/2)) - ((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 + 4*C^2*a^6*b^2*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*

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(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f
^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*
e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6
*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b
^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*C^2*a^(9/2)*f*(a*c)^(1/2)*(2*a^2*f^2 + b^
2*e^2)^2)/(b^2*c*e^2*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3
/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
- ((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^1
0*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^
2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2
*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*
f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^
2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4
*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2
)^(1/2)))*(b^10*e^10*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^8*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^6*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^4*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2) + a^8*b^2*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2)))/(16*
C^2*a^8*f^4 + 4*C^2*a^4*b^4*e^4 + 16*C^2*a^6*b^2*e^2*f^2))))/(2*(a*f + b*e)
^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b^2*(a^2*f^2 + 2*b^2*e
^2)*(2*atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/
((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b
^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*atan((((((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))*(((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b
^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*
b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c
^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^
4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 -
b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e
^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*
A^2*b^3*(a^2*f^2 + 2*b^2*e^2)^2)/(e*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2
- a^2*c*f^2)^(3/2)) - (A*b*(a^2*f^2 + 2*b^2*e^2)*(4*A*a^(13/2)*b^2*c*f^7*(a
*c)^(1/2) + 8*A*a^(1/2)*b^8*c*e^6*f*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*c*e^4*f^
3*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*
(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^
6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))))/((a + b*x)^(1/2) - a^(1/2))
 + ((((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e
^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e
^2*f^8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10
+ 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a
^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b*(a^2*f^2 + 2
*b^2*e^2)*(4*A*a^(13/2)*b^2*f^7*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*e^4*f^3*(a*c
)^(1/2) + 8*A*a^(1/2)*b^8*e^6*f*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2
```

)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*
a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 - ((
((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e^10 -
 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^
8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*
a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^
2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^
10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^
2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*A^2
*a^(1/2)*b^2*f*(a*c)^(1/2)*(a^2*f^2 + 2*b^2*e^2)^2)/(c*e^2*(a*f + b*e)^4*(a
*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - ((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*
c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^
6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^
2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^
4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f
+ b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f
^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*
c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))*(b^8*e^10*(a^2*c*f^2 - b^2*
c*e^2) + a^8*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^6*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^4*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^2*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2)))/(16*A^2*b^6*e^4 + 4*A^2*a^4*b^2*f^4 + 16*A^2*a
^2*b^4*e^2*f^2))))/(2*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(
1/2)) + (3*B*a^2*b^2*e*f*(2*atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^
2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)
f(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2
))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/
2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)
*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(
1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - 2*atan(((((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2
)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/(2*(a*f +
 b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2))

3.27 $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 501

$$\begin{aligned}
&\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx \\
&= -\frac{(16a^2Cf^2-b^2(3Ce^2-5f(3Be+4Af)))\,(e+fx)^2\,(a^2-b^2x^2)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad+\frac{(Ce-5Bf)(e+fx)^3\,(a^2-b^2x^2)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C(e+fx)^4\,(a^2-b^2x^2)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad-\frac{(4(16a^4Cf^4+4a^2b^2f^2(13Ce^2+5f(3Be+Af))-b^4e^2(3Ce^2-5f(3Be+16Af)))+b^2f(a^2f^2(71Ce+\cdots}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad+\frac{(4A(2b^4e^3+3a^2b^2ef^2)+a^2(3a^2f^2(3Ce+Bf)+4b^2e^2(Ce+3Bf)))\,\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}$$

```
[Out] -1/60*(16*a^2*C*f^2-b^2*(3*C*e^2-5*f*(4*A*f+3*B*e)))*(f*x+e)^2*(-b^2*x^2+a^
2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/20*(-5*B*f+C*e)*(f*x+e)^3*(-b^2
*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/5*C*(f*x+e)^4*(-b^2*x^2+
a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/120*(64*a^4*C*f^4+16*a^2*b^2*
f^2*(13*C*e^2+5*f*(A*f+3*B*e))-4*b^4*e^2*(3*C*e^2-5*f*(16*A*f+3*B*e))+b^2*f
*(a^2*f^2*(45*B*f+71*C*e)-2*b^2*e*(3*C*e^2-5*f*(10*A*f+3*B*e)))*x)*(-b^2*x^
2+a^2)/b^6/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/8*(4*A*(3*a^2*b^2*e*f^2+2*b
^4*e^3)+a^2*(3*a^2*f^2*(B*f+3*C*e)+4*b^2*e^2*(3*B*f+C*e)))*arctan(b*x*c^(1/
2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^5/c^(1/2)/(b*x+a)^(
1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1624, 1668, 847, 794, 223, 209}

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{(a^2-b^2x^2)\,(e+fx)^2\left(-\frac{16a^2Cf^2}{b^2}-5f(4Af+3Be)+3Ce^2\right)}{60b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{(a^2-b^2x^2)\,(e+fx)^3(Ce-5Bf)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)\,(e+fx)^4}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)(3a^4f^2(Bf+3Ce)+4A(3a^2b^2ef^2+2b^4e^3)+4a^2b^2e^2(3Bf+Ce))}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$-\frac{(a^2-b^2x^2)\,(b^2fx(a^2f^2(45Bf+71Ce)-b^2(6Ce^3-10ef(10Af+3Be)))+4(16a^4Cf^4+4a^2b^2f^2(5f(}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((3*C*e^2 - (16*a^2*C*f^2)/b^2 - 5*f*(3*B*e + 4*A*f))*(e + f*x)^2*(a^2 - b^
2*x^2))/(60*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e - 5*B*f)*(e + f*
x)^3*(a^2 - b^2*x^2))/(20*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - (C*(e +
f*x)^4*(a^2 - b^2*x^2))/(5*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - ((4*(16
*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2 + 5*f*(3*B*e + A*f)) - b^4*e^2*(3*C*e^
2 - 5*f*(3*B*e + 16*A*f))) + b^2*f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^
3 - 10*e*f*(3*B*e + 10*A*f)))*x)*(a^2 - b^2*x^2))/(120*b^6*f*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]) + ((3*a^4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f
) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*e*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sq
rt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^4(a^2-b^2x^2)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3(-c(5Ab^2+4a^2C)f^2+b^2cf(Ce-5Bf)x)}{\sqrt{a^2c-b^2cx^2}}\,dx}{5b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(b^2c^2f^2\left(20Ab^2e+a^2(13Ce+15Bf)\right)+b^2c^2f\left(4\left(5Ab^2+4a^2C\right)f^2-3b^2e(Ce-5Bf)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{20b^4c^2f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(-b^2c^3f^2\left(32a^4Cf^2+3a^2b^2e(11Ce+25Bf)+20A\left(3b^4e^2+2a^2b^2f^2\right)\right)-b^4c^3f\left(a^2f^2(71Ce+45Bf)-b^2\left(\right.\right.\right.}{\sqrt{a^2c-b^2cx^2}}}{60b^6c^3f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(16a^4Cf^4+4a^2b^2f^2\left(13Ce^2+5f(3Be+Af)\right)-b^4e^2\left(3Ce^2-5f(3Be+16Af)\right)\right)+b^2f\left(a^2f^2\right.\right.}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(3a^4f^2(3Ce+Bf)+4a^2b^2e^2(Ce+3Bf)+4A\left(2b^4e^3+3a^2b^2ef^2\right)\right)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}}{8b^4\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(16a^4Cf^4+4a^2b^2f^2\left(13Ce^2+5f(3Be+Af)\right)-b^4e^2\left(3Ce^2-5f(3Be+16Af)\right)\right)+b^2f\left(a^2f^2\right.\right.}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(3a^4f^2(3Ce+Bf)+4a^2b^2e^2(Ce+3Bf)+4A\left(2b^4e^3+3a^2b^2ef^2\right)\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{}{1+\right.}{8b^4\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(16a^4Cf^4+4a^2b^2f^2\left(13Ce^2+5f(3Be+Af)\right)-b^4e^2\left(3Ce^2-5f(3Be+16Af)\right)\right)+b^2f\left(a^2f^2\right.\right.}{120b^6f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(3a^4f^2(3Ce+Bf)+4a^2b^2e^2(Ce+3Bf)+4A\left(2b^4e^3+3a^2b^2ef^2\right)\right)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}}{\sqrt{a^2c-b}}\right.}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.56

$$\int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-\big((a-bx)\sqrt{a+bx}(64a^4Cf^3+a^2b^2f(5f(48Be+16Af+9Bfx)+C(240e^2+135efx+32f^2x^2))+2b^4(}{}$$

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-((a - b*x)*Sqrt[a + b*x]*(64*a^4*C*f^3 + a^2*b^2*f*(5*f*(48*B*e + 16*A*f
+ 9*B*f*x) + C*(240*e^2 + 135*e*f*x + 32*f^2*x^2)) + 2*b^4*(10*A*f*(18*e^2
+ 9*e*f*x + 2*f^2*x^2) + 15*B*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) +
 3*C*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*b*(3*a^4*f^
2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*
e*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(120*b^6*Sqrt[c*
(a - b*x)])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(24C\,f^3x^4b^4+30B\,b^4f^3x^3+90C\,b^4e\,f^2x^3+40A\,b^4f^3x^2+120B\,b^4e\,f^2x^2+32C\,a^2b^2f^3x^2+120C\,b^4e^2f\,x^2+180A\,b^4e\,f^2x+45B\,a^2b^2f^3}{12}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\,\Big(-24C\,b^4f^3x^4\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-30B\,b^4f^3x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-90C\,b^4e\,f^2x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+180}{}$

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/120*(24*C*b^4*f^3*x^4+30*B*b^4*f^3*x^3+90*C*b^4*e*f^2*x^3+40*A*b^4*f^3*x
^2+120*B*b^4*e*f^2*x^2+32*C*a^2*b^2*f^3*x^2+120*C*b^4*e^2*f*x^2+180*A*b^4*e
*f^2*x+45*B*a^2*b^2*f^3*x+180*B*b^4*e^2*f*x+135*C*a^2*b^2*e*f^2*x+60*C*b^4*
e^3*x+80*A*a^2*b^2*f^3+360*A*b^4*e^2*f+240*B*a^2*b^2*e*f^2+120*B*b^4*e^3+64
*C*a^4*f^3+240*C*a^2*b^2*e^2*f)*(b*x+a)^(1/2)/b^6*(-b*x+a)/(-c*(b*x-a))^(1/
2)+1/8*(12*A*a^2*b^2*e*f^2+8*A*b^4*e^3+3*B*a^4*f^3+12*B*a^2*b^2*e^2*f+9*C*a
^4*e*f^2+4*C*a^2*b^2*e^3)/b^4/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*
x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.40

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{15\,(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\,(Ca^2b^3+2\,Ab^5)e^3+3\,(3\,Ca^4b+4\,Aa^2b^3)ef^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{\right.}}{}\right.$$
$$-\frac{15\,(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\,(Ca^2b^3+2\,Ab^5)e^3+3\,(3\,Ca^4b+4\,Aa^2b^3)ef^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx-}}{b^2cx^2-a^2}\right.}{}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/240*(15*(12*B*a^2*b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e
^3 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 + 12
0*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*
(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*
b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4
*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*
x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(12*B*a^2*b^3*e^2*
f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e^3 + 3*(3*C*a^4*b + 4*A*a^2*b^
3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*
c*x^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 120*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 +
 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3
*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a
^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2
*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)
)/(b^6*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-b^2cx^2+a^2c}Cf^3x^4}{5\,b^2c} - \frac{4\,\sqrt{-b^2cx^2+a^2c}Ca^2f^3x^2}{15\,b^4c} + \frac{Ae^3\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be^3}{b^2c} - \frac{3\,\sqrt{-b^2cx^2+a^2c}Ae^2f}{b^2c} - \frac{8\,\sqrt{-b^2cx^2+a^2c}Ca^4f^3}{15\,b^6c} - \frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)x^3}{4\,b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{3\,b^2c} + \frac{3\,(3\,Cef^2+Bf^3)a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{3\,\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)x}{2\,b^2c} - \frac{2\,\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{3\,b^4c}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/5*sqrt(-b^2*c*x^2 + a^2*c)*C*f^3*x^4/(b^2*c) - 4/15*sqrt(-b^2*c*x^2 + a^
2*c)*C*a^2*f^3*x^2/(b^4*c) + A*e^3*arcsin(b*x/a)/(b*sqrt(c)) - sqrt(-b^2*c*
x^2 + a^2*c)*B*e^3/(b^2*c) - 3*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*f/(b^2*c) - 8
/15*sqrt(-b^2*c*x^2 + a^2*c)*C*a^4*f^3/(b^6*c) - 1/4*sqrt(-b^2*c*x^2 + a^2*
c)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*
f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) + 3/8*(3*C*e*f^2 + B*f^3)*a^4*arcsin(b*x
/a)/(b^5*sqrt(c)) + 1/2*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*arcsin(b*x/a)/(
b^3*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*
c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c)
 - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.14

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$
$$-\frac{\left(\left(2\left(3\left(\frac{4(bx+a)Cf^3}{c}+\frac{15\,Cbc^4ef^2-16\,Cac^4f^3+5\,Bbc^4f^3}{c^5}\right)(bx+a)+\frac{60\,Cb^2c^4e^2f-135\,Cabc^4ef^2+60\,Bb^2c^4ef^2+88\,Ca^2c^4f^3-}{c^5}\right.\right.\right.}{}$$

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/120*(((2*(3*(4*(b*x + a)*C*f^3/c + (15*C*b*c^4*e*f^2 - 16*C*a*c^4*f^3 +
5*B*b*c^4*f^3)/c^5)*(b*x + a) + (60*C*b^2*c^4*e^2*f - 135*C*a*b*c^4*e*f^2 +
 60*B*b^2*c^4*e*f^2 + 88*C*a^2*c^4*f^3 - 45*B*a*b*c^4*f^3 + 20*A*b^2*c^4*f^
3)/c^5)*(b*x + a) + 5*(12*C*b^3*c^4*e^3 - 48*C*a*b^2*c^4*e^2*f + 36*B*b^3*c
^4*e^2*f + 81*C*a^2*b*c^4*e*f^2 - 48*B*a*b^2*c^4*e*f^2 + 36*A*b^3*c^4*e*f^2
 - 32*C*a^3*c^4*f^3 + 27*B*a^2*b*c^4*f^3 - 16*A*a*b^2*c^4*f^3)/c^5)*(b*x +
a) - 15*(4*C*a*b^3*c^4*e^3 - 8*B*b^4*c^4*e^3 - 24*C*a^2*b^2*c^4*e^2*f + 12*
B*a*b^3*c^4*e^2*f - 24*A*b^4*c^4*e^2*f + 15*C*a^3*b*c^4*e*f^2 - 24*B*a^2*b^
2*c^4*e*f^2 + 12*A*a*b^3*c^4*e*f^2 - 8*C*a^4*c^4*f^3 + 5*B*a^3*b*c^4*f^3 -
8*A*a^2*b^2*c^4*f^3)/c^5)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 30*(4*
C*a^2*b^3*e^3 + 8*A*b^5*e^3 + 12*B*a^2*b^3*e^2*f + 9*C*a^4*b*e*f^2 + 12*A*a
^2*b^3*e*f^2 + 3*B*a^4*b*f^3)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c))/b^6
```

Mupad [B] (verification not implemented)

Time = 151.65 (sec) , antiderivative size = 4167, normalized size of antiderivative = 8.32

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((((23*B*a^4*c*f^3)/2 - 18*B*a^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))^13) + (((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^15*((3*B*a^4*f^3)/2 + 6*B*a^2*b^2*e^2*f))/(b^5*((a + b*x)^(1/
2) - a^(1/2))^15) - (((3*B*a^4*c^7*f^3)/2 + 6*B*a^2*b^2*c^7*e^2*f)*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*x)^(1/2) - a^(1/2))) - (((23*B*a^
4*c^6*f^3)/2 - 18*B*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) + (((333*B*a^4*c^5*f^3)/2 + 90*B*a^2
```

*b^2*c^5*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((333*B*a^4*c^2*f^3)/2 + 90*B*a^2*b^2*c^2*e^2*f)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))^11) - (((
671*B*a^4*c^4*f^3)/2 - 66*B*a^2*b^2*c^4*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((671*B*a^4*c^3*f^3)/2 -
66*B*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^5*((a + b
x)^(1/2) - a^(1/2))^9) + (a^(1/2)(a*c)^(1/2)*(48*B*b^2*c^5*e^3 + 192*B*a^
2*c^5*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) -
 a^(1/2))^4) + (a^(1/2)*(a*c)^(1/2)*(160*B*b^2*c^3*e^3 + 128*B*a^2*c^3*e*f^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^
8) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^4*e^3 + 256*B*a^2*c^4*e*f^2)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1
/2)*(a*c)^(1/2)*(120*B*b^2*c^2*e^3 + 256*B*a^2*c^2*e*f^2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^10)/(b^4*((a + b*x)^(1/2) - a^(1/2))^10) + (a^(1/2)*(a*c
)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12*(48*B*b^2*c*e^3 + 192*B*a^2*
c*e*f^2))/(b^4*((a + b*x)^(1/2) - a^(1/2))^12) + (8*B*a^(1/2)*e^3*(a*c)^(1/
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(b^2*((a + b*x)^(1/2) - a^(1/2))
^14) + (8*B*a^(1/2)*c^6*e^3*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^16/((a + b*x)^(1/2) - a^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^
2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) -
 ((a^(1/2)*(a*c)^(1/2)*(64*A*a^2*c^3*f^3 + 96*A*b^2*c^3*e^2*f)*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(
a*c)^(1/2)*((128*A*a^2*c^2*f^3)/3 - 144*A*b^2*c^2*e^2*f)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a*c)^(
1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8*(64*A*a^2*c*f^3 + 96*A*b^2*c*e^2
f))/(b^4((a + b*x)^(1/2) - a^(1/2))^8) + (6*A*a^2*e*f^2*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) - (6*A*a^2*c^5*
e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))
) - (30*A*a^2*c*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x
)^(1/2) - a^(1/2))^9) + (24*A*a^(1/2)*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(1/2))^10) + (30*A*a^2*c^4*e
f^2((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2)
)^3) + (36*A*a^2*c^3*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a
+ b*x)^(1/2) - a^(1/2))^5) - (36*A*a^2*c^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (24*A*a^(1/2)*c^4*e^2*f*
(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*((a + b*x)^(1/2) -
a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(
1/2))^12 + c^6 + (6*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1

/2) - a^(1/2))^10 + (6*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x
)^(1/2) - a^(1/2))^2 + (15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a +
b*x)^(1/2) - a^(1/2))^4 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(
(a + b*x)^(1/2) - a^(1/2))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
8)/((a + b*x)^(1/2) - a^(1/2))^8) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^1
9*((9*C*a^4*e*f^2)/2 + 2*C*a^2*b^2*e^3))/(b^5*((a + b*x)^(1/2) - a^(1/2))^1
9) - ((2*C*a^2*b^2*c*e^3 - (87*C*a^4*c*e*f^2)/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^17)/(b^5*((a + b*x)^(1/2) - a^(1/2))^17) - (((9*C*a^4*c^9*e*f^2)/
2 + 2*C*a^2*b^2*c^9*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*
x)^(1/2) - a^(1/2))) - (((87*C*a^4*c^8*e*f^2)/2 - 2*C*a^2*b^2*c^8*e^3)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - ((
42*C*a^4*c^6*e*f^2 - 88*C*a^2*b^2*c^6*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + ((42*C*a^4*c^3*e*f^2 - 88*C*a^
2*b^2*c^3*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^13) + ((426*C*a^4*c^7*e*f^2 + 40*C*a^2*b^2*c^7*e^3)*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b*x)^(1/2) - a^(1/2))^5) - ((426*C*
a^4*c^2*e*f^2 + 40*C*a^2*b^2*c^2*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^1
5)/(b^5*((a + b*x)^(1/2) - a^(1/2))^15) - ((507*C*a^4*c^5*e*f^2 - 52*C*a^2*
b^2*c^5*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^5*((a + b*x)^(1/2) -
a^(1/2))^9) + ((507*C*a^4*c^4*e*f^2 - 52*C*a^2*b^2*c^4*e^3)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))^11) + (a^(1/2)*(
a*c)^(1/2)*((2048*C*a^4*c^6*f^3)/3 + 640*C*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^6)/(b^6*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a
c)^(1/2)((2048*C*a^4*c^2*f^3)/3 + 640*C*a^2*b^2*c^2*e^2*f)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^14)/(b^6*((a + b*x)^(1/2) - a^(1/2))^14) - (a^(1/2)*(
a*c)^(1/2)*((4096*C*a^4*c^5*f^3)/3 - 832*C*a^2*b^2*c^5*e^2*f)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^8)/(b^6*((a + b*x)^(1/2) - a^(1/2))^8) - (a^(1/2)*(a
c)^(1/2)((4096*C*a^4*c^3*f^3)/3 - 832*C*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^12)/(b^6*((a + b*x)^(1/2) - a^(1/2))^12) + (a^(1/2)*(
a*c)^(1/2)*((12288*C*a^4*c^4*f^3)/5 + 768*C*a^2*b^2*c^4*e^2*f)*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^10)/(b^6*((a + b*x)^(1/2) - a^(1/2))^10) + (192*C*a
^(5/2)*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/(b^4*((a
+ b*x)^(1/2) - a^(1/2))^16) + (192*C*a^(5/2)*c^7*e^2*f*(a*c)^(1/2)*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4))/(((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^20/((a + b*x)^(1/2) - a^(1/2))^20 + c^10 + (
10*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^18)/((a + b*x)^(1/2) - a^(1/2))^18
+ (10*c^9*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2
))^2 + (45*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^
(1/2))^4 + (120*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2)
- a^(1/2))^6 + (210*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^
(1/2) - a^(1/2))^8 + (252*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a +
b*x)^(1/2) - a^(1/2))^10 + (210*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12
)/((a + b*x)^(1/2) - a^(1/2))^12 + (120*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14 + (45*c^2*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16) - (2*A*e*atan((A*e*(3*a^2*f^

```
2 + 2*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)*(2*A*b^2*e^3 +
 3*A*a^2*e*f^2)*((a + b*x)^(1/2) - a^(1/2))))*(3*a^2*f^2 + 2*b^2*e^2))/(b^3
*c^(1/2)) - (3*B*a^2*f*atan((B*a^2*f*(a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2)))/(c^(1/2)*(B*a^4*f^3 + 4*B*a^2*b^2*e^2*f)*((a + b*x)^(1
/2) - a^(1/2))))*(a^2*f^2 + 4*b^2*e^2))/(2*b^5*c^(1/2)) - (C*a^2*e*atan((C*
a^2*e*(9*a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)
*(9*C*a^4*e*f^2 + 4*C*a^2*b^2*e^3)*((a + b*x)^(1/2) - a^(1/2))))*(9*a^2*f^2
 + 4*b^2*e^2))/(2*b^5*c^(1/2))
```

3.28 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 368

$$
\begin{aligned}
&\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx \\
&= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad - \frac{\left(4(4a^2f^2(2Ce+Bf)-b^2e(Ce^2-4f(Be+3Af)))+f(9a^2Cf^2-b^2(2Ce^2-4f(2Be+3Af)))\,x\right)\left(a^2\right.}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad + \frac{\left(4A(2b^4e^2+a^2b^2f^2)+a^2(3a^2Cf^2+4b^2e(Ce+2Bf))\right)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{cx}}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

```
[Out] 1/12*(-4*B*f+C*e)*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)-1/4*C*(f*x+e)^3*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2
)-1/24*(16*a^2*f^2*(B*f+2*C*e)-4*b^2*e*(C*e^2-4*f*(3*A*f+B*e))+f*(9*a^2*C*f
^2-b^2*(2*C*e^2-4*f*(3*A*f+2*B*e)))*x)*(-b^2*x^2+a^2)/b^4/f/(b*x+a)^(1/2)/(
-b*c*x+a*c)^(1/2)+1/8*(4*A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(3*a^2*C*f^2+4*b^2*e
*(2*B*f+C*e)))*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2
*c)^(1/2)/b^5/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1624, 1668, 847, 794, 223, 209}

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{(a^2-b^2x^2)\left(4\left(4a^2f^2(Bf+2Ce)-\frac{1}{4}b^2(4Ce^3-16ef(3Af+Be))\right)+fx(9a^2Cf^2-b^2(2Ce^2-4f(3Af\ \cdots}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{(a^2-b^2x^2)(e+fx)^2(Ce-4Bf)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C(a^2-b^2x^2)(e+fx)^3}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$+\frac{\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)(3a^4Cf^2+4A(a^2b^2f^2+2b^4e^2)+4a^2b^2e(2Bf+Ce))}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*(a^2 - b^2*x^2))/(12*b^2*f*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]) - (C*(e + f*x)^3*(a^2 - b^2*x^2))/(4*b^2*f*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]) - ((4*(4*a^2*f^2*(2*C*e + B*f) - (b^2*(4*C*e^3 - 16*e*f*(B*e +
 3*A*f)))/4) + f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*f)))*x)*(a^
2 - b^2*x^2))/(24*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((3*a^4*C*f^2 +
4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2
*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[
a + b*x]*Sqrt[a*c - b*c*x])
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 223</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

<u>Rule 794</u>

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(-c\left(4Ab^2+3a^2C\right)f^2+b^2cf(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{4b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad+\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(b^2c^2f^2\left(12Ab^2e+a^2(7Ce+8Bf)\right)+b^2c^2f\left(9a^2Cf^2-2b^2\left(Ce^2-2f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{12b^4c^2f^2\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}$$

$$= \frac{(Ce-4Bf)(e+fx)^2(a^2-b^2x^2)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3(a^2-b^2x^2)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)-\frac{1}{4}b^2(4Ce^3-16ef(Be+3Af))\right)+f(9a^2Cf^2-b^2(2Ce^2-4f(2Be+3A\ldots}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left((3a^4Cf^2+4a^2b^2e(Ce+2Bf)+4A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{8b^4\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-4Bf)(e+fx)^2(a^2-b^2x^2)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3(a^2-b^2x^2)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)-\frac{1}{4}b^2(4Ce^3-16ef(Be+3Af))\right)+f(9a^2Cf^2-b^2(2Ce^2-4f(2Be+3A\ldots}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left((3a^4Cf^2+4a^2b^2e(Ce+2Bf)+4A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{\ldots}{\sqrt{a^2c\ldots}}\right.}{8b^4\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-4Bf)(e+fx)^2(a^2-b^2x^2)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3(a^2-b^2x^2)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce+Bf)-\frac{1}{4}b^2(4Ce^3-16ef(Be+3Af))\right)+f(9a^2Cf^2-b^2(2Ce^2-4f(2Be+3A\ldots}{24b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(3a^4Cf^2+4a^2b^2e(Ce+2Bf)+4A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^5\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.54

$$\int\frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{-b(a-bx)\sqrt{a+bx}(a^2f(32Ce+16Bf+9Cfx)+2b^2(6Af(4e+fx)+4B(3e^2+3efx+f^2x^2)+Cx(6e^2\ldots}{24b^5\sqrt{c(a\ldots}}$$

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-(b*(a - b*x)*Sqrt[a + b*x]*(a^2*f*(32*C*e + 16*B*f + 9*C*f*x) + 2*b^2*(6*
A*f*(4*e + f*x) + 4*B*(3*e^2 + 3*e*f*x + f^2*x^2) + C*x*(6*e^2 + 8*e*f*x +
3*f^2*x^2)))) + 6*(3*a^4*C*f^2 + 4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2
 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(24*b^5
*Sqrt[c*(a - b*x)])
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{(6C f^2x^3b^2+8B b^2f^2x^2+16C b^2ef x^2+12A b^2f^2x+24B b^2efx+9C a^2f^2x+12C b^2e^2x+48A b^2ef+16B a^2f^2+24B b^2e^2+32C a^2e}{24b^4\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-6C b^2f^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)a^2b^2cf^2+24A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)b^4}{}$

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24*(6*C*b^2*f^2*x^3+8*B*b^2*f^2*x^2+16*C*b^2*e*f*x^2+12*A*b^2*f^2*x+24*B
*b^2*e*f*x+9*C*a^2*f^2*x+12*C*b^2*e^2*x+48*A*b^2*e*f+16*B*a^2*f^2+24*B*b^2*
e^2+32*C*a^2*e*f)*(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/8*(4*A*a^
2*b^2*f^2+8*A*b^4*e^2+8*B*a^2*b^2*e*f+3*C*a^4*f^2+4*C*a^2*b^2*e^2)/b^4/(b^2
*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x
-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.31

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$=\left[-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}\right.}{}\right.$$
$$-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+(6\,Cb^3f^2}{}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/48*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^2 + (3*C*a^4 + 4*A*a
^2*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*
b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 + 16*B*a^2*b*f^2
+ 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)*x^2 + 3*(4*C*b
^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqr
t(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^
```

```
2 + (3*C*a^4 + 4*A*a^2*b^2)*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x
 + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 +
16*B*a^2*b*f^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)
*x^2 + 3*(4*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b
*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = &-\frac{\sqrt{-b^2cx^2+a^2c}Cf^2x^3}{4\,b^2c} + \frac{Ae^2\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} \\ &+ \frac{3\,Ca^4f^2\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} - \frac{3\sqrt{-b^2cx^2+a^2c}Ca^2f^2x}{8\,b^4c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}Be^2}{b^2c} - \frac{2\sqrt{-b^2cx^2+a^2c}Aef}{b^2c} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(2\,Cef+Bf^2)x^2}{3\,b^2c} \\ &+ \frac{(Ce^2+2\,Bef+Af^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} \\ &- \frac{\sqrt{-b^2cx^2+a^2c}(Ce^2+2\,Bef+Af^2)x}{2\,b^2c} \\ &- \frac{2\sqrt{-b^2cx^2+a^2c}(2\,Cef+Bf^2)a^2}{3\,b^4c}\end{aligned}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
[Out] -1/4*sqrt(-b^2*c*x^2 + a^2*c)*C*f^2*x^3/(b^2*c) + A*e^2*arcsin(b*x/a)/(b*sq
rt(c)) + 3/8*C*a^4*f^2*arcsin(b*x/a)/(b^5*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 +
a^2*c)*C*a^2*f^2*x/(b^4*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^2/(b^2*c) - 2*sqr
t(-b^2*c*x^2 + a^2*c)*A*e*f/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f
```

```
 + B*f^2)*x^2/(b^2*c) + 1/2*(C*e^2 + 2*B*e*f + A*f^2)*a^2*arcsin(b*x/a)/(b^
3*sqrt(c)) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*x/(b^2*
c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f + B*f^2)*a^2/(b^4*c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.99

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(2\left(\frac{3(bx+a)Cf^2}{c}+\frac{8\,Cbc^3ef-9\,Cac^3f^2+4\,Bbc^3f^2}{c^4}\right)(bx+a)+\frac{12\,Cb^2c^3e^2-32\,Cabc^3ef+24\,Bb^2c^3ef+27\,Ca^2c^3f^2-16\,Babc^3f^2+}{c^4}\right.\right.}{}$$

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/24*(((2*(3*(b*x + a)*C*f^2/c + (8*C*b*c^3*e*f - 9*C*a*c^3*f^2 + 4*B*b*c^
3*f^2)/c^4)*(b*x + a) + (12*C*b^2*c^3*e^2 - 32*C*a*b*c^3*e*f + 24*B*b^2*c^3
*e*f + 27*C*a^2*c^3*f^2 - 16*B*a*b*c^3*f^2 + 12*A*b^2*c^3*f^2)/c^4)*(b*x +
a) - 3*(4*C*a*b^2*c^3*e^2 - 8*B*b^3*c^3*e^2 - 16*C*a^2*b*c^3*e*f + 8*B*a*b^
2*c^3*e*f - 16*A*b^3*c^3*e*f + 5*C*a^3*c^3*f^2 - 8*B*a^2*b*c^3*f^2 + 4*A*a*
b^2*c^3*f^2)/c^4)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(4*C*a^2*b^2
*e^2 + 8*A*b^4*e^2 + 8*B*a^2*b^2*e*f + 3*C*a^4*f^2 + 4*A*a^2*b^2*f^2)*log(a
bs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^5
```

Mupad [B] (verification not implemented)

Time = 81.28 (sec) , antiderivative size = 2799, normalized size of antiderivative = 7.61

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((a^(1/2)*(a*c)^(1/2)*(64*B*a^2*c*f^2 + 32*B*b^2*c*e^2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^
(1/2)*(64*B*a^2*c^3*f^2 + 32*B*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((128*B*
a^2*c^2*f^2)/3 - 48*B*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(
b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (4*B*a^2*e*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) + (8*B*a^(1/2)*e^2*(a*c
```

)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(
1/2))^10) + (20*B*a^2*c^4*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*(
(a + b*x)^(1/2) - a^(1/2))^3) + (24*B*a^2*c^3*e*f*((a*c - b*c*x)^(1/2) - (a
c)^(1/2))^5)/(b^3((a + b*x)^(1/2) - a^(1/2))^5) - (24*B*a^2*c^2*e*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (8*
B*a^(1/2)*c^4*e^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*(
(a + b*x)^(1/2) - a^(1/2))^2) - (4*B*a^2*c^5*e*f*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (20*B*a^2*c*e*f*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))^9))/(((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^12 + c^6 + (6*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (6*c
^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (
15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4
 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2
))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^
(1/2))^8) - ((2*A*a^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a +
 b*x)^(1/2) - a^(1/2))^7) + (14*A*a^2*c^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) - (2*A*a^2*c^3*f^2*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (14*A*a^2*c*
f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))
^5) + (16*A*a^(1/2)*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/
(b^2*((a + b*x)^(1/2) - a^(1/2))^6) + (32*A*a^(1/2)*c*e*f*(a*c)^(1/2)*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*((a + b*x)^(1/2) - a^(1/2))^4) + (16
*A*a^(1/2)*c^2*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*
((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a
+ b*x)^(1/2) - a^(1/2))^8 + c^4 + (4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
6)/((a + b*x)^(1/2) - a^(1/2))^6 + (4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^5*((333*C*a^4*c^5*f^2)/2 + 30*C*a^2*b^2*c^5*e^2))/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*((23*C*a^4*c^6*f^2
)/2 - 6*C*a^2*b^2*c^6*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - (((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))*((3*C*a^4*c^7*f^2)/2 + 2*C*a^2*b^2*c^7*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11*(
(333*C*a^4*c^2*f^2)/2 + 30*C*a^2*b^2*c^2*e^2))/(b^5*((a + b*x)^(1/2) - a^(1
/2))^11) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7*((671*C*a^4*c^4*f^2)/2 -
22*C*a^2*b^2*c^4*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^9*((671*C*a^4*c^3*f^2)/2 - 22*C*a^2*b^2*c^3*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))^9) + (((23*C*a^4*c*f^2)/2 - 6*C*a^2*b^2*c*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))
^13) + (((3*C*a^4*f^2)/2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^15)/(b^5*((a + b*x)^(1/2) - a^(1/2))^15) + (128*C*a^(5/2)*c*e*f*(a*c)^
(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/(b^4*((a + b*x)^(1/2) - a^(1/
2))^12) + (128*C*a^(5/2)*c^5*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) + (512*C*a^(5/2)*c^4*e*f*(a*c)

```
^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(3*b^4*((a + b*x)^(1/2) - a^(
1/2))^6) + (256*C*a^(5/2)*c^3*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^8)/(3*b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (512*C*a^(5/2)*c^2*e*f*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(3*b^4*((a + b*x)^(1/2) -
 a^(1/2))^10))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16/((a + b*x)^(1/2) - a
^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^
(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b
*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a
 + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)
/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
)^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^2*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) - (2*A*atan((A*(a^2*f^2 + 2
*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)*(A*a^2*f^2 + 2*A*b^
2*e^2)*((a + b*x)^(1/2) - a^(1/2))))*(a^2*f^2 + 2*b^2*e^2))/(b^3*c^(1/2)) -
 (C*a^2*atan((C*a^2*(3*a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2)))/(c^(1/2)*(3*C*a^4*f^2 + 4*C*a^2*b^2*e^2)*((a + b*x)^(1/2) - a^(1/2)))
)*(3*a^2*f^2 + 4*b^2*e^2))/(2*b^5*c^(1/2)) - (4*B*a^2*e*f*atan(((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2
))
```

3.29 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 246

$$\begin{aligned}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx &= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad - \frac{(2(2a^2Cf^2-b^2(Ce^2-3f(Be+Af)))-b^2f(Ce-3Bf)x)\left(a^2-b^2x^2\right)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} \\ &\quad + \frac{(2Ab^2e+a^2(Ce+Bf))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

```
[Out] -1/3*C*(f*x+e)^2*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/6*
(4*a^2*C*f^2-2*b^2*(C*e^2-3*f*(A*f+B*e))-b^2*f*(-3*B*f+C*e)*x)*(-b^2*x^2+a^
2)/b^4/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2*e+a^2*(B*f+C*e))*arc
tan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/c^(1
/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used

$= \{1624, 1668, 794, 223, 209\}$

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+2Ab^2e\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C\left(a^2-b^2x^2\right)(e+fx)^2}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -1/3*(C*(e + f*x)^2*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
) - ((2*(2*a^2*C*f^2 - (b^2*(2*C*e^2 - 6*f*(B*e + A*f)))/2) - b^2*f*(C*e -
3*B*f)*x)*(a^2 - b^2*x^2))/(6*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*
A*b^2*e + a^2*(C*e + B*f))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqr
t[a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2c-b^2cx^2}\int\frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2(a^2-b^2x^2)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int\frac{(e+fx)(-c(3Ab^2+2a^2C)f^2+b^2cf(Ce-3Bf)x)}{\sqrt{a^2c-b^2cx^2}}\,dx}{3b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2(a^2-b^2x^2)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)-b^2f(Ce-3Bf)x\right)(a^2-b^2x^2)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left((2Ab^2e+a^2(Ce+Bf))\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2(a^2-b^2x^2)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)-b^2f(Ce-3Bf)x\right)(a^2-b^2x^2)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left((2Ab^2e+a^2(Ce+Bf))\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2(a^2-b^2x^2)}{3b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2(2Ce^2-6f(Be+Af))\right)-b^2f(Ce-3Bf)x\right)(a^2-b^2x^2)}{6b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(2Ab^2e+a^2(Ce+Bf))\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.52

$$\int \frac{(e+fx)\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-\left((a-bx)\sqrt{a+bx}(4a^2Cf+b^2(6Be+6Af+3Cex+3Bfx+2Cfx^2))\right)+6b(2Ab^2e+a^2(Ce+Bf))\sqrt{}}{6b^4\sqrt{c(a-bx)}}$$

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x
]
```

```
[Out] (-((a - b*x)*Sqrt[a + b*x]*(4*a^2*C*f + b^2*(6*B*e + 6*A*f + 3*C*e*x + 3*B*
f*x + 2*C*f*x^2))) + 6*b*(2*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a - b*x]*ArcTan
[Sqrt[a + b*x]/Sqrt[a - b*x]])/(6*b^4*Sqrt[c*(a - b*x)])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(2Cf\,x^2b^2+3B\,b^2fx+3C\,b^2ex+6A\,b^2f+6B\,b^2e+4a^2Cf)\sqrt{bx+a}\,(-bx+a)}{6b^4\sqrt{-c(bx-a)}}+\frac{(2A\,b^2e+B\,a^2f+C\,a^2e)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{}}{2b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)b^4ce+3B\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2cf+3C\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2c\right.}{}$

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/6*(2*C*b^2*f*x^2+3*B*b^2*f*x+3*C*b^2*e*x+6*A*b^2*f+6*B*b^2*e+4*C*a^2*f)*
(b*x+a)^(1/2)/b^4*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2*e+B*a^2*f+C*a^2*
e)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*
x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{3\left(Ba^2bf+(Ca^2b+2Ab^3)e\right)\sqrt{-c}\log\left(2b^2cx^2-2\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(2Cb^2fx^2+\right.}{12b^4c}\right.$$
$$-\frac{3\left(Ba^2bf+(Ca^2b+2Ab^3)e\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{cx}}{b^2cx^2-a^2c}\right)+\left(2Cb^2fx^2+6Bb^2e+2\left(2Ca^2+3Ab^2\right)f\right.}{6b^4c}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2*
sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2 +
 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e + B*b^2*f)*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)
*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a
^2*c)) + (2*C*b^2*f*x^2 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e
+ B*b^2*f)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-b^2cx^2+a^2c}Cfx^2}{3\,b^2c} + \frac{Ae\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} + \frac{(Ce+Bf)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be}{b^2c} - \frac{2\sqrt{-b^2cx^2+a^2c}Ca^2f}{3\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}Af}{b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)x}{2\,b^2c}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*sqrt(-b^2*c*x^2 + a^2*c)*C*f*x^2/(b^2*c) + A*e*arcsin(b*x/a)/(b*sqrt(c
)) + 1/2*(C*e + B*f)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) - sqrt(-b^2*c*x^2 + a^
2*c)*B*e/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*f/(b^4*c) - sqrt(-b^2
*c*x^2 + a^2*c)*A*f/(b^2*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e + B*f)*x/(b
^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(\frac{2\,(bx+a)Cf}{c} + \frac{3\,Cbc^2e-4\,Cac^2f+3\,Bbc^2f}{c^3}\right)(bx+a) - \frac{3\,\left(Cabc^2e-2\,Bb^2c^2e-2\,Ca^2c^2f+Babc^2f-2\,Ab^2c^2f\right)}{c^3}\right)\sqrt{-(bx+a)c}}{6\,b^4}$$

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] -1/6*(((2*(b*x + a)*C*f/c + (3*C*b*c^2*e - 4*C*a*c^2*f + 3*B*b*c^2*f)/c^3)*
(b*x + a) - 3*(C*a*b*c^2*e - 2*B*b^2*c^2*e - 2*C*a^2*c^2*f + B*a*b*c^2*f -
2*A*b^2*c^2*f)/c^3)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(C*a^2*b*e
 + 2*A*b^3*e + B*a^2*b*f)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c))/b^4
```

Mupad [B] (verification not implemented)

Time = 37.95 (sec) , antiderivative size = 1011, normalized size of antiderivative = 4.11

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\frac{2\,B\,a^2\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,B\,a^2\,c^3\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,B\,a^2\,c\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,B\,a^2\,c^2\,f\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}} - \frac{\frac{2\,C\,a^2\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,C\,a^2\,c^3\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,C\,a^2\,c^2\,e\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}} - \frac{\sqrt{a\,c-b\,c\,x}\,\left(\frac{2\,C\,a^3\,f}{3\,b^4\,c}+\frac{C\,f\,x^3}{3\,b\,c}+\frac{C\,a\,f\,x^2}{3\,b^2\,c}+\frac{2\,C\,a^2\,f\,x}{3\,b^3\,c}\right)}{\sqrt{a+b\,x}} - \frac{4\,A\,e\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}} - \frac{A\,f\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c} - \frac{B\,e\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c} - \frac{2\,B\,a^2\,f\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}} - \frac{2\,C\,a^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}$$

```
[In] int(((e + f*x)*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] - ((2*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(
1/2))^7 - (2*B*a^2*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1
/2) - a^(1/2)) - (14*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a +
 b*x)^(1/2) - a^(1/2))^5 + (14*B*a^2*c^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((2*C
*a^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/2))^7
 - (2*C*a^2*c^3*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a
^(1/2)) - (14*C*a^2*c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^(
1/2) - a^(1/2))^5 + (14*C*a^2*c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/
((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((a*c - b*c*
x)^(1/2)*((2*C*a^3*f)/(3*b^4*c) + (C*f*x^3)/(3*b*c) + (C*a*f*x^2)/(3*b^2*c)
 + (2*C*a^2*f*x)/(3*b^3*c)))/(a + b*x)^(1/2) - (4*A*e*atan((b*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^2*
```

```
c)^(1/2) - (A*f*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (B*e*(a*c -
b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (2*B*a^2*f*atan(((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (2
*C*a^2*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2)
 - a^(1/2)))))/(b^3*c^(1/2))
```

3.30 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 177

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx=-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{(2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] -B*(-b^2*x^2+a^2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-1/2*C*x*(-b^2*x^2+a^
2)/b^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*A*b^2+C*a^2)*arctan(b*x*c^(1
/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b^3/c^(1/2)/(b*x+a)^
(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {915, 1829, 655, 223, 209}

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx=\frac{(a^2C+2Ab^2)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -((B*(a^2 - b^2*x^2))/(b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - (C*x*(a^2 -
b^2*x^2))/(2*b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2 + a^2*C)*Sqrt
```

```
[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b^3*S
qrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2c-b^2cx^2}\int\frac{A+Bx+Cx^2}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{\sqrt{a^2c-b^2cx^2}\int\frac{-c(2Ab^2+a^2C)-2b^2Bcx}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2c\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad+\frac{\left((2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad + \frac{\left((2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx, x, \frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= -\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad + \frac{(2Ab^2+a^2C)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx \\
&= \frac{b(-a+bx)\sqrt{a+bx}(2B+Cx)+2(2Ab^2+a^2C)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{2b^3\sqrt{c(a-bx)}}
\end{aligned}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (b*(-a + b*x)*Sqrt[a + b*x]*(2*B + C*x) + 2*(2*A*b^2 + a^2*C)*Sqrt[a - b*x]
*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(2*b^3*Sqrt[c*(a - b*x)])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(Cx+2B)\sqrt{bx+a}\,(-bx+a)}{2b^{2}\sqrt{-c(bx-a)}}+\frac{(2b^{2}A+C\,a^{2})\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)\sqrt{-(bx+a)c(bx-a)}}{2b^{2}\sqrt{b^{2}c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(2A\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}}\right)b^{2}c+C\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}}\right)a^{2}c-C\sqrt{c(-b^{2}x^{2}+a^{2})}\,\sqrt{b^{2}c}\,x-2B\sqrt{b^{2}c}\,\sqrt{c(-b^{2}\cdots}\right.}{2b^{2}\sqrt{c(-b^{2}x^{2}+a^{2})}\,c\sqrt{b^{2}c}}$

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(C*x+2*B)*(b*x+a)^(1/2)/b^2*(-b*x+a)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2+C
*a^2)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-
(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \left[-\frac{(Ca^2+2\,Ab^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+ac}}{4\,b^3c}\right.$$

$$\left.-\frac{(Ca^2+2\,Ab^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+(Cbx+2\,Bb)\sqrt{-bcx+ac}\sqrt{bx+a}}{2\,b^3c}\right]$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sq
rt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{Ca^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} + \frac{A\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Cx}{2\,b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}B}{b^2c}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*C*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) + A*arcsin(b*x/a)/(b*sqrt(c)) - 1/2*s
qrt(-b^2*c*x^2 + a^2*c)*C*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B/(b^2*c)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left(\frac{(bx+a)C}{c}-\frac{Cac-2\,Bbc}{c^2}\right)+\frac{2\,(Ca^2+2\,Ab^2)\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}}{2\,b^3}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/2*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*C/c - (C*a*c - 2*
B*b*c)/c^2) + 2*(C*a^2 + 2*A*b^2)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(
b*x + a)*c + 2*a*c)))/sqrt(-c))/b^3
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\frac{2\,C\,a^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^7}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7}-\frac{2\,C\,a^2\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^5}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}+\frac{14\,C\,a^2\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^3}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}}{b^3\,c^4+\frac{b^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^8}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^8}+\frac{4\,b^3\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{6\,b^3\,c^2\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^4}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}+\frac{4\,b^3\,c\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^6}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}}-\frac{4\,A\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}-\frac{2\,C\,a^2\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}-\frac{B\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c}$$

```
[In] int((A + B*x + C*x^2)/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] - ((2*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/
2))^7 - (2*C*a^2*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2)
- a^(1/2)) - (14*C*a^2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^
(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(
(a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^
```

```
(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - (4*A*atan((b*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1
/2)))))/(b^2*c)^(1/2) - (2*C*a^2*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(
c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (B*(a*c - b*c*x)^(1/
2)*(a + b*x)^(1/2))/(b^2*c)
```

3.31 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 278

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] -C*(-b^2*x^2+a^2)/b^2/f/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)-(-B*f+C*e)*arctan(
b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2
)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(A*f^2-B*e*f+C*e^2)*arctan((b^2*e*x+a^2*
f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a
^2*c)^(1/2)/f^2/c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used

$= \{1624, 1668, 858, 223, 209, 739, 210\}$

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)}\,dx$$
$$= \frac{\sqrt{a^2c - b^2cx^2}(Af^2 - Bef + Ce^2)\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a + bx}\sqrt{ac - bcx}\sqrt{b^2e^2 - a^2f^2}}$$
$$- \frac{\sqrt{a^2c - b^2cx^2}(Ce - Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a + bx}\sqrt{ac - bcx}} - \frac{C(a^2 - b^2x^2)}{b^2f\sqrt{a + bx}\sqrt{ac - bcx}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad + \frac{\left((Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx, x, \frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}\\
&\quad - \frac{\left((Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx, x, \frac{a^2cf+b^2cex}{\sqrt{a^2c-b^2cx^2}}\right)}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

$$= -\frac{C(a^2-b^2x^2)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce^2-Bef+Af^2)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2} - \frac{2(Ce-Bf)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} + \frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] ((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[S
qrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x
]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/
(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^2\sqrt{-c(bx-a)}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{f\sqrt{b^2c}}-\frac{\left(A\,f^2-Bef+C\,e^2\right)\ln\left(\frac{\frac{2c\left(a^2f^2-b^2e^2\right)}{f^2}+\frac{2b^2ce\left(x+\frac{e}{f}\right)}{f}+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\cdots}{x+\cdots}\right)}{f^2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\cdots}\cdots\right)}{f\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\left(-A\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)b^2c\,f^2\sqrt{b^2c}+B\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{c\left(a^2f^2-b^2e^2\right)}{f^2}}\sqrt{c\left(-b^2x^2+a^2\right)}\,f}{fx+e}\right)\cdots\right.}{\cdots}$

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURN
VERBOSE)
```

[Out] -C*(b*x+a)^(1/2)*(-b*x+a)/f/b^2/(-c*(b*x-a))^(1/2)+1/f*((B*f-C*e)/f/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-b^2*c*(x+e/f)^2+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)}\, dx = \text{Timed out}$$

[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)}\, dx = \int \frac{A + Bx + Cx^2}{\sqrt{-c(-a + bx)}\sqrt{a + bx}\,(e + fx)}\, dx$$

[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)

[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)}\, dx = \text{Exception raised: ValueError}$$

[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for more detai

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9298, normalized size of antiderivative = 33.45

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)} \, dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
```

) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
 + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a
e((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*
e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5
/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*
B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(2
4*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c
)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2
f^3)((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) -
a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))
/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2
))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e
^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a +
 b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2)
 - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(
17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144
*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^
(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a
^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a
^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/
(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a
^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^
2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B
*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
(B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*
c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*
a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a
 + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b
^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4
*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x
)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^
(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a
^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1
/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e
f^4(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6
*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (
16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*
(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(

1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b
^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8
e^6((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2))
 + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
+ b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a*e*(
(4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*
(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2)
- 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*B^2*
a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(24*B*
a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3
/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1
/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^
6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2
) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f
^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x
)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 9
0*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) -
a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2
)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (
4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144*B^2
*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a^4*c
^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/
2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (917504*B^4*a^4*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*((a + b*x)^(1/2) - a^(1/2))^2)))*2i
)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) - (C*e^2*atan(((C*e^2*((4096*(32*C^
3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2
)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e
^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2
) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((409
6*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/
2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))
+ (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b
^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*
f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f
^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^
(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^
4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*
((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (409

6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^
2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2)
 - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*
c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) -
(4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*
c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*
x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c
*e^2)^(1/2)) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C
^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(16*C^
2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) + (C*e^2*((4096*(
24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(
3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2
*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2
)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a
^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))
/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^
7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a
^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*
(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*
f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*
b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^
2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
)^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*
(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4
c^5((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(
1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)))/((131072*C^4*a^4*c^5)/(b^8
*f^4) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3
/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c
^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^
(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/
(b^8*e^4*f^4) + (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/
(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c
^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*(
(a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(
11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2)
- a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*
f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e
^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)

^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^
2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*
a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(
b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4
*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*
c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32
*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(
3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))
)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e
^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^
4) - (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e
^4*f^4) + (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2
)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^
7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*
f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6)
)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2
)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096
*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)
^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 +
 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)
+ (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*
c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x
)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C
^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1
/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f
^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (91
7504*C^4*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))*2i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4*B*atan(
(67108864*B^5*a^16*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)
^(1/2) - a^(1/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c
^(15/2)*e^4 - 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) + (37748736*B^5*a^1
2*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1
/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 -
 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) - (100663296*B^5*a^14*b^2*c^7*e^
2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67
108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 - 1006632

```
96*B^5*a^14*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f) - (A*a*atan((a*c*(a*c -
b*c*x)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - (a*c)^(3/2)*(a^4*c*f^2
- a^2*b^2*c*e^2)^(1/2)*1i + a*c*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/
2)*1i + b*c*x*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - a^(1/2)*c*
(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*(a + b*x)^(1/2)*2i)/(2*a^(5/2
)*b*c^2*e - 2*a^3*c^2*f*(a + b*x)^(1/2) - 2*a^2*b*c^2*e*(a + b*x)^(1/2) + 2
*a^(5/2)*b*c^2*f*x + 2*a^(5/2)*c*f*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) - 2*a^(3
/2)*b*c*e*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) + 2*a*b*c*e*(a*c - b*c*x)^(1/2)*(
a*c)^(1/2)*(a + b*x)^(1/2)))*2i)/(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2) + (4*C*e
*atan((67108864*C^5*a^8*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a +
 b*x)^(1/2) - a^(1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^
4*c^(15/2)*e^4 - 100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) + (37748736*C^5*a
^4*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 -
100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) - (100663296*C^5*a^6*b^2*c^7*e^2*f
^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67108
864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 - 100663296*C^
5*a^6*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f^2) - (8*C*a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*f*((a + b*x)^(1/2) - a^(1/2))^2*
(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2
+ (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
))
```

3.32 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} dx$

Optimal result

Integrand size = 40, antiderivative size = 322

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} dx = \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)/(b*x+a)^(1
/2)/(-b*c*x+a*c)^(1/2)+C*arctan(b*x*c^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2
*c*x^2+a^2*c)^(1/2)/b/f^2/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+(a^2*f^2
*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^
2+b^2*e^2)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/f^2/(-a
^2*f^2+b^2*e^2)^(3/2)/c^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used

$= \{1624, 1665, 858, 223, 209, 739, 210\}$

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$
$$= \frac{\sqrt{a^2c-b^2cx^2}(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{3/2}}$$
$$+ \frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)} + \frac{C\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2c-b^2cx^2}\int\frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{\sqrt{a^2c-b^2cx^2}\int\frac{c(Ab^2e+a^2(Ce-Bf))+cC\left(\frac{b^2e^2}{f}-a^2f\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{c\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad+\frac{\left(\left(-cCe\left(\frac{b^2e^2}{f}-a^2f\right)+cf(Ab^2e+a^2(Ce-Bf))\right)\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{cf\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}} \\
&= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} \\
&\quad+\frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{1+b^2cx^2}\,dx,x,\frac{x}{\sqrt{a^2c-b^2cx^2}}\right)}{f\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}} \\
&\quad-\frac{\left(\left(-cCe\left(\frac{b^2e^2}{f}-a^2f\right)+cf(Ab^2e+a^2(Ce-Bf))\right)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx,\right.}{cf\,(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}}
\end{aligned}
$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{(b^2e^2-a^2f^2)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{\sqrt{c}f^2(b^2e^2-a^2f^2)^{3/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)} + \frac{C\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b} - \frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] (2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a*
f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a -
b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*x]*
ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((
b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. 2(290) = 580.

Time = 1.68 (sec) , antiderivative size = 1166, normalized size of antiderivative = 3.62

method	result	size
default	Expression too large to display	1166

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*
f^4*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*a^2*
c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x
^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*ln(2*(b
^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f
```

```
)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^
2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^3*f*x*(b
^2*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^
(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x
/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*
ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^4
*(b^2*c)^(1/2)-A*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/
2)*(b^2*c)^(1/2)+B*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))
^(1/2)*(b^2*c)^(1/2)-C*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2
+a^2))^(1/2)*(b^2*c)^(1/2))/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2
+a^2))^(1/2)/(a*f+b*e)/(b^2*c)^(1/2)/(a*f-b*e)/(f*x+e)/(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)/f^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c\,(-a+bx)}\sqrt{a+bx}\,(e+fx)^2}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e detai
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$

$$= \frac{\frac{2\,(Cb^3\sqrt{-c}e^3-2\,Ca^2b\sqrt{-c}ef^2-Ab^3\sqrt{-c}ef^2+Ba^2b\sqrt{-c}f^3)\arctan\left(-\frac{2\,bce-\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f}{2\sqrt{-b^2e^2+a^2f^2c}}\right)}{(b^2e^2f^2-a^2f^4)\sqrt{-b^2e^2+a^2f^2c}} - \frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx\cdots\right.\right.}{\sqrt{-c}f^2\cdots}}{\cdots}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] (2*(C*b^3*sqrt(-c)*e^3 - 2*C*a^2*b*sqrt(-c)*e*f^2 - A*b^3*sqrt(-c)*e*f^2 +
B*a^2*b*sqrt(-c)*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^2 -
a^2*f^4)*sqrt(-b^2*e^2 + a^2*f^2)*c) - C*log((sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(C*b^3*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3 - B*b^3*(sqrt(b*x + a)*sqrt(
-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^2*f - 2*C*a^2*b^2*sqrt(-c)*c
*e^2*f + A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt
(-c)*e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c*f^3)/((b
^2*e^2*f^2 - a^2*f^4)*(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^2*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f - 4
*a^2*c^2*f)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx=\frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}+\frac{(2a^2f^2(2Ce-Bf)-b^2e(Ce^2+f(Be-3Af)))\left(a^2-b^2x^2\right)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}+\frac{(A(2b^4e^2+a^2b^2f^2)+a^2(2a^2Cf^2+b^2e(Ce-3Bf)))\sqrt{a^2c-b^2cx^2}\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}\left(b^2e^2-a^2f^2\right)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] 1/2*f*(A+e*(-B*f+C*e)/f^2)*(-b^2*x^2+a^2)/(-a^2*f^2+b^2*e^2)/(f*x+e)^2/(b*x
+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*e*(C*e^2+f*(-3
*A*f+B*e)))*(-b^2*x^2+a^2)/f/(-a^2*f^2+b^2*e^2)^2/(f*x+e)/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(1/2)+1/2*(A*(a^2*b^2*f^2+2*b^4*e^2)+a^2*(2*a^2*C*f^2+b^2*e*(-3*B
*f+C*e)))*arctan((b^2*e*x+a^2*f)*c^(1/2)/(-a^2*f^2+b^2*e^2)^(1/2)/(-b^2*c*x
^2+a^2*c)^(1/2))*(-b^2*c*x^2+a^2*c)^(1/2)/(-a^2*f^2+b^2*e^2)^(5/2)/c^(1/2)/
(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {1624, 1665, 821, 739, 210}

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{f(a^2-b^2x^2)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)} + \frac{(a^2-b^2x^2)\,(2a^2f^2(2Ce-Bf)-b^2(ef(Be-3Af)+Ce^3))}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^2} + \frac{\sqrt{a^2c-b^2cx^2}(2a^4Cf^2+A(a^2b^2f^2+2b^4e^2)+a^2b^2e(Ce-3Bf))\arctan\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\,(b^2e^2-a^2f^2)^{5/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2c-b^2cx^2}\int\frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int\frac{2c\left(Ab^2e+a^2(Ce-Bf)\right)-c\left(2a^2Cf-b^2\left(Be+\frac{Ce^2}{f}-Af\right)\right)x}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{2c\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af))\right)(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left((2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{2\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af))\right)(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} - \frac{\left((2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int\frac{1}{-b^2ce^2+a^2cf^2-x^2}\,dx,x\right.}{2\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$
= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)(a^2-b^2x^2)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af))\right)(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2)\right)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{\sqrt{c}(a^2f+b^2ex)}{\sqrt{b^2e^2-a^2f^2}\sqrt{a^2c-b^2cx^2}}\right)}{2\sqrt{c}\left(b^2e^2-a^2f^2\right)^{5/2}\sqrt{a+bx}\sqrt{ac-bcx}}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.69

$$
\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{\frac{(-a+bx)\sqrt{a+bx}\left(b^2e\left(Ce^2x+Be(2e+fx)-Af(4e+3fx)\right)+a^2f\left(-Ce(3e+4fx)+f(Af+B(e+2fx))\right)\right)}{2(be-af)^2(be+af)^2(e+fx)^2} + \frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+}{(be-af)^{5/2}}}{\sqrt{c(a-bx)}}
$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] (((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e + 3
*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e -
 a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f
) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt
[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e + a*f)
^(5/2)))/Sqrt[c*(a - b*x)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. 2(333) = 666.

Time = 5.68 (sec) , antiderivative size = 1794, normalized size of antiderivative = 4.94

method	result	size
default	Expression too large to display	1794

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2*(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^2
*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b
```

```
^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*e*
f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^
2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c
*e^3*f+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2
*x^2+a^2))^(1/2)*f)/(f*x+e))*a^4*c*f^4*x^2+2*B*a^2*f^4*x*(c*(-b^2*x^2+a^2))
^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*A*b^2*e^2*f^2*(c*(-b^2*x^2+a^2))^(
1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+A*a^2*f^4*(c*(-b^2*x^2+a^2))^(1/2)*(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^
2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^4+2*C*ln(2*(b^2*
c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(
f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^4+B*a^2*e*f^3*(c*(-b^
2*x^2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*B*b^2*e^3*f*(c*(-b^2*x^
2+a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*C*a^2*e^2*f^2*(c*(-b^2*x^2+
a^2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*A*b^2*e*f^3*x*(c*(-b^2*x^2+a^
2))^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+B*b^2*e^2*f^2*x*(c*(-b^2*x^2+a^2)
)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-4*C*a^2*e*f^3*x*(c*(-b^2*x^2+a^2))^
(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+C*b^2*e^3*f*x*(c*(-b^2*x^2+a^2))^(1/2
)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-3*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b
^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e*f^3*x^2
+C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2
))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2*x^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(
a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*
e*f^3*x-6*B*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^
2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2*x+2*C*ln(2*(b^2*c*e*x+a^2*c
*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2
*b^2*c*e^3*f*x)/c*(b*x+a)^(1/2)*(c*(-b*x+a))^(1/2)/(c*(-b^2*x^2+a^2))^(1/2)
/(a*f+b*e)/(a*f-b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)/f
```

Fricas [A] (verification not implemented)

none

Time = 40.27 (sec) , antiderivative size = 1355, normalized size of antiderivative = 3.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2
)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 +
```

```
A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f
 - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*a^2*
b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f^2)*x
^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2*e^3*f^
2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*
A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4
*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*
sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*
b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^
4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3
*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2
 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a
^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2
*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sq
rt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f^2)*(b^2*e*x + a^2
*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^4*c*f^2 - (b^4*c*e^
2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*e^3*f^2 - B*a^4*e*f^4 -
 A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*A*a^2*b^2)*e^2*f^
3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4*f^5 - (5*C*a^2*b
^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*b^2*c*e^4*f^4 - a
^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^4*b^2*c*e^2*f^6 -
 a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^
5 - a^6*c*e*f^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(335) = 670$.

Time = 0.72 (sec) , antiderivative size = 1425, normalized size of antiderivative = 3.93

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -((C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2 - 3*B*a^2*b^3*sqrt(-c)*e*f
 + 2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2)*arctan(-1/2*(2*b*c*e -
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 +
 a^2*f^2)*c))/((b^4*e^4 - 2*a^2*b^2*e^2*f^2 + a^4*f^4)*sqrt(-b^2*e^2 + a^2*
f^2)*c) + 2*(4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^
4*sqrt(-c)*c*e^5 - 2*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*
a*c))^6*sqrt(-c)*e^4*f + 4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*
c + 2*a*c))^4*sqrt(-c)*c*e^4*f - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt
(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^4*f - 14*C*a^2*b^4*(sqrt(b*x + a)*
sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 12*A*b^6*(sqr
t(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 16
*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)
*c^2*e^3*f^2 + 8*C*a^4*b^4*sqrt(-c)*c^3*e^3*f^2 + 5*C*a^2*b^3*(sqrt(b*x + a
)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 + 2*A*b^5*(sqrt
(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 + 10*B*
a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*
e^2*f^3 + 44*C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*c^2*e^2*f^3 + 40*A*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 8*B*a^4*b^4*sqrt(-c)*c^3*e^2*f^
```

```
3 - 3*B*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqr
t(-c)*e*f^4 - 8*C*a^4*b^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^4*sqrt(-c)*c*e*f^4 - 6*A*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^4*sqrt(-c)*c*e*f^4 - 20*B*a^4*b^3*(sqrt(b*x + a)*sqrt(-c)
- sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e*f^4 - 32*C*a^6*b^2*sqrt(-c)*
c^3*e*f^4 - 24*A*a^4*b^4*sqrt(-c)*c^3*e*f^4 + A*a^2*b^3*(sqrt(b*x + a)*sqrt
(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*f^5 + 4*B*a^4*b^2*(sqrt(b*x +
 a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*f^5 - 4*A*a^4*b^3*(
sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*f^5 + 1
6*B*a^6*b^2*sqrt(-c)*c^3*f^5)/((b^4*e^4*f^2 - 2*a^2*b^2*e^2*f^4 + a^4*f^6)*
(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c*e - (sqrt(b*
x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f - 4*a^2*c^2*f)^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9344, normalized size of antiderivative = 25.74

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
```

c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c
f(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2)
 - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2*
b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(
1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4
 - 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x
)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a
^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*
f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^
2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a
^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2)
 - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(
a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4
*c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8
 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2
*b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6*
f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^
(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c
*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3
*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a +
 b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1
/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1
/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) +
 (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*(
(a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2
f)((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(
b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2
*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^
(1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1
/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^
4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2*
a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^
4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4*
c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/
2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) - (6*B*a^2*b*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(a^4*f^4 +

b^4*e^4 - 2*a^2*b^2*e^2*f^2)) + (6*B*a^2*b*c^3*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(a^4*f^4 + b^4*e^4 - 2*a^2*b^2*e^2
*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^
8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^
2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*
e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(
1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1/2) - a^(1/2))^7)
 + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*(
(a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f
*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) -
 a^(1/2))^3)) + (C*a^2*(2*a^2*f^2 + b^2*e^2)*(2*atan(((((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2
*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2
*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*
atan((((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 + 4*C^2*a^6*b^2*e^2*f^2))/(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e
^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 -
52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)
*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a
^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (C*a^(3/2)*(2
*a^2*f^2 + b^2*e^2)*(8*C*a^(17/2)*f^7*(a*c)^(1/2) - 12*C*a^(13/2)*b^2*e^2*f
^5*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*e^6*f*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1/
2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4
*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)
))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e
^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e
^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e
^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b
^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b
e)^4(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2
 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^
2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2)/(b*e*(a*f
 + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)) - (C*a^(3/2)*(2*a^2*
f^2 + b^2*e^2)*(8*C*a^(17/2)*c*f^7*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*c*e^6*f*(a
*c)^(1/2) - 12*C*a^(13/2)*b^2*c*e^2*f^5*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1
/2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 -
4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8
))))/((a + b*x)^(1/2) - a^(1/2)) - ((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 +
 4*C^2*a^6*b^2*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4
 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*

```
(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f
^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*
e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6
*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b
^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*C^2*a^(9/2)*f*(a*c)^(1/2)*(2*a^2*f^2 + b^
2*e^2)^2)/(b^2*c*e^2*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3
/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
- ((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^1
0*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^
2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2
*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*
f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^
2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4
*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2
)^(1/2)))*(b^10*e^10*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^8*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^6*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^4*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2) + a^8*b^2*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2)))/(16*
C^2*a^8*f^4 + 4*C^2*a^4*b^4*e^4 + 16*C^2*a^6*b^2*e^2*f^2))))/(2*(a*f + b*e)
^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b^2*(a^2*f^2 + 2*b^2*e
^2)*(2*atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/
((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b
^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*atan((((((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))*(((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b
^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*
b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c
^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^
4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 -
b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e
^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*
A^2*b^3*(a^2*f^2 + 2*b^2*e^2)^2)/(e*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2
- a^2*c*f^2)^(3/2)) - (A*b*(a^2*f^2 + 2*b^2*e^2)*(4*A*a^(13/2)*b^2*c*f^7*(a
*c)^(1/2) + 8*A*a^(1/2)*b^8*c*e^6*f*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*c*e^4*f^
3*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*
(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^
6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))))/((a + b*x)^(1/2) - a^(1/2))
 + ((((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e
^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e
^2*f^8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10
+ 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a
^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b*(a^2*f^2 + 2
*b^2*e^2)*(4*A*a^(13/2)*b^2*f^7*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*e^4*f^3*(a*c
)^(1/2) + 8*A*a^(1/2)*b^8*e^6*f*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2
```

```
)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*
a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 - ((
((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e^10 -
 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^
8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*
a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^
2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^
10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^
2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*A^2
*a^(1/2)*b^2*f*(a*c)^(1/2)*(a^2*f^2 + 2*b^2*e^2)^2)/(c*e^2*(a*f + b*e)^4*(a
*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - ((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*
c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^
6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^
2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^
4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f
+ b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f
^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*
c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))*(b^8*e^10*(a^2*c*f^2 - b^2*
c*e^2) + a^8*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^6*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^4*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^2*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2)))/(16*A^2*b^6*e^4 + 4*A^2*a^4*b^2*f^4 + 16*A^2*a
^2*b^4*e^2*f^2))))/(2*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(
1/2)) + (3*B*a^2*b^2*e*f*(2*atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^
2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2
))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/
2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)
*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(
1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - 2*atan(((((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2
)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/(2*(a*f +
 b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2))
```

3.34 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 87

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{cx^2\sqrt{-1+dx}\sqrt{1+dx}}{3d^2} + \frac{\sqrt{-1+dx}\sqrt{1+dx}(2(2c+3ad^2)+3bd^2x)}{6d^4} + \frac{b\operatorname{arccosh}(dx)}{2d^3}$$

```
[Out] 1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*d
^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1624, 1823, 794, 223, 212}

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(1-d^2x^2)\,(2(3ad^2+2c)+3bd^2x)}{6d^4\sqrt{dx-1}\sqrt{dx+1}} + \frac{b\sqrt{d^2x^2-1}\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}} - \frac{cx^2(1-d^2x^2)}{3d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(c*x^2*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c +
3*a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) +
 (b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 +
d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{x(a+bx+cx^2)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\ &= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int \frac{x(2c+3ad^2+3bd^2x)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)\,(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)\,(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx, x, \frac{x}{\sqrt{-1+d^2x^2}}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{cx^2(1-d^2x^2)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(2(2c+3ad^2)+3bd^2x)\,(1-d^2x^2)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx \\
&= \frac{\sqrt{-1+dx}\sqrt{1+dx}(3d^2(2a+bx)+2c(2+d^2x^2))+6bd\,\text{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}
\end{aligned}
$$

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*b
*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

method	result
risch	$\frac{(2c\,d^2x^2+3b\,d^2x+6a\,d^2+4c)\sqrt{dx+1}\,\sqrt{dx-1}}{6d^4}+\frac{b\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\,\text{csgn}(d)c\,d^2x^2\sqrt{d^2x^2-1}+3\sqrt{d^2x^2-1}\,\text{csgn}(d)b\,d^2x+6\sqrt{d^2x^2-1}\,\text{csgn}(d)a\,d^2+4\sqrt{d^2x^2-1}\,\text{csgn}(d)c+3\ln\left(\left(\sqrt{d^2x^2-1}\right.\right.\right.}{6d^4\sqrt{d^2x^2-1}}$

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*d^2*x^2+3*b*d^2*x+6*a*d^2+4*c)*(d*x+1)^(1/2)*(d*x-1)^(1/2)/d^4+1/2
*b/d^2*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1)
)^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log(\sqrt{dx+1}-\sqrt{dx-1})}{d^2}}{6\,d}$$

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 16.03 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.66

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d\,x-1}\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,(\sqrt{d\,x-1}-\mathrm{i})^3}{(\sqrt{d\,x+1}-1)^3}+\frac{14\,b\,(\sqrt{d\,x-1}-\mathrm{i})^5}{(\sqrt{d\,x+1}-1)^5}+\frac{2\,b\,(\sqrt{d\,x-1}-\mathrm{i})^7}{(\sqrt{d\,x+1}-1)^7}+\frac{2\,b\,(\sqrt{d\,x-1}-\mathrm{i})}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,(\sqrt{d\,x-1}-\mathrm{i})^2}{(\sqrt{d\,x+1}-1)^2}+\frac{6\,d^3\,(\sqrt{d\,x-1}-\mathrm{i})^4}{(\sqrt{d\,x+1}-1)^4}-\frac{4\,d^3\,(\sqrt{d\,x-1}-\mathrm{i})^6}{(\sqrt{d\,x+1}-1)^6}+\frac{d^3\,(\sqrt{d\,x-1}-\mathrm{i})^8}{(\sqrt{d\,x+1}-1)^8}} + \frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.35 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}} dx$

Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}} dx = \frac{(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}}{2d^2} + \frac{(c+2ad^2)\operatorname{arccosh}(dx)}{2d^3}$$

```
[Out] 1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/
d^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(52) = 104.

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {915, 1829, 655, 223, 212}

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}} dx = \frac{\sqrt{d^2x^2-1}(2ad^2+c)\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}} - \frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 915

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[(d + e*x)^FracPart[m]*((f + g*x)^Fr
acPart[m]/(d*f + e*g*x^2)^FracPart[m]), Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left((c+2ad^2)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad+\frac{\left((c+2ad^2)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx,x,\frac{x}{\sqrt{-1+d^2x^2}}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}$$

$$= -\frac{b(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx(1-d^2x^2)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{(c+2ad^2)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{d(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}+2(c+2ad^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt[
(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.

Time = 5.70 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.85

method	result
risch	$\frac{(cx+2b)\sqrt{dx-1}\,\sqrt{dx+1}}{2d^2}+\frac{(2a\,d^2+c)\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(dx+1)(dx-1)}}{2d^2\sqrt{d^2}\,\sqrt{dx-1}\,\sqrt{dx+1}}$
default	$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(\sqrt{d^2x^2-1}\ \operatorname{csgn}(d)dcx+2\ln\left(\left(\sqrt{d^2x^2-1}\ \operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)a\,d^2+2\ \operatorname{csgn}(d)d\sqrt{d^2x^2-1}\,b+\ln\left(\left(\sqrt{d^2x^2-1}\ \operatorname{csgn}(d\right.\right.}{2d^3\sqrt{d^2x^2-1}}$

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/2*(2*a*d^2+c)/d^2*ln(x*d^2/
(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^
(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{dx-1}-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 312, normalized size of antiderivative = 6.00

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - \frac{\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}}$$

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

3.36 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+\frac{b\operatorname{arccosh}(dx)}{d}+a\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/d^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1624, 1823, 858, 223, 212, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{a\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}-\frac{c(1-d^2x^2)}{d^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int\frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\sqrt{-1+d^2x^2}\int\frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(a\sqrt{-1+d^2x^2}\right)\int\frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(b\sqrt{-1+d^2x^2}\right)\int\frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad+\frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\frac{1}{1-d^2x^2}\,dx,x,\frac{x}{\sqrt{-1+d^2x^2}}\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad+\frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int\frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx,x,\sqrt{-1+d^2x^2}\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}\\
&= -\frac{c(1-d^2x^2)}{d^2\sqrt{-1+dx}\sqrt{1+dx}}+\frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\\
&\quad+\frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+2a\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)+\frac{2b\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)
]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(-\operatorname{csgn}(d)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)c+\ln\left(\left(\sqrt{(dx+1)(dx-1)}\,\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)bd\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{d^2\sqrt{d^2x^2-1}}$

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*csgn(d)*c+ln(
(((d*x+1)*(d*x-1))^(1/2)*csgn(d)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) - \frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

3.37 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx=\frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x}+\frac{c\operatorname{arccosh}(dx)}{d}+b\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/x
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 135 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1624, 1821, 858, 223, 212, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx=-\frac{a(1-d^2x^2)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\operatorname{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(c\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-d^2x^2}\,dx, x, \frac{x}{\sqrt{-1+d^2x^2}}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx, x, \sqrt{-1+d^2x^2}\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} \\
&= -\frac{a(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} \\
&\quad + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x} + 2b\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\text{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
 + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result	siz
risch	$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x}+\frac{\left(\frac{c\ln\left(\frac{x\,d^2}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)}{\sqrt{d^2}}-b\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	95
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\text{csgn}(d)dbx+\sqrt{d^2x^2-1}\,\text{csgn}(d)da+\ln\left(\left(\sqrt{d^2x^2-1}\,\text{csgn}(d)+dx\right)\text{csgn}(d)\right)cx\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\text{csgn}(d)}{\sqrt{d^2x^2-1}\,xd}$	96

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+(c*ln(x*d^2/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/
(d^2)^(1/2)-b*arctan(1/(d^2*x^2-1)^(1/2)))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^
(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1\right.}}{dx}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.63 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
```

```
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}a}{x}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

3.38 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{x}+\frac{1}{2}(2c+ad^2)\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] 1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1624, 1821, 821, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}(ad^2+2c)\arctan\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*(a*(1 - d^2*x^2))/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*Arc
Tan[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+(2c+ad^2)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{4\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left((2c+ad^2)\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx, x, \sqrt{-1+d^2x^2}\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{(2c+ad^2)\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(a+2bx)\sqrt{-1+dx}\sqrt{1+dx}}{2x^2} + (2c+ad^2)\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
[Out] ((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[S
qrt[(-1 + d*x)/(1 + d*x)]]
```

Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(2bx+a)}{2x^{2}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\sqrt{(dx+1)(dx-1)}}{\sqrt{dx-1}\,\sqrt{dx+1}}$	76
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)^{2}\left(\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)a\,d^{2}x^{2}+2\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)c\,x^{2}-2\sqrt{d^{2}x^{2}-1}\,bx-\sqrt{d^{2}x^{2}-1}\,a\right)}{2\sqrt{d^{2}x^{2}-1}\,x^{2}}$	103

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(2*b*x+a)/x^2-(c+1/2*a*d^2)*arctan(1/(d^2*x
^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right) - c\arcsin\left(\frac{1}{d|x|}\right) + \frac{\sqrt{d^2x^2-1}b}{x} + \frac{\sqrt{d^2x^2-1}a}{2\,x^2}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) + \frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.81

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}} - c\left(\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i} - \frac{a\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2} + \frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.39 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{(3c+2ad^2)\sqrt{-1+dx}\sqrt{1+dx}}{3x}+\frac{1}{2}bd^2\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

```
[Out] 1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1
/2)*(d*x+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1624, 1821, 849, 821, 272, 65, 211}

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{(1-d^2x^2)(2ad^2+3c)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\arctan\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(a*(1 - d^2*x^2))/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2
))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(
```

```
3*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-
1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+(3c+2ad^2)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2(3c+2ad^2)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}\,dx}{6\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{4\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}+\frac{x^2}{d^2}}\,dx, x, \sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a(1-d^2x^2)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b(1-d^2x^2)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{(3c+2ad^2)(1-d^2x^2)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{bd^2\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+dx}\sqrt{1+dx}(3x(b+2cx)+a(2+4d^2x^2))}{6x^3} + bd^2\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]

[Out] (Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^3
) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,(4a\,d^2x^2+6c\,x^2+3bx+2a)}{6x^3}-\frac{b\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(dx+1)(dx-1)}}{2\sqrt{dx-1}\,\sqrt{dx+1}}$	89
default	$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)^2\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)b\,d^2x^3-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d^2x^2-1}\,a\right)}{6\sqrt{d^2x^2-1}\,x^3}$	123

[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(4*a*d^2*x^2+6*c*x^2+3*b*x+2*a)/x^3-1/2*b*d
^2*arctan(1/(d^2*x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1
)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\,(2\,ad^3+3\,cd)x^3+(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}\sqrt{d}}{6\,x^3}$$

[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right) + \frac{2\sqrt{d^2x^2-1}ad^2}{3\,x} + \frac{\sqrt{d^2x^2-1}c}{x} + \frac{\sqrt{d^2x^2-1}b}{2\,x^2} + \frac{\sqrt{d^2x^2-1}a}{3\,x^3}$$

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(92) = 184.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) + \frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{8}-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-\cdots}\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{\cdots}\right.\right.}}{3\,d}$$

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.62

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}} - \frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2} + \frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}} + \frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

3.40 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 199

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3} \, dx = -\frac{(cd^2-bde+ae^2)\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)(d+ex)^2} + \frac{(cd^3+bd^2e-(3a+4c)de^2+2be^3)\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)^2(d+ex)} + \frac{((2a+c)d^2-3bde+(a+2c)e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+e}\sqrt{1+x}}{\sqrt{d-e}\sqrt{-1+x}}\right)}{(d-e)^{5/2}(d+e)^{5/2}}$$

```
[Out] ((2*a+c)*d^2-3*b*d*e+(a+2*c)*e^2)*arctanh((d+e)^(1/2)*(1+x)^(1/2)/(d-e)^(1/
2)/(-1+x)^(1/2))/(d-e)^(5/2)/(d+e)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(-1+x)^(1/
2)*(1+x)^(1/2)/e/(d^2-e^2)/(e*x+d)^2+1/2*(c*d^3+b*d^2*e-(3*a+4*c)*d*e^2+2*b
*e^3)*(-1+x)^(1/2)*(1+x)^(1/2)/e/(d^2-e^2)^2/(e*x+d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1624, 1665, 821, 739, 212}

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx \\ = -\frac{\sqrt{x^2-1}\text{arctanh}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a(2d^2+e^2)+3bde-c(d^2+2e^2)\right)}{2\sqrt{x-1}\sqrt{x+1}\,(d^2-e^2)^{5/2}} \\ +\frac{(1-x^2)\,(ae^2-bde+cd^2)}{2e\sqrt{x-1}\sqrt{x+1}\,(d^2-e^2)\,(d+ex)^2} \\ -\frac{(1-x^2)\,(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))}{2e\sqrt{x-1}\sqrt{x+1}\,(d^2-e^2)^2\,(d+ex)}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(1 - x^2))/(2*e*(d^2 - e^2)*Sqrt[-1 + x]*Sqrt[1 +
x]*(d + e*x)^2) - ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*(1 -
 x^2))/(2*e*(d^2 - e^2)^2*Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)) - ((3*b*d*e -
 a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Sqrt[-1 + x^2]*ArcTanh[(e + d*x)/(Sqrt[
d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2)*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1624

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m]), Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-1+x^2}\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{(cd^2-bde+ae^2)(1-x^2)}{2e(d^2-e^2)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{\sqrt{-1+x^2}\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2(d^2-e^2)\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{(cd^2-bde+ae^2)(1-x^2)}{2e(d^2-e^2)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))(1-x^2)}{2e(d^2-e^2)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)} - \frac{\left(\left(-2d(ad+cd-be)-e\left(-bd-\frac{cd^2}{e}+ae+2ce\right)\right)\sqrt{-1+x^2}\right)\int \frac{1}{(d+ex)\sqrt{-1+x^2}}\,dx}{2(d^2-e^2)^2\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{(cd^2-bde+ae^2)(1-x^2)}{2e(d^2-e^2)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))(1-x^2)}{2e(d^2-e^2)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)} + \frac{\left(\left(-2d(ad+cd-be)-e\left(-bd-\frac{cd^2}{e}+ae+2ce\right)\right)\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{d^2-e^2-x^2}\,dx, x, \frac{-e-dx}{\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^2\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{(cd^2-bde+ae^2)(1-x^2)}{2e(d^2-e^2)\sqrt{-1+x}\sqrt{1+x}(d+ex)^2} - \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))(1-x^2)}{2e(d^2-e^2)^2\sqrt{-1+x}\sqrt{1+x}(d+ex)} - \frac{(3bde-a(2d^2+e^2)-c(d^2+2e^2))\sqrt{-1+x^2}\tanh^{-1}\left(\frac{e+dx}{\sqrt{d^2-e^2}\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^{5/2}\sqrt{-1+x}\sqrt{1+x}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$$
$$=\frac{\sqrt{-1+x}\sqrt{1+x}(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{2(d-e)^2(d+e)^2(d+ex)^2}$$
$$-\frac{(-3bde+a(2d^2+e^2)+c(d^2+2e^2))\arctan\left(\frac{\sqrt{d-e}\sqrt{\frac{-1+x}{1+x}}}{\sqrt{-d-e}}\right)}{(-d-e)^{5/2}(d-e)^{5/2}}$$

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[1 + x]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2
*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(2*(d - e)^2*(d + e
)^2*(d + e*x)^2) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTan[(
Sqrt[d - e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[-d - e]])/((-d - e)^(5/2)*(d - e)^
(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(175) = 350$.

Time = 1.69 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.50

method	result	size
default	Expression too large to display	1095

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*e
^2*x^2+4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^
3*e*x+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d*e
^3*x-6*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d^2*
e^2*x+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^3
*e*x+4*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d*e^
3*x+4*a*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-2*b*d^3*e*(x^2-1)^(1/2)
*((d^2-e^2)/e^2)^(1/2)-b*d*e^3*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+3*c*d^2*
e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/
e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2*x^2-3*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^
2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d*e^3*x^2+3*a*d*e^3*x*(x^2-1)^(1/2)*((d^2
-e^2)/e^2)^(1/2)-b*d^2*e^2*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-c*d^3*e*x*
(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+4*c*d*e^3*x*(x^2-1)^(1/2)*((d^2-e^2)/e^
```

```
2)^(1/2)-2*b*e^4*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+ln(-2*(-(x^2-1)^(1/2
)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*e^4*x^2+2*ln(-2*(-(x^2-1)^(1/2)
*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*e^4*x^2+ln(-2*(-(x^2-1)^(1/2)*((
d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2-3*ln(-2*(-(x^2-1)^(1/2)*((d
^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*b*d^3*e+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-
e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^2*e^2-a*e^4*(x^2-1)^(1/2)*((d^2-e^2)/
e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*
a*d^4+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)*e+d*x+e)/(e*x+d))*c*d^4)*
(-1+x)^(1/2)*(1+x)^(1/2)/(x^2-1)^(1/2)/(d+e)/(d-e)/(d^2-e^2)/(e*x+d)^2/((d^
2-e^2)/e^2)^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(175) = 350$.

Time = 0.32 (sec) , antiderivative size = 1186, normalized size of antiderivative = 5.96

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*sqrt(x + 1)*sqrt(x - 1) + sqr
t(d^2 - e^2)*(d*x + e))/(e*x + d)) + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 - b
*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3 -
 (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x +
 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4
+ (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^
2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e
^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*
d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a
 + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a +
c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e
^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*
e^5)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*e*sqrt(x + 1)*sqrt(x - 1
) - sqrt(-d^2 + e^2)*(e*x + d))/(d^2 - e^2)) + (2*b*d^5*e^2 - (4*a + 3*c)*d
^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b
*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x
)*sqrt(x + 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 +
```

```
b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^
4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3
 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((e-d)*(e+d)>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \int \frac{cx^2+bx+a}{(ex+d)^3\sqrt{x+1}\sqrt{x-1}}\,dx$$

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 72.89 (sec) , antiderivative size = 7235, normalized size of antiderivative = 36.36

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x + c*x^2)/((x - 1)^(1/2)*(x + 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] ((((x - 1)^(1/2) - 1i)^2*(2*c*e^3 + c*d^2*e)*12i)/(d^2*((x + 1)^(1/2) - 1)^
2*(d^4 + e^4 - 2*d^2*e^2)) - (2*(7*c*d^4 + 14*c*d^2*e^2)*((x - 1)^(1/2) - 1
i))/(7*d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x - 1)^(1/2) -
 1i)^4*(2*c*e^3 - c*d^2*e)*24i)/(d^2*((x + 1)^(1/2) - 1)^4*(d^4 + e^4 - 2*d
^2*e^2)) - (2*(21*c*d^4 - 102*c*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(3*d^3*((x
 + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) - (2*(35*c*d^4 - 170*c*d^2*e^2)
*((x - 1)^(1/2) - 1i)^3)/(5*d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^
2)) + (c*((x - 1)^(1/2) - 1i)^7*(d^2*1i + e^2*2i)*2i)/(d*((x + 1)^(1/2) - 1
)^7*(d^4 + e^4 - 2*d^2*e^2)) + (12*c*e*((x - 1)^(1/2) - 1i)^6*(d^2*1i + e^2
*2i))/(d^2*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2)
- 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/
2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x
 - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)
^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(d^2
*((x + 1)^(1/2) - 1)^4) + 1) - ((2*((x - 1)^(1/2) - 1i)^3*(16*b*e^3 + 11*b*
d^2*e))/(d^2*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*((x -
1)^(1/2) - 1i)^7)/(((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*
((x - 1)^(1/2) - 1i))/(((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x
- 1)^(1/2) - 1i)^4*(2*b*e^4 - 2*b*d^4 + 3*b*d^2*e^2)*8i)/(d^3*((x + 1)^(1/2
) - 1)^4*(d^4 + e^4 - 2*d^2*e^2)) + (b*((x - 1)^(1/2) - 1i)^2*(2*d^4 + 2*e^
4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 - 2*d^2*e^2)) + (b
*((x - 1)^(1/2) - 1i)^6*(2*d^4 + 2*e^4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2)
 - 1)^6*(d^4 + e^4 - 2*d^2*e^2)) + (2*b*e*((x - 1)^(1/2) - 1i)^5*(11*d^2 +
16*e^2))/(d^2*((x + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/
2) - 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^
(1/2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*
((x - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) -
1i)^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e
^2))/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(
d^2*((x + 1)^(1/2) - 1)^4) + 1) + ((2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2
) - 1i))/(d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) - (((x - 1)^(1/2
) - 1i)^4*(2*a*e^5 - 9*a*d^2*e^3 + 4*a*d^4*e)*8i)/(d^4*((x + 1)^(1/2) - 1)^
4*(d^4 + e^4 - 2*d^2*e^2)) + (2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2) - 1i
```

)^7)/(d^3*((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (2*(2*a*e^4 - 29
*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^3)/(d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 -
2*d^2*e^2)) - (2*(2*a*e^4 - 29*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(d^3*((x
+ 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^2*(4*
a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4i)/(d^4*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 -
2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^6*(4*a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4
i)/(d^4*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2) - 1
i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/2)
- 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x -
1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)^7*
8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(d^2
((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4(6*d^2 - 32*e^2))/(d^2*((
x + 1)^(1/2) - 1)^4) + 1) - (c*atan(((c*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2
*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2
) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(((x
+ 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) -
(c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4
*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10
+ d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2
*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/((
(x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)))
)/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (c
*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8
- 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i
- c*d^2*e^5*12i + c*d^6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*
d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1
i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*
e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*
d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^
4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/
(2*(d + e)^(5/2)*(d - e)^(5/2)))/((8*(c^2*d^4 + 4*c^2*e^4 + 4*c^2*d^2*e^2))
/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (8*((x - 1)^(1/2) -
1i)^2*(c^2*d^4 + 4*c^2*e^4 + 4*c^2*d^2*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10
+ d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (c*(d^2 + 2*e^2)*((4*(c*e
^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^
4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i - c*d^2*e^5*12i + c*d^
6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4
*d^8*e^2)) - (c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/
2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^
8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^
(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28
*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 -
4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(

5/2)) + (c*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2
*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^
2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (c*(d^2 + 2*e^2)*((e*((x - 1)
^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^
8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^
6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e
^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 +
d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5
/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))*(d^2 + 2*e^2)*1i)/((d + e)^(5/2)*(
d - e)^(5/2)) - (a*atan(((a*(2*d^2 + e^2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a
*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x -
 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(((x + 1)^(1/2)
- 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (a*(2*d^2 +
e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4
*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 1
2*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2
) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)
^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (a*(2*d^2 + e^
2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6
 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3
*12i + a*d^6*e*8i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*
d^6*e^4 - 4*d^8*e^2)) + (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*(
(x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*
e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (
4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d
^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 +
6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(
5/2)*(d - e)^(5/2)))/((8*(4*a^2*d^4 + a^2*e^4 + 4*a^2*d^2*e^2))/(d^10 + d^2
*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (8*((x - 1)^(1/2) - 1i)^2*(4*a^
2*d^4 + a^2*e^4 + 4*a^2*d^2*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (a*(2*d^2 + e^2)*((4*(a*e^7*4i - a*d^
4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^
2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(((
x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) -
 (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (
4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^1
0 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^
2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(
((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))
))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (a*(
2*d^2 + e^2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i -
 a*d^4*e^3*12i + a*d^6*e*8i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^

4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)
64i)/(d((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^
6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^
8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*
e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*
d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d
 + e)^(5/2)*(d - e)^(5/2))))*(2*d^2 + e^2)*1i)/((d + e)^(5/2)*(d - e)^(5/2)
) + (b*d*e*atan(((b*d*e*((4*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/
(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) -
1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/(((x + 1)^(1/2) - 1)^2
*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (3*b*d*e*((e*((x -
 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2
*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6
*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^
2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)
^(5/2)))*3i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (b*d*e*((4*(b*d^5*e^2*12i -
b*d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d
^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^
6*12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*
d^8*e^2)) + (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1))
- (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(
d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1
i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2)
)/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^
2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*3i)/(2*(d + e)^(5/2)*(d - e)^(5/2)))
/((72*b^2*d^2*e^2)/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (
72*b^2*d^2*e^2*((x - 1)^(1/2) - 1i)^2)/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e
^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (3*b*d*e*((4*(b*d^5*e^2*12i - b*
d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8
e^2) + (4((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*
12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^
8*e^2)) - (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) -
(4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^
10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)
^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/
(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)
)))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (3*
b*d*e*((4*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*1
2i - b*d^3*e^4*24i + b*d*e^6*12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64
i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 +
 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e
^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6

```
 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4
*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d +
e)^(5/2)*(d - e)^(5/2))))*3i)/((d + e)^(5/2)*(d - e)^(5/2))
```

3.41 $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1348

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+9\right.}{}$$
$$+\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+9c^2def^2}{}$$
$$-\frac{(2aCdf-b(4Bdf-3C(de+cf)))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$
$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$
$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38cdef+3\right.}{}$$
$$-\frac{(de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f\right.}{}$$

```
[Out] -1/20*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e)))*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)
^(3/2)/b/d^2/f^2+1/6*C*(b*x+a)^3*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/960*(d
*x+c)^(3/2)*(f*x+e)^(3/2)*(64*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-7*C*(
c*f+d*e))-8*a*b^2*d*f*(C*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A*d*f
-5*B*(c*f+d*e)))+b^3*(7*C*(15*c^3*f^3+17*c^2*d*e*f^2+17*c*d^2*e^2*f+15*d^3*
e^3)+4*d*f*(50*A*d*f*(c*f+d*e)-B*(35*c^2*f^2+38*c*d*e*f+35*d^2*e^2)))+6*b*d
*f*(10*b*d*f*(-4*A*b*d*f+C*a*c*f+C*a*d*e+2*C*b*c*e)+(4*a*d*f-7*b*(c*f+d*e))
*(2*a*C*d*f-b*(4*B*d*f-3*C*(c*f+d*e))))*x)/b/d^4/f^4-1/512*(-c*f+d*e)^2*(8*
a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))
-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*
d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4*f^4+28*c^3
```

```
*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2*A*d*f*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e
^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(11/2)/f^(11/
2)+1/256*(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B
*(c*f+d*e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)
+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^
4*f^4+28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2
*A*d*f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e
^2*f+7*d^3*e^3))))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^5/f^4+1/512*(-c*f+d*e)*(8*
a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))
-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*
d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4*f^4+28*c^3
*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2*A*d*f*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e
^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^5/f^5
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 1345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1629, 158, 152, 52, 65, 223, 212}

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \frac{C(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^3}{6bdf}$$

$$+\frac{(4bBdf-2aCdf-3bC(de+cf))(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^2}{20bd^2f^2}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3)+4df(50Adf(de+cf)-B(35d^2\right.\right.}{}$$

$$-\frac{(de-cf)^2\left(\left(C(21d^4e^4+28cd^3fe^3+30c^2d^2f^2e^2+28c^3df^3e+21c^4f^4)+4df(2Adf(5d^2e^2+6cdfe+5c^2\right.\right.}{}$$

$$+\frac{\left(\left(C(21d^4e^4+28cd^3fe^3+30c^2d^2f^2e^2+28c^3df^3e+21c^4f^4)+4df(2Adf(5d^2e^2+6cdfe+5c^2f^2)-B(7\right.\right.}{}$$

$$+\frac{(de-cf)\left(\left(C(21d^4e^4+28cd^3fe^3+30c^2d^2f^2e^2+28c^3df^3e+21c^4f^4)+4df(2Adf(5d^2e^2+6cdfe+5c^2\right.\right.}{}$$

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*
(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*
e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f
^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f +
 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*
```

```
Sqrt[c + d*x]*Sqrt[e + f*x])/(512*d^5*f^5) + ((8*a^2*d^2*f^2*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*
(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d
*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 +
28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*
(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f
 + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(256*d^5*f^
4) + ((4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/
2)*(e + f*x)^(3/2))/(20*b*d^2*f^2) + (C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*
x)^(3/2))/(6*b*d*f) - ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 -
8*a^2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 +
 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*
(15*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d
*f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c*
f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(960*b*d^4*f^4) - ((d*
e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2
*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2
 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5
*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*Ar
cTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(512*d^(11/2)*f^(11
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
```

```
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$
$$+\frac{\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3}{2}b(2bcCe+aCde+acCf-4Abdf)+\frac{3}{2}b(4bBdf-2aCdf-3bC(de+c\right.}{6b^2df}$$

$$=\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$
$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$
$$+\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3}{4}b(10adf(2bcCe+aCde+acCf-4Abdf)+(4bce+3a(de+cf)\right.}{}$$

$$=\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$
$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$
$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38c\right.}{}$$
$$+\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+}{}$$

$$=\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+9c}{}$$
$$+\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$
$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$
$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38c\right.}{}$$
$$+\frac{((de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9c\right.}{}$$

$$+\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+}{}$$

$$+\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38\right.}{}$$

$$-\frac{((de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9c\right.}{}$$

$$+\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+}{}$$

$$+\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38\right.}{}$$

$$-\frac{((de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9c\right.}{}$$

$$+\frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+}{}$$

$$+\frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38\right.}{}$$

$$-\frac{((de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd\right.}{}$$

$$+ \frac{(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f+}{}$$

$$+ \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{20bd^2f^2}$$

$$+ \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$- \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(de+cf))-8ab^2df(C(35d^2e^2+38c\right.}{}$$

$$- \frac{(de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+\right.}{}$$

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 1253, normalized size of antiderivative = 0.93

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(40a^2d^2f^2(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2}{}$$

$$- \frac{(de-cf)^2\left(8a^2d^2f^2(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-8abdf(C(7d^3e^3+9cd^2e^2f\right.}{}$$

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(40*a^2*d^2*f^2*(C*(15*c^3*f^3 - c^2*d*f^2*(7*
e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2
*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) +
B*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) +
 8*a*b*d*f*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-
17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*
x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*
x^3 + 384*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2
*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x)
 + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*
f^2*x^2 + 48*f^3*x^3)))) + b^2*(C*(315*c^5*f^5 - 105*c^4*d*f^4*(e + 2*f*x)
+ 2*c^3*d^2*f^3*(-41*e^2 + 28*e*f*x + 84*f^2*x^2) - 2*c^2*d^3*f^2*(41*e^3 -
 26*e^2*f*x + 20*e*f^2*x^2 + 72*f^3*x^3) + c*d^4*f*(-105*e^4 + 56*e^3*f*x -
 40*e^2*f^2*x^2 + 32*e*f^3*x^3 + 128*f^4*x^4) + d^5*(315*e^5 - 210*e^4*f*x
+ 168*e^3*f^2*x^2 - 144*e^2*f^3*x^3 + 128*e*f^4*x^4 + 1280*f^5*x^5)) + 4*d*
```

```
f*(10*A*d*f*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*
f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) +
B*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 + 11*
e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24*f^3
*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 384*f^
4*x^4))))))/(7680*d^5*f^5) - ((d*e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 +
6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7
*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e
 + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28
*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2
*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f +
 9*c^2*d*e*f^2 + 7*c^3*f^3))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqr
t[c + d*x])])/(512*d^(11/2)*f^(11/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5733 vs. 2(1304) = 2608.

Time = 1.68 (sec) , antiderivative size = 5734, normalized size of antiderivative = 4.25

method	result	size
default	Expression too large to display	5734

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 1.58 (sec) , antiderivative size = 3096, normalized size of antiderivative = 2.30

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/30720*(15*(21*C*b^2*d^6*e^6 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)
*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b
+ A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*
(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*
b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B
*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^
```

5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(1280*C*b^2*d^6*f^6*x^5 + 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)*d^6)*e^4*f^2 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*b + B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*b)*d^6)*e^2*f^4 - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)*c*d^5)*e*f^5 + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*d^4)*f^6 + 128*(C*b^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*f^6)*x^4 - 16*(9*C*b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d^6)*e*f^5 + 3*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 28*(2*C*a*b + B*b^2)*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*C*a*b + B*b^2)*c^2*d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 + 2*A*a*b)*d^6)*f^6)*x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*C*a*b + B*b^2)*d^6)*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*d^5 - 100*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*(2*C*a*b + B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2 + 2*A*a*b)*d^6)*e*f^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b + B*b^2)*c^3*d^3 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*f^6)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6), 1/15360*(15*(21*C*b^2*d^6*e^6 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(1280*C*b^2*d^6*f^6*x^5 + 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)*d^6)*e^4*f^2 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*b + B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*b)*d^6)*e^2*f^4 - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)

```
*c*d^5)*e*f^5 + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)
*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*
d^4)*f^6 + 128*(C*b^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*
f^6)*x^4 - 16*(9*C*b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d
^6)*e*f^5 + 3*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*
B*a*b + A*b^2)*d^6)*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 2
8*(2*C*a*b + B*b^2)*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c
*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*
C*a*b + B*b^2)*c^2*d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 +
2*A*a*b)*d^6)*f^6)*x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*
C*a*b + B*b^2)*d^6)*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*
d^5 - 100*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*
(2*C*a*b + B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2
+ 2*A*a*b)*d^6)*e*f^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b +
 B*b^2)*c^3*d^3 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*
b)*c*d^5)*f^6)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6)]
```

Sympy [F]

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (a+bx)^2\,\sqrt{c+dx}\,\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4656 vs. $2(1304) = 2608$.

Time = 0.86 (sec) , antiderivative size = 4656, normalized size of antiderivative = 3.45

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/7680*(7680*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*
e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)*sqrt(d*x + c))*A*a^2*c*abs(d)/d^2 - 320*(sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4
)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4))
- 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*
sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*C*
a^2*c*abs(d)/d^2 - 640*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(
2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3*(d
^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 + c*d^
2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqr
t(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*a*b*c*abs(d)/d^2 -
80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x +
 c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c
*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*
e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) +
3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*
f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
))/(sqrt(d*f)*d^2*f^3))*C*a*b*c*abs(d)/d^2 - 320*(sqrt(d^2*e + (d*x + c)*d*
f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*
d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7
*f^4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqr
t(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f
^2))*A*b^2*c*abs(d)/d^2 - 40*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x +
 c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)
) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(
5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d
^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2
+ 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*
e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*B*b^2*c*abs(d)/d^2 - 4*(s
qrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c
)/d^4 + (d^20*e*f^7 - 41*c*d^19*f^8)/(d^23*f^8)) - (7*d^21*e^2*f^6 + 26*c*d
^20*e*f^7 - 513*c^2*d^19*f^8)/(d^23*f^8)) + 5*(7*d^22*e^3*f^5 + 19*c*d^21*e
^2*f^6 + 37*c^2*d^20*e*f^7 - 447*c^3*d^19*f^8)/(d^23*f^8))*(d*x + c) - 15*(
```

```
7*d^23*e^4*f^4 + 12*c*d^22*e^3*f^5 + 18*c^2*d^21*e^2*f^6 + 28*c^3*d^20*e*f^
7 - 193*c^4*d^19*f^8)/(d^23*f^8))*sqrt(d*x + c) - 15*(7*d^5*e^5 + 5*c*d^4*e
^4*f + 6*c^2*d^3*e^3*f^2 + 10*c^3*d^2*e^2*f^3 + 35*c^4*d*e*f^4 - 63*c^5*f^5
)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/
(sqrt(d*f)*d^3*f^4))*C*b^2*c*abs(d)/d^2 - 320*(sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5
*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^
4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d
*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2)
)*B*a^2*abs(d)/d - 40*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*
(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*
d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*
e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6
))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^
3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*
x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*a^2*abs(d)/d - 640*(sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^
6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^
2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^
3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))
/(sqrt(d*f)*d*f^2))*A*a*b*abs(d)/d - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)
/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*
f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d
^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d
^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c)
+ sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*B*a*b*abs(d)/d
 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(
d*x + c)/d^4 + (d^20*e*f^7 - 41*c*d^19*f^8)/(d^23*f^8)) - (7*d^21*e^2*f^6 +
 26*c*d^20*e*f^7 - 513*c^2*d^19*f^8)/(d^23*f^8)) + 5*(7*d^22*e^3*f^5 + 19*c
*d^21*e^2*f^6 + 37*c^2*d^20*e*f^7 - 447*c^3*d^19*f^8)/(d^23*f^8))*(d*x + c)
 - 15*(7*d^23*e^4*f^4 + 12*c*d^22*e^3*f^5 + 18*c^2*d^21*e^2*f^6 + 28*c^3*d^
20*e*f^7 - 193*c^4*d^19*f^8)/(d^23*f^8))*sqrt(d*x + c) - 15*(7*d^5*e^5 + 5*
c*d^4*e^4*f + 6*c^2*d^3*e^3*f^2 + 10*c^3*d^2*e^2*f^3 + 35*c^4*d*e*f^4 - 63*
c^5*f^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*
d*f)))/(sqrt(d*f)*d^3*f^4))*C*a*b*abs(d)/d - 40*(sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d
^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6
)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 -
93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f +
 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d
*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*A*b^2*
abs(d)/d - 4*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x +
 c)*(8*(d*x + c)/d^4 + (d^20*e*f^7 - 41*c*d^19*f^8)/(d^23*f^8)) - (7*d^21*e
^2*f^6 + 26*c*d^20*e*f^7 - 513*c^2*d^19*f^8)/(d^23*f^8)) + 5*(7*d^22*e^3*f^
```

```
5 + 19*c*d^21*e^2*f^6 + 37*c^2*d^20*e*f^7 - 447*c^3*d^19*f^8)/(d^23*f^8))*(
d*x + c) - 15*(7*d^23*e^4*f^4 + 12*c*d^22*e^3*f^5 + 18*c^2*d^21*e^2*f^6 + 2
8*c^3*d^20*e*f^7 - 193*c^4*d^19*f^8)/(d^23*f^8))*sqrt(d*x + c) - 15*(7*d^5*
e^5 + 5*c*d^4*e^4*f + 6*c^2*d^3*e^3*f^2 + 10*c^3*d^2*e^2*f^3 + 35*c^4*d*e*f
^4 - 63*c^5*f^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*
d*f - c*d*f)))/(sqrt(d*f)*d^3*f^4))*B*b^2*abs(d)/d - (sqrt(d^2*e + (d*x + c
)*d*f - c*d*f)*(2*(4*(2*(d*x + c)*(8*(d*x + c)*(10*(d*x + c)/d^5 + (d^30*e*
f^9 - 61*c*d^29*f^10)/(d^34*f^10)) - 3*(3*d^31*e^2*f^8 + 14*c*d^30*e*f^9 -
417*c^2*d^29*f^10)/(d^34*f^10)) + (21*d^32*e^3*f^7 + 77*c*d^31*e^2*f^8 + 18
3*c^2*d^30*e*f^9 - 3481*c^3*d^29*f^10)/(d^34*f^10))*(d*x + c) - 5*(21*d^33*
e^4*f^6 + 56*c*d^32*e^3*f^7 + 106*c^2*d^31*e^2*f^8 + 176*c^3*d^30*e*f^9 - 2
279*c^4*d^29*f^10)/(d^34*f^10))*(d*x + c) + 15*(21*d^34*e^5*f^5 + 35*c*d^33
*e^4*f^6 + 50*c^2*d^32*e^3*f^7 + 70*c^3*d^31*e^2*f^8 + 105*c^4*d^30*e*f^9 -
 793*c^5*d^29*f^10)/(d^34*f^10))*sqrt(d*x + c) + 15*(21*d^6*e^6 + 14*c*d^5*
e^5*f + 15*c^2*d^4*e^4*f^2 + 20*c^3*d^3*e^3*f^3 + 35*c^4*d^2*e^2*f^4 + 126*
c^5*d*e*f^5 - 231*c^6*f^6)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e +
(d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^4*f^5))*C*b^2*abs(d)/d - 1920*(sqrt(d
^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*
x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x
+ c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*f))*B*a^2*c*abs(d)/
d^3 - 3840*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e*f - 5*c
*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*log(abs(-s
qrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*f
))*A*a*b*c*abs(d)/d^3 - 1920*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x +
2*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2
*d*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)))/(sqrt(d*f)*f))*A*a^2*abs(d)/d^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

```
[In] int((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.42 $\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 721

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2def\right.\right.\right.}{128d^4f^4}$$
$$+\frac{\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2def^2+7c^3\right.\right.\right.}{64d^4f^3}$$
$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$
$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2\left(C\left(35d^2e^2+38cdef+35c^2f^2\right)\right.\right.}{240bd^3f^3}$$
$$-\frac{(de-cf)^2\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2\right.\right.\right.}{128d^{9/2}f^{9/2}}$$

```
[Out] 1/5*C*(b*x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/240*(d*x+c)^(3/2)*(f*x+
e)^(3/2)*(48*a^2*C*d^2*f^2-10*a*b*d*f*(8*B*d*f-5*C*(c*f+d*e))-b^2*(C*(35*c^
2*f^2+38*c*d*e*f+35*d^2*e^2)+10*d*f*(8*A*d*f-5*B*(c*f+d*e)))+6*b*d*f*(6*a*C
*d*f-b*(10*B*d*f-7*C*(c*f+d*e)))*x)/b/d^3/f^3-1/128*(-c*f+d*e)^2*(2*a*d*f*(
C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*
f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^
2*f^2+6*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/d^(9/2)/f^(9/2)+1/64*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*
d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^
3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/d^4/f^3+1/128*(-c*f+d*e)*(2*a*d*f*(C*(5*c^2*f^2+6*c*d
*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+
9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*
d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^4/f^4
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1629, 152, 52, 65, 223, 212}

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx =$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(c\right.}{240bd^3f^3}$$

$$-\frac{(de-cf)^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2adf(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2))-b(2df(8Ad\right.}{128d^{9/2}f^{9/2}}$$

$$+\frac{(c+dx)^{3/2}\sqrt{e+fx}(2adf(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2))-b(2df(8Adf(cf+de}{64d^4f^3}$$

$$+\frac{\sqrt{c+dx}\sqrt{e+fx}(de-cf)\left(2adf(8df(2Adf-B(cf+de))+C(5c^2f^2+6cdef+5d^2e^2))-b(2df(8Adf(\right.}{128d^4f^4}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

```
[In] Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d
*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*
c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^
2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(128*d^4*f^4) + ((2*a*d*f*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e
^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f
) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*(c + d*x)^(3/2)*Sqrt[e + f*x])
/(64*d^4*f^3) + (C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) -
 ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f -
 5*C*(d*e + c*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*
(8*A*d*f - 5*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e
 + c*f))*x))/(240*b*d^3*f^3) - ((d*e - c*f)^2*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$+\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{1}{2}b(4bcCe+3aCde+3acCf-10Abdf)+\frac{1}{2}b(10bBdf-6aCdf-7bC(de\right.}{5b^2df}$$

$$=\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+3\right.}{240bd^3f^3}$$

$$+\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def^2}{32d^3f^3}$$

$$=\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def^2+}{64d^4f^3}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+3\right.}{240bd^3f^3}$$

$$+\frac{((de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f\right.}{128d^4f^3}$$

$$=\frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9\right.}{128d^4f^4}$$

$$+\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def^2}{64d^4f^3}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+3\right.}{240bd^3f^3}$$

$$-\frac{((de-cf)^2\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2\right.}{256d^4}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+}{128d^4f^4}$$

$$+\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def}{64d^4f^3}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+\right.}{240bd^3f^3}$$

$$-\frac{((de-cf)^2\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e\right.}{}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+}{128d^4f^4}$$

$$+\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def}{64d^4f^3}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+\right.}{240bd^3f^3}$$

$$-\frac{((de-cf)^2\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e\right.}{}$$

$$= \frac{(de-cf)\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+}{128d^4f^4}$$

$$+\frac{(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2f+9c^2def}{64d^4f^3}$$

$$+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-10abdf(8Bdf-5C(de+cf))-b^2(C(35d^2e^2+38cdef+\right.}{240bd^3f^3}$$

$$-\frac{(de-cf)^2\left(2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))-b(C(7d^3e^3+9cd^2e^2\right.}{128d^{9/}}$$

Mathematica [A] (verified)

Time = 2.73 (sec) , antiderivative size = 662, normalized size of antiderivative = 0.92

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(10adf(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2fx}{} + \frac{(de-cf)^2\left(-2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))+b(C(7d^3e^3+9cd^2e^2f+9c^2}{128d^{9/2}f^{9/2}}$$

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(10*a*d*f*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10
*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x +
 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*
c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) + b*(C*
(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 + 11*e*
f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24*f^3*x
^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 384*f^4*
x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e
*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*f*(-7
*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2 + 48*f
^3*x^3))))))/(1920*d^4*f^4) + ((d*e - c*f)^2*(-2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) + b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqr
t[f]*Sqrt[c + d*x])])/(128*d^(9/2)*f^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3024 vs. 2(683) = 1366.

Time = 1.67 (sec) , antiderivative size = 3025, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	3025

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-32*C*b*c*d^3*e*f^3*x^2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+200*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^2*f
^2*x+480*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*a*d^5*e^2*f^3-240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
```

d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*f^5-960*A*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e*f^4+240*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b*c^2*d^3*e*f^4+200*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e^2*f^2*x-1
40*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^3*d*f^4*x+240*B*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e^2*f
^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*b*c^3*d^2*e*f^4+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^4*f-960*A*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*a*c*d^3*f^4-960*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e*
f^3+480*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d^2*f^4+480*A*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^2*f^2+480*B*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)*a*c^2*d^2*f^4-300*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^3*f
-1920*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*f^4*x-768*C*b*d^4*f^4*x^4
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-960*B*b*d^4*f^4*x^3*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)-960*C*a*d^4*f^4*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-1
280*A*b*d^4*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-1280*B*a*d^4*f^4*x^
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+210*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)*b*c^4*f^4+210*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^4+480*A*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
*c^2*d^3*f^5+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b*c*d^4*e^2*f^3+140*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)*b*c^2*d^2*e*f^3-68*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d^2*e^2*f
^2-80*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^3*e^3*f+140*C*((d*x+c)*(f
x+e))^(1/2)(d*f)^(1/2)*a*c*d^3*e^2*f^2-80*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)*b*c^3*d*e*f^3+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c^2*d^2*e*
f^3+200*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c^2*d^2*f^4*x-80*B*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^3*e*f^3*x-80*C*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)*a*c*d^3*e*f^3*x+150*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^4*d*f^5-240*B*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^3*f^2-105*C*ln(
1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*
c^5*f^5-105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*b*d^5*e^5+150*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*d*f^5-240*A*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^3*f^2-240*B*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c
^3*d^2*f^5+150*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*a*d^5*e^4*f+150*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^4*f+200*B*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)*b*c^2*d^2*f^4*x-320*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a
*c*d^3*f^4*x-320*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e*f^3*x-320*A*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e*f^3*x-320*A*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)*b*c*d^3*f^4*x+140*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*
c*d^3*e^2*f^2-320*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^3*e*f^3-320*B

*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^3*e*f^3-140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^4*e^3*f*x+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*d*e*f^4+30*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*e^2*f^3+30*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^3*f^2-60*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^2*f^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^3*f^2-120*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d^2*e*f^4-60*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e^2*f^3-120*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e^3*f^2+480*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e^2*f^2-300*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^3*d*f^4-300*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c^3*d*f^4-300*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^4*e^3*f+240*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e*f^4+44*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d^2*e*f^3*x+44*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^3*e^2*f^2*x-96*C*b*c*d^3*f^4*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-96*C*b*d^4*e*f^3*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-160*B*b*c*d^3*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-160*B*b*d^4*e*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-160*C*a*c*d^3*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-160*C*a*d^4*e*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+112*C*b*c^2*d^2*f^4*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+112*C*b*d^4*e^2*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/d^4/f^4/(d*f)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 1620, normalized size of antiderivative = 2.25

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f - 2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*c^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a + A*b)*c^3*d^2)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b
```

```
*c*d^4 + 15*(C*a + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c
*d^4 - 120*(B*a + A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C
*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*
c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*
e*f^4 + (C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 -
2*(C*b*c*d^4 + 5*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c
*d^4 - 80*(B*a + A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4
 + 50*(C*a + B*b)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 8
0*(B*a + A*b)*d^5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c
^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^
5), -1/3840*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f -
2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*
c^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3
 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2
*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a
+ A*b)*c^3*d^2)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x
)) - 2*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b*c*d^4 + 15*(C*a
 + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c*d^4 - 120*(B*a
+ A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C*a + B*b)*c^2*d^
3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*c*d^4 - 10*(C*a
+ B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*e*f^4 + (C*b*c*d
^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 - 2*(C*b*c*d^4 + 5
*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c*d^4 - 80*(B*a +
 A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4 + 50*(C*a + B*b
)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 80*(B*a + A*b)*d^
5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c^2*d^3 + 16*(B*a
 + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5)]
```

Sympy [F]

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2592 vs. 2(683) = 1366.

Time = 0.62 (sec) , antiderivative size = 2592, normalized size of antiderivative = 3.60

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] -1/1920*(1920*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*
e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)*sqrt(d*x + c))*A*a*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(
d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3
*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqr
t(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*C*a*c
*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x
 + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^2
*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 + c*d^2*e^2*
f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*
e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*b*c*abs(d)/d^2 - 10*(sqrt
(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3
+ (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*
f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4
+ 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4
*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log
(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt
(d*f)*d^2*f^3))*C*b*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*
```

sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d
^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*
(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt
(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*a*ab
s(d)/d - 10*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*
(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f
^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 +
9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*
x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3
- 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*a*abs(d)/d - 80*(sqrt(d^2*e + (d*x + c)*
d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*
c*d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d
^7*f^4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-s
qrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d
*f^2))*A*b*abs(d)/d - 10*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*
(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) -
(5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^
14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*
f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20
*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e +
(d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*B*b*abs(d)/d - (sqrt(d^2*e +
(d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)/d^4 + (d^2
0*e*f^7 - 41*c*d^19*f^8)/(d^23*f^8)) - (7*d^21*e^2*f^6 + 26*c*d^20*e*f^7 -
513*c^2*d^19*f^8)/(d^23*f^8)) + 5*(7*d^22*e^3*f^5 + 19*c*d^21*e^2*f^6 + 37*
c^2*d^20*e*f^7 - 447*c^3*d^19*f^8)/(d^23*f^8))*(d*x + c) - 15*(7*d^23*e^4*f
^4 + 12*c*d^22*e^3*f^5 + 18*c^2*d^21*e^2*f^6 + 28*c^3*d^20*e*f^7 - 193*c^4*
d^19*f^8)/(d^23*f^8))*sqrt(d*x + c) - 15*(7*d^5*e^5 + 5*c*d^4*e^4*f + 6*c^2
*d^3*e^3*f^2 + 10*c^3*d^2*e^2*f^3 + 35*c^4*d*e*f^4 - 63*c^5*f^5)*log(abs(-s
qrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d
^3*f^4))*C*b*abs(d)/d - 480*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2
*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*
d*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*
f)))/(sqrt(d*f)*f))*B*a*c*abs(d)/d^3 - 480*(sqrt(d^2*e + (d*x + c)*d*f - c*
d*f)*(2*d*x + 2*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d
^2*e*f - 3*c^2*d*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f)))/(sqrt(d*f)*f))*A*b*c*abs(d)/d^3 - 480*(sqrt(d^2*e + (d*
x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (
d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqr
t(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*f))*A*a*abs(d)/d^2)/d

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Hanged}$$

```
[In] int((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] \text{Hanged}
```

3.43 $\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 330

$$\begin{aligned}
&\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&\quad+\frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad-\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad-\frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{64d^{7/2}f^{7/2}}
\end{aligned}$$

```
[Out] -1/24*(-8*B*d*f+11*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/d^2/f^2+1/4*C
*(d*x+c)^(5/2)*(f*x+e)^(3/2)/d^2/f-1/64*(-c*f+d*e)^2*(C*(5*c^2*f^2+6*c*d*e*
f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/32*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)
+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^3/f^2+1/64*(-c*
f+d*e)*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x
+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {965, 81, 52, 65, 223, 212}

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=-\frac{(de-cf)^2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{64d^{7/2}f^{7/2}}$$
$$+\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{32d^3f^2}$$
$$+\frac{\sqrt{c+dx}\sqrt{e+fx}(de-cf)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{64d^3f^3}$$
$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{24d^2f^2}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$

```
[In] Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^3) + ((C*(5*d^2*e^2 + 6*
c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(32*d^3*f^2) - ((5*C*d*e + 11*c*C*f - 8*B*d*f)*(c + d*x)^(3/2)*
(e + f*x)^(3/2))/(24*d^2*f^2) + (C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*
f) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])]
)/(64*d^(7/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

integral

$$= \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{\int \sqrt{c+dx}\sqrt{e+fx}\left(\frac{1}{2}(-5cCde-3c^2Cf+8Ad^2f)-\frac{1}{2}d(5Cde+11cCf-8Bdf)x\right)\,dx}{4d^2f}$$

$$= -\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))\int \sqrt{c+dx}\sqrt{e+fx}\,dx}{16d^2f^2}$$

$$
\begin{aligned}
&= \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad - \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad + \frac{\left((de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\,dx}{64d^3f^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&\quad + \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad - \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad - \frac{\left((de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{128d^3f^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&\quad + \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad - \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad - \frac{\left((de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right.}{64d^4f^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&\quad + \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad - \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad - \frac{\left((de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\right)\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{64d^4f^3}
\end{aligned}
$$

$$= \frac{(de-cf)\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3}$$
$$+ \frac{\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2}$$
$$- \frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$
$$- \frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{64d^{7/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2fx+8e}{192}$$
$$- \frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{64d^{7/2}f^{7/2}}$$

```
[In] Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*
d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x
^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*f^2 +
2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(192*d^3*f^3) -
((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*
(d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(64
*d^(7/2)*f^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. 2(292) = 584.

Time = 1.66 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1207

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-6*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*f^2+15*C*ln(1/2*(2*d*
```

```
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+15*
C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*d^4*e^4-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*c^3*d*e*f^3-16*C*c*d^2*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)-16*C*d^3*e*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-32*B*(d*f)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2*x+20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*c^2*d*f^3*x-96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*f^3-96*A
*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e*f^2+48*B*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*c^2*d*f^3+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e^2*f-9
6*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*c*d^3*e*f^3-192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*f^3*x-8*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e*f^2*x+48*A*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*f^4+48*A*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4
*e^2*f^2-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*c^3*d*f^4-24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*c^3*f^3-30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^3*e^3+24*B*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d
^2*e*f^3+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*c*d^3*e^2*f^2-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e^3*f+20*C*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*d^3*e^2*f*x-32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*d^2*e*f
^2+14*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*d*e*f^2+14*C*(d*f)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*c*d^2*e^2*f-32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*
c*d^2*f^3*x-96*C*d^3*f^3*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-128*B*d^3*
f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/d^3/f^
3/(d*f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.55

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$=\left[\frac{3\,(5\,Cd^4e^4-4\,(Ccd^3+2\,Bd^4)e^3f-2\,(Cc^2d^2-4\,Bcd^3-8\,Ad^4)e^2f^2-4\,(Cc^3d-2\,Bc^2d^2+8\,Acd^3)ef^3-}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*c^2*d^2 - 4*B*c
*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*e*f^3 + (5*
```

```
C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^
2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*s
qrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^
3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4
)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (
C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*
f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x +
 e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*
c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d
^3)*e*f^3 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2
*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2
+ c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^3*f
- (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4)*e*
f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (C*c*
d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*f^3
+ (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))
/(d^4*f^4)]
```

Sympy [F]

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(292) = 584$.

Time = 0.43 (sec) , antiderivative size = 1073, normalized size of antiderivative = 3.25

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] -1/192*(192*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*
sqrt(d*x + c))*A*c*abs(d)/d^2 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt
(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f
^4)) - 3*(d^7*e^2*f^2 + 2*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3
*e^3 + c*d^2*e^2*f + 3*c^2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x
 + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*C*c*abs(d)
/d^2 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4
*(d*x + c)/d^2 + (d^6*e*f^3 - 13*c*d^5*f^4)/(d^7*f^4)) - 3*(d^7*e^2*f^2 + 2
*c*d^6*e*f^3 - 11*c^2*d^5*f^4)/(d^7*f^4)) - 3*(d^3*e^3 + c*d^2*e^2*f + 3*c^
2*d*e*f^2 - 5*c^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f)))/(sqrt(d*f)*d*f^2))*B*abs(d)/d - (sqrt(d^2*e + (d*x + c
)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 2
5*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^1
1*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f
^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^
3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*s
qrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C
*abs(d)/d - 48*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e*f -
 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*log(ab
s(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*
f)*f))*B*c*abs(d)/d^3 - 48*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*
c + (d*e*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d
*f^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)))/(sqrt(d*f)*f))*A*abs(d)/d^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Hanged}$$

```
[In] int((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

3.44 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 450

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$
$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\sqrt{e+fx}}{8b^3d^2f^2}$$
$$-\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$
$$-\frac{(16a^3Cd^3f^3-8a^2bd^2f^2(Cde+cCf+2Bdf)-2ab^2df(C(de-cf)^2-4df(Bde+Bcf+2Adf))-b^3(}{8b^4d^{5/2}f^{5/2}}$$
$$-\frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\sqrt{be-af}\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4}$$

```
[Out] 1/3*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/8*(16*a^3*C*d^3*f^3-8*a^2*b*d^2*f
^2*(2*B*d*f+C*c*f+C*d*e)-2*a*b^2*d*f*(C*(-c*f+d*e)^2-4*d*f*(2*A*d*f+B*c*f+B
*d*e))-b^3*(C*(-c*f+d*e)^2*(c*f+d*e)-2*d*f*(B*(-c*f+d*e)^2-4*A*d*f*(c*f+d*e
))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(5/2)/f^(5/
2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^
(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)*(-a*f+b*e)^(1/2)/b^4-1/4*(2*a*C*d*f+b
*(-2*B*d*f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/d/f^2+1/8*(4*b*d*f
*(2*A*b*d*f-a*C*(c*f+d*e))+(4*a*d*f-b*c*f+b*d*e)*(2*a*C*d*f+b*(-2*B*d*f+C*c
*f+C*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^2/f^2
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1629, 159, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx =$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df(C(de-cf)^2-4df(2Adf+B}{8b^4d^{5/2}f^{5/2}}$$

$$-\frac{2\sqrt{bc-ad}\sqrt{be-af}(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^4}$$

$$+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(\frac{(4adf-bcf+bde)(2aCdf+b(-2Bdf+cCf+Cde))}{bdf}-4aC(cf+de)+8Abdf\right)}{8b^2df}$$

$$-\frac{\sqrt{c+dx}(e+fx)^{3/2}(2aCdf+b(-2Bdf+cCf+Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((8*A*b*d*f - 4*a*C*(d*e + c*f) + ((b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b
*(C*d*e + c*C*f - 2*B*d*f)))/(b*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(8*b^2*d
*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3
/2))/(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) - ((16*a
^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*e
 + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*b^4*d^(5/2)*f^(5/2)) - (2*(A*b^
2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b^4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
```

```
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(\frac{3}{2}b(2Abdf-aC(de+cf))-\frac{3}{2}b(2aCdf+b(Cde+cCf-2Bdf))x\right)}{a+bx}\,dx}{3b^2df}$$

$$= -\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{\sqrt{e+fx}\left(\frac{3}{4}b(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf)))+\frac{3}{4}b(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4ad\ldots}{(a+bx)\sqrt{c+dx}}}{6b^3df^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\sqrt{e-\ldots}}{8b^3d^2f^2} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{3}{8}b\left(16Ab^3cd^2ef^2-8a^3Cd^2f^2(de+cf)+2a^2bdf\left(4Bdf(de+cf)+C\left(d^2e^2+6cdef+c^2f^2\right)\right)+ab^2\left(C(de-cf)^2(de+cf)-2df\left(4Adf(de+c\ldots\right.\right.\right.}{\ldots}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\sqrt{e-\ldots}}{8b^3d^2f^2} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\left(\left(Ab^2-a(bB-aC)\right)(bc-ad)(be-af)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^4} - \frac{\left(16a^3Cd^3f^3-8a^2bd^2f^2(Cde+cCf+2Bdf)-2ab^2df\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)\ldots\right.}{16b^4d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))\sqrt{c+dx}\sqrt{e-\ldots}}{8b^3d^2f^2} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\left(2\left(Ab^2-a(bB-aC)\right)(bc-ad)(be-af)\right)\text{Subst}\left(\int \frac{1}{-bc+ad-(-be+af)x^2}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^4} - \frac{\left(16a^3Cd^3f^3-8a^2bd^2f^2(Cde+cCf+2Bdf)-2ab^2df\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)\ldots\right.}{\ldots}$$

$$= \frac{(4bdf(2Abdf - aC(de + cf)) + (bde - bcf + 4adf)(2aCdf + b(Cde + cCf - 2Bdf)))\sqrt{c + dx}\sqrt{e}}{8b^3d^2f^2}$$
$$- \frac{(2aCdf + b(Cde + cCf - 2Bdf))\sqrt{c + dx}(e + fx)^{3/2}}{4b^2df^2} + \frac{C(c + dx)^{3/2}(e + fx)^{3/2}}{3bdf}$$
$$- \frac{2(Ab^2 - a(bB - aC))\sqrt{bc - ad}\sqrt{be - af}\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4}$$
$$- \frac{(16a^3Cd^3f^3 - 8a^2bd^2f^2(Cde + cCf + 2Bdf) - 2ab^2df(C(de - cf)^2 - 4df(Bde + Bcf + 2Adf}{8}$$

$$= \frac{(4bdf(2Abdf - aC(de + cf)) + (bde - bcf + 4adf)(2aCdf + b(Cde + cCf - 2Bdf)))\sqrt{c + dx}\sqrt{e}}{8b^3d^2f^2}$$
$$- \frac{(2aCdf + b(Cde + cCf - 2Bdf))\sqrt{c + dx}(e + fx)^{3/2}}{4b^2df^2} + \frac{C(c + dx)^{3/2}(e + fx)^{3/2}}{3bdf}$$
$$- \frac{(16a^3Cd^3f^3 - 8a^2bd^2f^2(Cde + cCf + 2Bdf) - 2ab^2df(C(de - cf)^2 - 4df(Bde + Bcf + 2Adf}{8b^4d^{5/2}f}$$
$$- \frac{2(Ab^2 - a(bB - aC))\sqrt{bc - ad}\sqrt{be - af}\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}(A + Bx + Cx^2)}{a + bx}\,dx$$
$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(24a^2Cd^2f^2 - 6abdf(cCf + 4Bdf + Cd(e+2fx)) + b^2(6df(Bcf + 4Adf + Bd(e+2fx)) + C(-3c^2f^2 + 2cdf(e+fx) + d^2(-3e^2 + 2efx + 8}{d^2f^2}}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 - 6*a*b*d*f*(c*C*f + 4*B*
d*f + C*d*(e + 2*f*x)) + b^2*(6*d*f*(B*c*f + 4*A*d*f + B*d*(e + 2*f*x)) + C
*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(
d^2*f^2) - 48*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]
*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])]
 + (3*(-16*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) + 2*a*
b^2*d*f*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) + b^3*(C*(d*e -
 c*f)^2*(d*e + c*f) + 2*d*f*(-(B*(d*e - c*f)^2) + 4*A*d*f*(d*e + c*f))))*Ar
cTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(5/2)*f^(5/2)))/
(24*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3897 vs. $2(406) = 812$.

Time = 5.72 (sec) , antiderivative size = 3898, normalized size of antiderivative = 8.66

method	result	size
default	Expression too large to display	3898

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*b^4*d^3*e^3+24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*a*b^3*d^3*e*f^2+24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*
c*d^2*f^3-3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^3*f^3-6*
C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^3*e^2*f+48*C*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^3*f^3-24*C*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^3*e*f^2-12*B*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*d^2*e*f+
48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a)
)*a^4*d^3*f^3-48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*
b*c*e)/(b*x+a))*a*b^3*c*d^2*f^3-6*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*a*b^3*c^2*d*f^3-24*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*d^2*f^2*x+48*B*(d*f)^(1/2)*ln((-2*a
*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^3*e*f^2-24*C*
((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*c*d^2*f^3-24*A*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*d^2*f^3-24*A*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^3*e*f^2+48*A*(d*f)^(1/2)*ln((-2*a*d*
f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^3*f^3-48*B*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
```

)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d^3*f^3+6*B*((a^2*d*f-a*b
*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*d*f^3+6*B*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b^4*d^3*e^2*f-48*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^3*f^3+48*A*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^3*f^3-48*A*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*d^2*f^2+12*C*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*a*b^3*c*d*f^2+12*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b^3*d^2*e*f-4*C*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*c*d*e*f-48*B*(
d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
e)/b^2)^(1/2)((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^
3*c*d^2*e*f^2+24*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)*a*b^3*d^2*f^2*x-4*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*c*d*f^2*x-4*C*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)*b^4*d^2*e*f*x+12*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*
c*d^2*e*f^2+48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b
*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*
c*e)/(b*x+a))*a^2*b^2*c*d^2*e*f^2+6*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*c^2*f^2+6*C*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*d^2
*e^2+48*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*a^2*b^2*c*d^2*f^3+3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b^4*c^2*d*e*f^2-12*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^4*c*d*f^2-48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*
c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^3*e*f^2+48*A*(d*f)^(1/2)
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d^2*e*f^
2+3*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*d^2*e^2*f-48*C*(
d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
e)/b^2)^(1/2)((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*
b*c*d^2*f^3-48*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)*a^2*b^2*d^2*f^2-48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c
*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^3*e*f^2+48*B*((a^2*d*f-a*

```
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b^3
*d^2*f^2-12*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*d^2*e*f^
2-16*C*b^4*d^2*f^2*x^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/b^5/d^2/f^2/(d*f)^(1
/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?` fo
r more
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Hanged}$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x),x)
```

```
[Out] \text{Hanged}
```

3.45 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 521

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$
$$= \frac{(12a^2Cdf^2 - abf(7Cde + cCf + 8Bdf) + b^2(4df(Be + Af) - Ce(de - cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be-af)}$$
$$+ \frac{(3a^2Cdf + b^2(cCe + 2Adf) - ab(Cde + cCf + 2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)}$$
$$+ \frac{(24a^2Cd^2f^2 - 8abdf(Cde + cCf + 2Bdf) - b^2(C(de-cf)^2 - 4df(Bde + Bcf + 2Adf)))\operatorname{arctanh}\Big(\frac{\sqrt{f}\sqrt{c}}{\sqrt{d}\sqrt{e}}}{4b^4d^{3/2}f^{3/2}}$$
$$+ \frac{(6a^3Cdf - b^3(2Bce + Ade + Acf) + ab^2(4cCe + 3Bde + 3Bcf + 2Adf) - a^2b(4Bdf + 5C(de+cf)))\ \mathrm{ar}}{b^4\sqrt{bc-ad}\sqrt{be-af}}$$

```
[Out] -(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b
*x+a)+1/4*(24*a^2*C*d^2*f^2-8*a*b*d*f*(2*B*d*f+C*c*f+C*d*e)-b^2*(C*(-c*f+d*
e)^2-4*d*f*(2*A*d*f+B*c*f+B*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f
*x+e)^(1/2))/b^4/d^(3/2)/f^(3/2)+(6*a^3*C*d*f-b^3*(A*c*f+A*d*e+2*B*c*e)+a*b
^2*(2*A*d*f+3*B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(4*B*d*f+5*C*(c*f+d*e)))*arctanh
((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b
*c)^(1/2)/(-a*f+b*e)^(1/2)+1/2*(3*a^2*C*d*f+b^2*(2*A*d*f+C*c*e)-a*b*(2*B*d*
f+C*c*f+C*d*e))*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+1/4
*(12*a^2*C*d*f^2-a*b*f*(8*B*d*f+C*c*f+7*C*d*e)+b^2*(4*d*f*(A*f+B*e)-C*e*(-c
*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a*f+b*e)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1627, 159, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

$$= \frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(24a^2Cd^2f^2 - 8abdf(2Bdf + cCf + Cde) - (b^2(C(de-cf)^2 - 4df(2Adf + Bcf + B}{4b^4d^{3/2}f^{3/2}}$$

$$+ \frac{\sqrt{c+dx}(e+fx)^{3/2}\left(3a^2Cdf - ab(2Bdf + cCf + Cde) + b^2(2Adf + cCe)\right)}{2b^2f(bc-ad)(be-af)}$$

$$+ \frac{\sqrt{c+dx}\sqrt{e+fx}(12a^2Cdf^2 - abf(8Bdf + cCf + 7Cde) + b^2(4df(Af+Be) - Ce(de-cf)))}{4b^3df(be-af)}$$

$$+ \frac{\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(6a^3Cdf - a^2b(4Bdf + 5C(cf+de)) + ab^2(2Adf + 3Bcf + 3Bde + 4cCe) - b^3}{b^4\sqrt{bc-ad}\sqrt{be-af}}$$

$$- \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2 - a(bB - aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] ((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A*
f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*d*f*(b*e - a*f))
 + ((3*a^2*C*d*f + b^2*(c*C*e + 2*A*d*f) - a*b*(C*d*e + c*C*f + 2*B*d*f))*S
qrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 -
 a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)
*(a + b*x)) + ((24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^
2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*b^4*d^(3/2)*f^(3/2)) + ((6*a^3*C*d
*f - b^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2
*A*d*f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[
c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[b*e -
 a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
```

```
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)+b^2(2Bce+Ade+Acf)-ab(2cCe+3Bde+3Bcf-2Adf)}{2b}+\left(-\frac{3a^2Cdf}{b}-b(cCe+2Adf)+a(Cde+cCf+2Bdf)\right)x\right)}{a+bx}\,dx}{(bc-ad)(be-af)}$$

$$= \frac{(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)} - \frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{\sqrt{e+fx}\left(-\frac{(bc-ad)\left(3a^2Cf(de+3cf)+2b^2f(2Bce+Ade+Acf)-ab(2Bf(de+3cf)+Ce(de+7cf))\right)}{2b}-\frac{(bc-ad)\left(12a^2Cdf^2-abf(7Cde+cCf+8Bd\ldots\right)}{2b}\ldots\right.}{(a+bx)\sqrt{c+dx}}\ldots}{2b(bc-ad)f(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be-af)} + \frac{(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)} - \frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{-\frac{(bc-ad)(be-af)\left(12a^2Cdf(de+cf)+4b^2df(2Bce+Ade+Acf)-ab\left(8Bdf(de+cf)+C\left(d^2e^2+14cdef+c^2f^2\right)\right)\right)}{4b}-\frac{(bc-ad)(be-af)\left(24a^2Cd^2f^2\ldots\right.}{\ldots}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\ldots}{2b^2d(bc-ad)f(be-af)}$$

$$= \frac{(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be-af)} + \frac{(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc-ad)f(be-af)} - \frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \frac{(6a^3Cdf-b^3(2Bce+Ade+Acf)+ab^2(4cCe+3Bde+3Bcf+2Adf)-a^2b(4Bdf+5C(de\ldots}{2b^4} + \frac{(24a^2Cd^2f^2-8abdf(Cde+cCf+2Bdf)-b^2(C(de-cf)^2-4df(Bde+Bcf+2Adf)))\int \frac{\ldots}{\sqrt{c\ldots}}}{8b^4df}$$

$$= \frac{(12a^2Cdf^2 - abf(7Cde + cCf + 8Bdf) + b^2(4df(Be + Af) - Ce(de - cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be - af)}$$
$$+ \frac{(3a^2Cdf + b^2(cCe + 2Adf) - ab(Cde + cCf + 2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc - ad)f(be - af)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc - ad)(be - af)(a + bx)}$$
$$- \frac{(6a^3Cdf - b^3(2Bce + Ade + Acf) + ab^2(4cCe + 3Bde + 3Bcf + 2Adf) - a^2b(4Bdf + 5C(de +}{b^4}$$
$$+ \frac{(24a^2Cd^2f^2 - 8abdf(Cde + cCf + 2Bdf) - b^2(C(de - cf)^2 - 4df(Bde + Bcf + 2Adf)))\,\mathrm{Subst}}{4b^4d^2f}$$

$$= \frac{(12a^2Cdf^2 - abf(7Cde + cCf + 8Bdf) + b^2(4df(Be + Af) - Ce(de - cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be - af)}$$
$$+ \frac{(3a^2Cdf + b^2(cCe + 2Adf) - ab(Cde + cCf + 2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc - ad)f(be - af)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc - ad)(be - af)(a + bx)}$$
$$+ \frac{(6a^3Cdf - b^3(2Bce + Ade + Acf) + ab^2(4cCe + 3Bde + 3Bcf + 2Adf) - a^2b(4Bdf + 5C(de +}{b^4\sqrt{bc - ad}\sqrt{be - af}}$$
$$+ \frac{(24a^2Cd^2f^2 - 8abdf(Cde + cCf + 2Bdf) - b^2(C(de - cf)^2 - 4df(Bde + Bcf + 2Adf)))\,\mathrm{Subst}}{4b^4d^2f}$$

$$= \frac{(12a^2Cdf^2 - abf(7Cde + cCf + 8Bdf) + b^2(4df(Be + Af) - Ce(de - cf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3df(be - af)}$$
$$+ \frac{(3a^2Cdf + b^2(cCe + 2Adf) - ab(Cde + cCf + 2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{2b^2(bc - ad)f(be - af)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc - ad)(be - af)(a + bx)}$$
$$+ \frac{(24a^2Cd^2f^2 - 8abdf(Cde + cCf + 2Bdf) - b^2(C(de - cf)^2 - 4df(Bde + Bcf + 2Adf)))\tanh^{-}}{4b^4d^{3/2}f^{3/2}}$$
$$+ \frac{(6a^3Cdf - b^3(2Bce + Ade + Acf) + ab^2(4cCe + 3Bde + 3Bcf + 2Adf) - a^2b(4Bdf + 5C(de +}{b^4\sqrt{bc - ad}\sqrt{be - af}}$$

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$
$$=\frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(-12a^2Cdf+ab(cCf+8Bdf+Cd(e-6fx))+b^2(-4Adf+x(cCf+4Bdf+Cd(e+2fx))))}{df(a+bx)}+\frac{4(-6a^3Cdf+b^3(2Bce+Ade+Acf)-a}{}}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*C*d*f + a*b*(c*C*f + 8*B*d*f + C*d
*(e - 6*f*x)) + b^2*(-4*A*d*f + x*(c*C*f + 4*B*d*f + C*d*(e + 2*f*x)))))/(d
*f*(a + b*x)) + (4*(-6*a^3*C*d*f + b^3*(2*B*c*e + A*d*e + A*c*f) - a*b^2*(4
*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*f) + a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*
ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])
/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((-24*a^2*C*d^2*f^2 + 8*a*b*d*f*(C*
d*e + c*C*f + 2*B*d*f) + b^2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*
d*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2)*f
^(3/2)))/(4*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4679 vs. 2(479) = 958.

Time = 1.69 (sec) , antiderivative size = 4680, normalized size of antiderivative = 8.98

method	result	size
default	Expression too large to display	4680

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/8*(8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^
2*d^2*f^2*(d*f)^(1/2)-16*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)-8*A*b^4*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*b^4*d^2*e^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+
4*C*b^4*d*f*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d
```

*e+b^2*c*e)/b^2)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*a^2*b^2*d^2*e*f*(d*f)^(1/2)+4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*f^2*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*B*b^4*d*f*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^4*c*f*x*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*
C*b^4*d*e*x*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)+16*B*a*b^3*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-24*C*a^2*b^2*d*f*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*a*b^3*
c*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)+2*C*a*b^3*d*e*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)-16*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*
e)/(b*x+a))*a^2*b^2*d^2*f^2*x*(d*f)^(1/2)+24*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e
x+2((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*
b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d^2*f^2*(d*f)^(1/2)+8*A*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*f^2*
((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-16*B*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^2*f^2*(d*f)^(1/2)-16*B*ln(1/2*(
2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2
*d^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+24*C*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d^2*f^2*
((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c^2*f^2*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d^2*e^2*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)+8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a*b^3*d^2*f^2*x*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*
c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d*f^2*x*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*
c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*d^2*e*f*x*(d*f)^(1/2)+4*B*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b
^4*c*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*B*ln(1/2*(2*d*
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d^2*e*f
x((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-20*C*ln((-2*a*d*f*x+b*c*f*
x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*d*f^2*(d*f)^(1/2)-20*C*ln((-
2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(

(d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^2*e*f*(d*f)^
(1/2)-8*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a^2*b^2*c*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-8*
C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a^2*b^2*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+24*C*ln((-
2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^2*f^2*x*(d*f
)^(1/2)+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*a^2*b^2*d^2*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*
d*f^2*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*a*b^3*d^2*e*f*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a
*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*d*f^2*(d*f)^(1/2)-12*C*a*b^3*d*f*x*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+
12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*d*
f^2*x*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(
b*x+a))*a*b^3*d^2*e*f*x*(d*f)^(1/2)-8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d*e*f*x*(d*f)^(1/2)-20*C*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*d*f^2*x*(d*f)^(1/2)-20*C
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^2*e*
f*x*(d*f)^(1/2)-8*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)-8*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*a*b^3*d^2*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+
2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*b^4*c*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-8*B*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*d*e*f*(d*f)^(
1/2)+16*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b
^2*c*d*e*f*(d*f)^(1/2)+16*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*a*b^3*c*d*e*f*x*(d*f)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*d*e*f*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2))*(f*x+e)^(1/2)*(d*x+c)^(1/2)/f/d/((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/(d*f)^(1/2)/(b*x+a)/((d*x+c)*(f*x+e))^(1
/2)/b^5

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?` fo
r more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1507 vs. $2(478) = 956$.

Time = 1.54 (sec) , antiderivative size = 1507, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"giac")
```

```
[Out] 1/4*sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d)
/(b^2*d^3) + (C*b^7*d^4*e*f*abs(d) - C*b^7*c*d^3*f^2*abs(d) - 8*C*a*b^6*d^4
*f^2*abs(d) + 4*B*b^7*d^4*f^2*abs(d))/(b^9*d^6*f^2)) + (4*sqrt(d*f)*C*a*b^2
*c*e*abs(d) - 2*sqrt(d*f)*B*b^3*c*e*abs(d) - 5*sqrt(d*f)*C*a^2*b*d*e*abs(d)
 + 3*sqrt(d*f)*B*a*b^2*d*e*abs(d) - sqrt(d*f)*A*b^3*d*e*abs(d) - 5*sqrt(d*f
)*C*a^2*b*c*f*abs(d) + 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) - sqrt(d*f)*A*b^3*c*f
*abs(d) + 6*sqrt(d*f)*C*a^3*d*f*abs(d) - 4*sqrt(d*f)*B*a^2*b*d*f*abs(d) + 2
*sqrt(d*f)*A*a*b^2*d*f*abs(d))*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f -
 (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt
(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/(sqrt(-b^2*c*d
*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*b^4*d) - 2*(sqrt(d*f)*C*a^2
*b*d^3*e^2*abs(d) - sqrt(d*f)*B*a*b^2*d^3*e^2*abs(d) + sqrt(d*f)*A*b^3*d^3*
e^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*e*f*abs(d) + 2*sqrt(d*f)*B*a*b^2*c*d
^2*e*f*abs(d) - 2*sqrt(d*f)*A*b^3*c*d^2*e*f*abs(d) + sqrt(d*f)*C*a^2*b*c^2*
d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f)*A*b^3*c^2*d*f
^2*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^2*C*a^2*b*d*e*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d*e*abs(d) - sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*d*e*abs(d) -
 sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*C*a^2*b*c*f*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^2*B*a*b^2*c*f*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*c*f*abs(d) + 2*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*d
*f*abs(d) - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*B*a^2*b*d*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^2*d*f*abs(d))/((b*d^4*e^2 - 2*
b*c*d^3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^4*b)*b^4) + 1/8*(C*b^2*d^2*e^2*abs(d) - 2*C*b^2
*c*d*e*f*abs(d) + 8*C*a*b*d^2*e*f*abs(d) - 4*B*b^2*d^2*e*f*abs(d) + C*b^2*c
^2*f^2*abs(d) + 8*C*a*b*c*d*f^2*abs(d) - 4*B*b^2*c*d*f^2*abs(d) - 24*C*a^2*
d^2*f^2*abs(d) + 16*B*a*b*d^2*f^2*abs(d) - 8*A*b^2*d^2*f^2*abs(d))*log((sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2)/(sqrt(d*f)*b
^4*d^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Hanged}$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^2,x)
[Out] \text{Hanged}
```

3.46 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 658

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx =$$
$$-\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) + ab^2(Bf(5de+3cf) + 4Ce(de+4cf)) - b^3(Adef + c(4}{4b^3(bc-ad)(be-af)^2}$$
$$+\frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de+cf)))}{4b^2(bc-ad)(be-af)^2(a+bx)}$$
$$-\frac{(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2}$$
$$-\frac{(6aCdf - b(Cde + cCf + 2Bdf))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^4\sqrt{d}\sqrt{f}}$$
$$-\frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce+Bf) + 2cde(4Ce+3Bf)) - 8a^3bdf(Bdf + 5C(de+cf)) - b^4}{}$$

```
[Out] -1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^2-1/4*(24*a^4*C*d^2*f^2-3*a*b^3*(B*d^2*e^2+c^2*f*(B*f+8*C*e)+2*c*
d*e*(3*B*f+4*C*e))-8*a^3*b*d*f*(B*d*f+5*C*(c*f+d*e))-b^4*(A*d^2*e^2-2*c*d*e
*(A*f+2*B*e)-c^2*(-A*f^2+4*B*e*f+8*C*e^2))+3*a^2*b^2*(4*B*d*f*(c*f+d*e)+C*(
5*c^2*f^2+22*c*d*e*f+5*d^2*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-
a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)-(6*a*C*
d*f-b*(2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^
(1/2))/b^4/d^(1/2)/f^(1/2)+1/4*(6*a^3*C*d*f-b^3*(-A*c*f-A*d*e+4*B*c*e)+a*b^
2*(-2*A*d*f+3*B*c*f+3*B*d*e+8*C*c*e)-a^2*b*(2*B*d*f+7*C*(c*f+d*e)))*(f*x+e)
^(3/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)-1/4*(12*a^3*C*d*f^
2-a^2*b*f*(4*B*d*f+11*C*c*f+17*C*d*e)+a*b^2*(B*f*(3*c*f+5*d*e)+4*C*e*(4*c*f
```

```
+d*e))-b^3*(A*d*e*f+c*(-A*f^2+4*B*e*f+4*C*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2
)/b^3/(-a*d+b*c)/(-a*f+b*e)^2
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1627, 154, 159, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx =$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(12a^3Cdf^2-a^2bf(4Bdf+11cCf+17Cde)+ab^2(Bf(3cf+5de)+4Ce(4cf+de))-}{4b^3(bc-ad)(be-af)^2}$$

$$+\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-\right.}{4b^2(a+bx)(bc-ad)(be-af)^2}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(24a^4Cd^2f^2-8a^3bdf(Bdf+5C(cf+de))+3a^2b^2(4Bdf(cf+de)+C(5c^2f^2+\right.}{}$$

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(6aCdf-b(2Bdf+cCf+Cde))}{b^4\sqrt{d}\sqrt{f}}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -1/4*((12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*
C*e^2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(
d*e + 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b^3*(b*c - a*d)*(b*e - a*f)^2)
 + ((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e
 + 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C*(d*e + c*f)))*Sqrt[c + d*x]*(e
 + f*x)^(3/2))/(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b
*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(2*b*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^2) - ((6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^4*Sqrt[d]*Sqrt[f]) - ((24*a^4*C*d
^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*B*f)
) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(2*B*e
 + A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f)
 + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^4*(b*c - a*d)^(3/2)*(b*e -
a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)+b^2(4Bce-Ade-Acf)-ab(4cCe+3Bde+3Bcf-4Adf)}{2b}+\left(aBdf-\frac{3a^2Cdf}{b}+2aC(de+cf)-b(2cCe+Adf)\right)x\right)}{(a+bx)^2}\,dx}{2(bc-ad)(be-af)}$$

$$= \frac{\left(6a^3Cdf-b^3(4Bce-Ade-Acf)+ab^2(8cCe+3Bde+3Bcf-2Adf)-a^2b(2Bdf+7C(de+c\right.}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{e+fx}\left(\frac{6a^3Cdf(de+3cf)+b^3\left(Ad^2e^2-2cde(2Be+Af)-c^2\left(8Ce^2+4Bef-Af^2\right)\right)+ab^2\left(d^2e(3Be-2Af)+3c^2f(8Ce+Bf)+2cd\left(8Ce^2+5Bef+A\right.\right.}{4b}\right.}{}}{}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + Adef + cf(4Be - Af)) + ab^2(Bf(5\cdots}{4b^3(bc-ad)(be-af)^2} + \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de\cdots}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\frac{d(be-af)\left(12a^3Cdf(de+cf)+ab^2\left(3Bd^2e^2+10cde(2Ce+Bf)+c^2f(20Ce+3Bf)\right)+b^3\left(Ad^2e^2-2cde(2Be+Af)-c^2\left(8Ce^2+4Bef-Af^2\right)\right)-a^2b\left(4\cdots\right.\right.}{4b}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2b^2d(bc-ad)(be-af)^2}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + Adef + cf(4Be - Af)) + ab^2(Bf(5\cdots}{4b^3(bc-ad)(be-af)^2} + \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de\cdots}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{(6aCdf - b(Cde + cCf + 2Bdf))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2b^4} + \frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce + Bf) + 2cde(4Ce + 3Bf)) - 8a^3bdf(Bdf + 5C(de + c\cdots}{\cdots}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + Adef + cf(4Be - Af)) + ab^2(Bf(5\cdots}{4b^3(bc-ad)(be-af)^2} + \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de\cdots}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{(6aCdf - b(Cde + cCf + 2Bdf))\text{Subst}\left(\int \frac{1}{\sqrt{e - \frac{cf}{d} + \frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^4d} + \frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce + Bf) + 2cde(4Ce + 3Bf)) - 8a^3bdf(Bdf + 5C(de + c\cdots}{\cdots}$$

$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + Adef + cf(4Be - Af)) + ab^2(Bf(5c}{4b^3(bc-ad)(be-af)^2}$$
$$+ \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de +}{4b^2(bc-ad)(be-af)^2(a+bx)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2}$$
$$- \frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce + Bf) + 2cde(4Ce + 3Bf)) - 8a^3bdf(Bdf + 5C(de + cf)}{}$$
$$- \frac{(6aCdf - b(Cde + cCf + 2Bdf))\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^4d}$$
$$= -\frac{(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3(4cCe^2 + Adef + cf(4Be - Af)) + ab^2(Bf(5c}{4b^3(bc-ad)(be-af)^2}$$
$$+ \frac{(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf) - a^2b(2Bdf + 7C(de +}{4b^2(bc-ad)(be-af)^2(a+bx)}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2}$$
$$- \frac{(6aCdf - b(Cde + cCf + 2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^4\sqrt{d}\sqrt{f}}$$
$$- \frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2 + c^2f(8Ce + Bf) + 2cde(4Ce + 3Bf)) - 8a^3bdf(Bdf + 5C(de + cf)}{}$$

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$
$$= \frac{b\sqrt{c+dx}\sqrt{e+fx}(12a^4Cdf + 4b^4cex(-B+Cx) + Ab^3(acf + ad(e+2fx) - b(2ce+dex+cfx)) + ab^3(-4Cx(-4ce+dex+cfx) + B(-2ce+5dex+5cfx)) + a}{(bc-ad)(be-af)(a+bx)^2}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] ((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(12*a^4*C*d*f + 4*b^4*c*e*x*(-B + C*x) + A*
b^3*(a*c*f + a*d*(e + 2*f*x) - b*(2*c*e + d*e*x + c*f*x)) + a*b^3*(-4*C*x*(
-4*c*e + d*e*x + c*f*x) + B*(-2*c*e + 5*d*e*x + 5*c*f*x)) + a^2*b^2*(3*B*d*
(e - 2*f*x) + C*d*x*(-17*e + 4*f*x) + c*(10*C*e + 3*B*f - 17*C*f*x)) - a^3*
b*(4*B*d*f + C*(11*d*e + 11*c*f - 18*d*f*x))))/((b*c - a*d)*(b*e - a*f)*(a
```

```
+ b*x)^2) - ((24*a^4*C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) +
 2*c*d*e*(4*C*e + 3*B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) + b^4*(-(
A*d^2*e^2) + 2*c*d*e*(2*B*e + A*f) + c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a
^2*b^2*(4*B*d*f*(d*e + c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcT
an[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b
*c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) + (4*(-6*a*C*d*f + b*(C*d*e + c*C*f +
 2*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(Sqrt[
d]*Sqrt[f]))/(4*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11203 vs. 2(614) = 1228.

Time = 1.69 (sec) , antiderivative size = 11204, normalized size of antiderivative = 17.03

method	result	size
default	Expression too large to display	11204

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8241 vs. $2(613) = 1226$.

Time = 4.81 (sec) , antiderivative size = 8241, normalized size of antiderivative = 12.52

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"giac")
```

```
[Out] -1/4*(8*sqrt(d*f)*C*b^4*c^2*e^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c*d*e^2*abs(d
) + 4*sqrt(d*f)*B*b^4*c*d*e^2*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*d^2*e^2*abs(d
) - 3*sqrt(d*f)*B*a*b^3*d^2*e^2*abs(d) - sqrt(d*f)*A*b^4*d^2*e^2*abs(d) - 2
4*sqrt(d*f)*C*a*b^3*c^2*e*f*abs(d) + 4*sqrt(d*f)*B*b^4*c^2*e*f*abs(d) + 66*
sqrt(d*f)*C*a^2*b^2*c*d*e*f*abs(d) - 18*sqrt(d*f)*B*a*b^3*c*d*e*f*abs(d) +
2*sqrt(d*f)*A*b^4*c*d*e*f*abs(d) - 40*sqrt(d*f)*C*a^3*b*d^2*e*f*abs(d) + 12
*sqrt(d*f)*B*a^2*b^2*d^2*e*f*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*c^2*f^2*abs(d)
 - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*abs(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs(d) - 40
*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d) + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2*abs(d) +
 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*abs(d))*ar
ctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b
*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e - a*b^5*d*e - a*b^5*c*f + a^2*b^4*d*f
)*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d) + 1/2*(8*
sqrt(d*f)*C*a*b^4*c*d^7*e^5*abs(d) - 4*sqrt(d*f)*B*b^5*c*d^7*e^5*abs(d) - 9
*sqrt(d*f)*C*a^2*b^3*d^8*e^5*abs(d) + 5*sqrt(d*f)*B*a*b^4*d^8*e^5*abs(d) -
sqrt(d*f)*A*b^5*d^8*e^5*abs(d) - 32*sqrt(d*f)*C*a*b^4*c^2*d^6*e^4*f*abs(d)
+ 16*sqrt(d*f)*B*b^5*c^2*d^6*e^4*f*abs(d) + 27*sqrt(d*f)*C*a^2*b^3*c*d^7*e^
4*f*abs(d) - 15*sqrt(d*f)*B*a*b^4*c*d^7*e^4*f*abs(d) + 3*sqrt(d*f)*A*b^5*c*
```

d^7*e^4*f*abs(d) + 10*sqrt(d*f)*C*a^3*b^2*d^8*e^4*f*abs(d) - 6*sqrt(d*f)*B*
a^2*b^3*d^8*e^4*f*abs(d) + 2*sqrt(d*f)*A*a*b^4*d^8*e^4*f*abs(d) + 48*sqrt(d
*f)*C*a*b^4*c^3*d^5*e^3*f^2*abs(d) - 24*sqrt(d*f)*B*b^5*c^3*d^5*e^3*f^2*abs
(d) - 18*sqrt(d*f)*C*a^2*b^3*c^2*d^6*e^3*f^2*abs(d) + 10*sqrt(d*f)*B*a*b^4*
c^2*d^6*e^3*f^2*abs(d) - 2*sqrt(d*f)*A*b^5*c^2*d^6*e^3*f^2*abs(d) - 40*sqrt
(d*f)*C*a^3*b^2*c*d^7*e^3*f^2*abs(d) + 24*sqrt(d*f)*B*a^2*b^3*c*d^7*e^3*f^2
*abs(d) - 8*sqrt(d*f)*A*a*b^4*c*d^7*e^3*f^2*abs(d) - 32*sqrt(d*f)*C*a*b^4*c
^4*d^4*e^2*f^3*abs(d) + 16*sqrt(d*f)*B*b^5*c^4*d^4*e^2*f^3*abs(d) - 18*sqrt
(d*f)*C*a^2*b^3*c^3*d^5*e^2*f^3*abs(d) + 10*sqrt(d*f)*B*a*b^4*c^3*d^5*e^2*f
^3*abs(d) - 2*sqrt(d*f)*A*b^5*c^3*d^5*e^2*f^3*abs(d) + 60*sqrt(d*f)*C*a^3*b
^2*c^2*d^6*e^2*f^3*abs(d) - 36*sqrt(d*f)*B*a^2*b^3*c^2*d^6*e^2*f^3*abs(d) +
12*sqrt(d*f)*A*a*b^4*c^2*d^6*e^2*f^3*abs(d) + 8*sqrt(d*f)*C*a*b^4*c^5*d^3*
e*f^4*abs(d) - 4*sqrt(d*f)*B*b^5*c^5*d^3*e*f^4*abs(d) + 27*sqrt(d*f)*C*a^2*
b^3*c^4*d^4*e*f^4*abs(d) - 15*sqrt(d*f)*B*a*b^4*c^4*d^4*e*f^4*abs(d) + 3*sq
rt(d*f)*A*b^5*c^4*d^4*e*f^4*abs(d) - 40*sqrt(d*f)*C*a^3*b^2*c^3*d^5*e*f^4*a
bs(d) + 24*sqrt(d*f)*B*a^2*b^3*c^3*d^5*e*f^4*abs(d) - 8*sqrt(d*f)*A*a*b^4*c
^3*d^5*e*f^4*abs(d) - 9*sqrt(d*f)*C*a^2*b^3*c^5*d^3*f^5*abs(d) + 5*sqrt(d*f
)*B*a*b^4*c^5*d^3*f^5*abs(d) - sqrt(d*f)*A*b^5*c^5*d^3*f^5*abs(d) + 10*sqrt
(d*f)*C*a^3*b^2*c^4*d^4*f^5*abs(d) - 6*sqrt(d*f)*B*a^2*b^3*c^4*d^4*f^5*abs(
d) + 2*sqrt(d*f)*A*a*b^4*c^4*d^4*f^5*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c*d^5*e^4*abs(d)
+ 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^2*B*b^5*c*d^5*e^4*abs(d) + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*d^6*e^4*abs(d) - 15*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*d
^6*e^4*abs(d) + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*A*b^5*d^6*e^4*abs(d) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^2*d^4*e^3*f*abs(d)
- 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^2*B*b^5*c^2*d^4*e^3*f*abs(d) + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c*d^5*e^3*f*abs(d) - 20*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B
*a*b^4*c*d^5*e^3*f*abs(d) - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*c*d^5*e^3*f*abs(d) - 80*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*d^6
*e^3*f*abs(d) + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*B*a^2*b^3*d^6*e^3*f*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*d^6*e^3*f*abs(d
) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*C*a*b^4*c^3*d^3*e^2*f^2*abs(d) - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c^3*d^3*e^2*f^2*abs(d) -
142*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^2*C*a^2*b^3*c^2*d^4*e^2*f^2*abs(d) + 70*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c^2*d^4*e^2*f^2*abs(d)
+ 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*

f))^2*A*b^5*c^2*d^4*e^2*f^2*abs(d) + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c*d^5*e^2*f^2*abs(d) - 4
4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^2*B*a^2*b^3*c*d^5*e^2*f^2*abs(d) + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c*d^5*e^2*f^2*abs(d) + 56*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C
*a^4*b*d^6*e^2*f^2*abs(d) - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^2*d^6*e^2*f^2*abs(d) + 8*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3
*d^6*e^2*f^2*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^4*d^2*e*f^3*abs(d) + 12*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c^4*d^2*
e*f^3*abs(d) + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*C*a^2*b^3*c^3*d^3*e*f^3*abs(d) - 20*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c^3*d^3*e*f
^3*abs(d) - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*A*b^5*c^3*d^3*e*f^3*abs(d) + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c^2*d^4*e*f^3*abs
(d) - 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*B*a^2*b^3*c^2*d^4*e*f^3*abs(d) + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^2*d^4*e*f^3*abs(d)
- 112*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*C*a^4*b*c*d^5*e*f^3*abs(d) + 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^2*c*d^5*e*f^3*abs(d) - 16*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*
A*a^2*b^3*c*d^5*e*f^3*abs(d) + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c^4*d^2*f^4*abs(d) - 15*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b
^4*c^4*d^2*f^4*abs(d) + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^2*A*b^5*c^4*d^2*f^4*abs(d) - 80*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c^3*d^3
*f^4*abs(d) + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^2*B*a^2*b^3*c^3*d^3*f^4*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^3*d^3*f^4*abs
(d) + 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*C*a^4*b*c^2*d^4*f^4*abs(d) - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^2*c^2*d^4*f^4*abs(d) + 8
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*A*a^2*b^3*c^2*d^4*f^4*abs(d) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^4*c*d^3*e^3*abs(d) - 12*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c
*d^3*e^3*abs(d) - 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^4*C*a^2*b^3*d^4*e^3*abs(d) + 15*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*d^4*e^3*abs(d)
- 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*

f))^4*A*b^5*d^4*e^3*abs(d) + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^4*c^2*d^2*e^2*f*abs(d) - 8*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c^
2*d^2*e^2*f*abs(d) - 109*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*c*d^3*e^2*f*abs(d) + 57*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c*d^3*
e^2*f*abs(d) - 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^4*A*b^5*c*d^3*e^2*f*abs(d) + 102*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^2*d^4*e^2*f*abs(d
) - 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^4*B*a^2*b^3*d^4*e^2*f*abs(d) + 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^4*d^4*e^2*f*abs(d) + 24*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a
*b^4*c^3*d*e*f^2*abs(d) - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c^3*d*e*f^2*abs(d) - 109*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*c^2
*d^2*e*f^2*abs(d) + 57*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^4*B*a*b^4*c^2*d^2*e*f^2*abs(d) - 5*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*c^2*d^2*e*f
^2*abs(d) + 228*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*C*a^3*b^2*c*d^3*e*f^2*abs(d) - 124*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c*d^3*e*f^2*
abs(d) + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^4*A*a*b^4*c*d^3*e*f^2*abs(d) - 152*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b*d^4*e*f^2*abs(d) + 8
8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^4*B*a^3*b^2*d^4*e*f^2*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^3*d^4*e*f^2*abs(d) - 27*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*
b^3*c^3*d*f^3*abs(d) + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c^3*d*f^3*abs(d) - 3*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*c^3*d*f^3*ab
s(d) + 102*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^4*C*a^3*b^2*c^2*d^2*f^3*abs(d) - 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c^2*d^2*f^3*abs(d)
 + 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^4*A*a*b^4*c^2*d^2*f^3*abs(d) - 152*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b*c*d^3*f^3*abs(d) + 88*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a
^3*b^2*c*d^3*f^3*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^3*c*d^3*f^3*abs(d) + 80*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*d^4*f^
3*abs(d) - 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^4*B*a^4*b*d^4*f^3*abs(d) + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^2*d^4*f^3*abs(d) - 8*sq

```
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C
*a*b^4*c*d*e^2*abs(d) + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^6*B*b^5*c*d*e^2*abs(d) + 9*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^3*d^2*e^2*abs(
d) - 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^6*B*a*b^4*d^2*e^2*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5*d^2*e^2*abs(d) - 8*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^4*c^2*e*f
*abs(d) + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^6*B*b^5*c^2*e*f*abs(d) + 38*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^3*c*d*e*f*abs(d) - 22*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*
b^4*c*d*e*f*abs(d) + 6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^6*A*b^5*c*d*e*f*abs(d) - 32*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^2*d^2*e*f*abs(d)
 + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^6*B*a^2*b^3*d^2*e*f*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^4*d^2*e*f*abs(d) + 9*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^3*
c^2*f^2*abs(d) - 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^6*B*a*b^4*c^2*f^2*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5*c^2*f^2*abs(d) - 32*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*
a^3*b^2*c*d*f^2*abs(d) + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^3*c*d*f^2*abs(d) - 8*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^4*c*d*f^2*
abs(d) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^6*C*a^4*b*d^2*f^2*abs(d) - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^2*d^2*f^2*abs(d) + 8*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a
^2*b^3*d^2*f^2*abs(d))/((b^6*c*e - a*b^5*d*e - a*b^5*c*f + a^2*b^4*d*f)*(b*
d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*b)^2) + sqrt(d^2*e + (d*x + c)*d*
f - c*d*f)*sqrt(d*x + c)*C*abs(d)/(b^3*d^2) - 1/2*(C*b*d*e*abs(d) + C*b*c*f
*abs(d) - 6*C*a*d*f*abs(d) + 2*B*b*d*f*abs(d))*log((sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2)/(sqrt(d*f)*b^4*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Hanged}$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^3,x)
[Out] \text{Hanged}
```

3.47 $\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1032

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$
$$-\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2f+9c^2d}{}$$
$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$
$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3Bcf-6A}{}$$
$$+\frac{(de-cf)\left(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e}{}$$

```
[Out] 1/128*(-c*f+d*e)*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2
+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f
+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))-b
^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+63*d^4*e^
4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f^3+9*c^2*d*e*f^2+
15*c*d^2*e^2*f+35*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)
^(1/2))/d^(9/2)/f^(11/2)-1/40*(4*a*C*d*f+b*(-10*B*d*f+7*C*c*f+9*C*d*e))*(b*
x+a)^2*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d^2/f^2+1/5*C*(b*x+a)^3*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/d/f-1/960*(d*x+c)^(3/2)*(96*a^3*C*d^3*f^3+8*a^2*b*d^2*f^2*(
-30*B*d*f+9*C*c*f+23*C*d*e)+20*a*b^2*d*f*(8*d*f*(-6*A*d*f+3*B*c*f+5*B*d*e)-
C*(15*c^2*f^2+22*c*d*e*f+35*d^2*e^2))+b^3*(C*(105*c^3*f^3+145*c^2*d*e*f^2+2
03*c*d^2*e^2*f+315*d^3*e^3)+10*d*f*(8*A*d*f*(3*c*f+5*d*e)-B*(15*c^2*f^2+22*
```

```
c*d*e*f+35*d^2*e^2)))+4*b*d*f*(8*b*d*f*(-10*A*b*d*f+C*a*c*f+3*C*a*d*e+6*C*b
*c*e)-(-4*a*d*f+5*b*c*f+7*b*d*e)*(4*a*C*d*f+b*(-10*B*d*f+7*C*c*f+9*C*d*e)))
*x)*(f*x+e)^(1/2)/b/d^4/f^4-1/128*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*
B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f
^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e
*f+5*d^2*e^2)))-b^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^
3*e^3*f+63*d^4*e^4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f
^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/
d^4/f^5
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1629, 158, 152, 52, 65, 223, 212}

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^3}{5bdf}$$
$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^2}{40bd^2f^2}$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}((C(315d^3e^3+203cd^2fe^2+145c^2df^2e+105c^3f^3)+10df(8Adf(5de+3cf)-B(35}{}$$
$$+\frac{(de-cf)\left(-((C(63d^4e^4+28cd^3fe^3+18c^2d^2f^2e^2+12c^3df^3e+7c^4f^4)+2df(8Adf(5d^2e^2+2cdfe+c^2}{}$$
$$-\frac{(-((C(63d^4e^4+28cd^3fe^3+18c^2d^2f^2e^2+12c^3df^3e+7c^4f^4)+2df(8Adf(5d^2e^2+2cdfe+c^2f^2)-B(3}{}$$

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -1/128*((16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 +
 2*c*d*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*
e*f + c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2
 + 12*c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^
2*f^2) - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Sqr
t[c + d*x]*Sqrt[e + f*x])/(d^4*f^5) - ((4*a*C*d*f + b*(9*C*d*e + 7*c*C*f -
10*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(40*b*d^2*f^2) + (C*(
a + b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((c + d*x)^(3/2)*Sqrt
[e + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*
f) + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22
*c*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d
*e*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 2
2*c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C
```

```
*f - 10*A*b*d*f) - (7*b*d*e + 5*b*c*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e +
7*c*C*f - 10*B*d*f)))*x))/(960*b*d^4*f^4) + ((d*e - c*f)*(16*a^2*d^2*f^2*(2
*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + 4
*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d
*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) - b^2*(C*
(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*
f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3*e^3 + 1
5*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(11/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{\int \frac{(a+bx)^2\sqrt{c+dx}\left(-\frac{1}{2}b(6bcCe+3aCde+acCf-10Abdf)-\frac{1}{2}b(4aCdf+b(9Cde+7cCf-10Bdf))x\right)}{\sqrt{e+fx}}\,dx}{5b^2df}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2} + \frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{\int \frac{(a+bx)\sqrt{c+dx}\left(-\frac{1}{4}b(8adf(6bcCe+3aCde+acCf-10Abdf)-(4bce+3ade+acf)(4aCdf+b(9Cde+7cCf-10Bdf)))-\frac{1}{4}b(8bdf(6bcC\ldots\right.}{\sqrt{e+fx}}}{20b^2d^2f^2}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\,(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3B$$

$$-\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2$$

$$=$$

$$-\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2$$

$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\,(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3B$$

$$+\,((de-cf)\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3$$

$$=$$

$$-\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2$$

$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\,(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3B$$

$$+\,((de-cf)\,(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e}{}$$

$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3}{}$$

$$+\frac{((de-cf)(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e}{}$$

$$= -\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e}{}$$

$$-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(96a^3Cd^3f^3+8a^2bd^2f^2(23Cde+9cCf-30Bdf)+20ab^2df(8df(5Bde+3}{}$$

$$+\frac{(de-cf)(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3}{}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3220 vs. 2(1032) = 2064.

Time = 16.92 (sec) , antiderivative size = 3220, normalized size of antiderivative = 3.12

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((-(b*e) + a*f)^2*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)^2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
```

```
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]])))/(2*d^3*f^6*Sqrt[c + d*x]*Sqrt[e +
 f*x]) + (2*b^2*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((
3*(35/(64*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((
d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d
*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f
]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]])))/(512*d^2*f^2*(c + d*
x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))))^4)))/(3*d^4*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(
7/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*b*(d*e - c*f)^2*(-4*b*C*e + b*B*
f + 2*a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5
/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)))]])))/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^4*(d/((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) +
 (2*(d*e - c*f)*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6*a*b*C*e*f + A*b^2*f^2 + 2*a*
b*B*f^2 + a^2*C*f^2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (
d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2
) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])]
)/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]
)))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*d^2*f^4*(d/((d^2*e)/(d*e - c*f) -
(c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) +
```

```
a*f)*(4*b*C*e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*(c + d*x)^(
3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSin
h[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)))]))))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^4*Sqrt[
d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*
f)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. 2(994) = 1988.

Time = 1.68 (sec) , antiderivative size = 3958, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	3958

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-4200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(
1/2)*a*b*d^4*e^3*f+90*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e^2*f^3+150*C*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e^3*f^2+640
*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*f^4*x-3200*B*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*e*f^3*x+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^2*f^3+1050*B*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*
d^5*e^4*f-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*a^2*d^5*e^3*f^2-300*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^4*d*f^5+75*C*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*d*e*f^
4+3840*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^4*f^4*x-1920*A*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^
4*e*f^4-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*b^2*c^3*d^2*e*f^4+300*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*
b^2*c^3*d*f^4+1890*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^4*e^4+105*C*
ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))
*b^2*c^5*f^5+720*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e^2*f^3+960*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
```

1/2)*a^2*c*d^3*f^4-945*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^5*e^5+4800*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*a*b*d^4*e^2*f^2-210*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^4*f^4
+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b^2*c^3*d^2*f^5-480*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^3*f^5-150*B*ln(1/2*(2*d*f*x+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*d*f^5-960*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*b*c^2*d^3*f^5-600*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^3*f^2-240*B*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)*b^2*c*d^3*e*f^3*x+1280*C*a^2*d^4*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)+1920*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*d^4*f^4*x+240*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
^2*c^2*d^3*e*f^4-480*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*c^2*d^2*f^4+
320*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*c*d^3*f^4*x-480*A*((d*x+c)*(f
x+e))^(1/2)(d*f)^(1/2)*b^2*c^2*d^2*f^4+240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^3*d^2*f^5-180*B*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*
c^2*d^3*e^2*f^3+1920*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*f^5-640*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)*a^2*c*d^3*e*f^3+600*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c^3*d*f^
4-220*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^3*d*e*f^3+1400*B*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^4*e^2*f^2*x+480*B*ln(1/2*(2*d*f*x+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e*f^4+1920
*C*a*b*d^4*f^4*x^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+96*C*b^2*c*d^3*f^4*x
^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-864*C*b^2*d^4*e*f^3*x^3*(d*f)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)+2560*B*a*b*d^4*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)+768*C*b^2*d^4*f^4*x^4*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+196*C*((d
x+c)(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e^2*f^2*x-1280*B*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*e*f^3+680*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)*a*b*c^2*d^2*e*f^3+1000*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^3*
e^2*f^2+2880*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*a*b*d^5*e^2*f^3+720*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^2*f^3+2100*C*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^
4*f+525*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*b^2*c*d^4*e^4*f+2400*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d
^4*e^2*f^2-2880*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^4*e*f^3-2100*B*
(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^4*e^3*f+2400*C*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*a^2*d^4*e^2*f^2-1600*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)*a^2*d^4*e*f^3*x+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^3*d*f^4*x
-1260*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^4*e^3*f*x+480*B*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*
d^2*f^5-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*a*b*c*d^4*e^3*f^2-5760*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2

```
)*a*b*d^4*e*f^3-960*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e*f^4-2400*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^5*e^3*f^2-240*C*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c
^3*d^2*e*f^4-360*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e^2*f^3+1920*A*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)*a*b*c*d^3*f^4-640*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e
*f^3+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*b^2*c^2*d^3*e*f^4+3840*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^
2*d^4*f^4+1280*A*b^2*d^4*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-1920*A
*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*a^2*d^5*e*f^4-1200*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b^2*d^5*e^3*f^2+960*B*b^2*d^4*f^4*x^3*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^4*e^2*f^3+2800*C*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)*a*b*d^4*e^2*f^2*x+320*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)*b^2*c*d^3*f^4*x-960*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c^2*d^2*f^4
+340*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e*f^3+500*B*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^3*e^2*f^2-480*C*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)*a*b*c*d^3*e*f^3*x-1120*B*b^2*d^4*e*f^3*x^2*(d*f)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)-112*C*b^2*c^2*d^2*f^4*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)-200*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*f^4*x-1600*A*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^4*e*f^3*x-420*C*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)*b^2*c*d^3*e^3*f+156*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^
2*c^2*d^2*e*f^3*x+1008*C*b^2*d^4*e^2*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)-400*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c^2*d^2*f^4*x-272*C*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e^2*f^2+160*B*b^2*c*d^3*f^4*x^
2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+320*C*a*b*c*d^3*f^4*x^2*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)-2240*C*a*b*d^4*e*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)-128*C*b^2*c*d^3*e*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2))/((d
*x+c)*(f*x+e))^(1/2)/f^5/d^4/(d*f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 3.08 (sec) , antiderivative size = 2176, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(63*C*b^2*d^5*e^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)
*e^4*f - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b
```

+ A*b^2)*d^5)*e^3*f^2 - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8
*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C
*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 1
28*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*
c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x
+ c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b^2*d^5*f^5*x^4 +
945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5)*e^3*f^2 -
2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2 + 2*B*a*b +
A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B*b^2)*c^2*d^3
+ 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*d^5)*e*f^4 - 1
5*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2
+ 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f^5 - 48*(9*C*b^2
*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5)*x^3 + 8*(63*C*b^
2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d^5)*e*f^4 - (7*C*b
^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*B*a*b + A*b^2)*d^5)
*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b^2*c*d^4 + 50*(2*C*a*b + B*b
^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*a*b + B*b^2)*c*d^4 - 400*(C*
a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*c^3*d^2 - 10*(2*C*a*b + B*b^
2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 96*(B*a^2 + 2*A*a*b)*d^5)
*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6), 1/3840*(15*(63*C*b^2*d^5*e
^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)*e^4*f - 10*(C*b^2*c^2*d^3 -
4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^3*f^2 - 6*(
C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c
d^4 + 16(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C*b^2*c^4*d - 128*A*a^2*d^5 -
8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B
*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b
+ B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*
b)*c^2*d^3)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqr
t(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) +
2*(384*C*b^2*d^5*f^5*x^4 + 945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C
*a*b + B*b^2)*d^5)*e^3*f^2 - 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*
d^4 - 600*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 1
7*(2*C*a*b + B*b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a
^2 + 2*A*a*b)*d^5)*e*f^4 - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b
+ B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a
*b)*c*d^4)*f^5 - 48*(9*C*b^2*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2
)*d^5)*f^5)*x^3 + 8*(63*C*b^2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b
+ B*b^2)*d^5)*e*f^4 - (7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C
*a^2 + 2*B*a*b + A*b^2)*d^5)*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b
^2*c*d^4 + 50*(2*C*a*b + B*b^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*
a*b + B*b^2)*c*d^4 - 400*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*
c^3*d^2 - 10*(2*C*a*b + B*b^2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4
+ 96*(B*a^2 + 2*A*a*b)*d^5)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6)

]

Sympy [F]

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)**2*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 1509, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
[Out] 1/1920*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(8
*(d*x + c)*C*b^2/(d^5*f) - (9*C*b^2*d^21*e*f^7 + 31*C*b^2*c*d^20*f^8 - 20*C
*a*b*d^21*f^8 - 10*B*b^2*d^21*f^8)/(d^25*f^9)) + (63*C*b^2*d^22*e^2*f^6 + 1
54*C*b^2*c*d^21*e*f^7 - 140*C*a*b*d^22*e*f^7 - 70*B*b^2*d^22*e*f^7 + 263*C*
b^2*c^2*d^20*f^8 - 340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*a^2*d
^22*f^8 + 160*B*a*b*d^22*f^8 + 80*A*b^2*d^22*f^8)/(d^25*f^9)) - 5*(63*C*b^2
```

```
*d^23*e^3*f^5 + 91*C*b^2*c*d^22*e^2*f^6 - 140*C*a*b*d^23*e^2*f^6 - 70*B*b^2
*d^23*e^2*f^6 + 109*C*b^2*c^2*d^21*e*f^7 - 200*C*a*b*c*d^22*e*f^7 - 100*B*b
^2*c*d^22*e*f^7 + 80*C*a^2*d^23*e*f^7 + 160*B*a*b*d^23*e*f^7 + 80*A*b^2*d^2
3*e*f^7 + 121*C*b^2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B*b^2*c^2*d
^21*f^8 + 112*C*a^2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^2*c*d^22*f^
8 - 96*B*a^2*d^23*f^8 - 192*A*a*b*d^23*f^8)/(d^25*f^9))*(d*x + c) + 15*(63*
C*b^2*d^24*e^4*f^4 + 28*C*b^2*c*d^23*e^3*f^5 - 140*C*a*b*d^24*e^3*f^5 - 70*
B*b^2*d^24*e^3*f^5 + 18*C*b^2*c^2*d^22*e^2*f^6 - 60*C*a*b*c*d^23*e^2*f^6 -
30*B*b^2*c*d^23*e^2*f^6 + 80*C*a^2*d^24*e^2*f^6 + 160*B*a*b*d^24*e^2*f^6 +
80*A*b^2*d^24*e^2*f^6 + 12*C*b^2*c^3*d^21*e*f^7 - 36*C*a*b*c^2*d^22*e*f^7 -
 18*B*b^2*c^2*d^22*e*f^7 + 32*C*a^2*c*d^23*e*f^7 + 64*B*a*b*c*d^23*e*f^7 +
32*A*b^2*c*d^23*e*f^7 - 96*B*a^2*d^24*e*f^7 - 192*A*a*b*d^24*e*f^7 + 7*C*b^
2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^21*f^8 + 16*C*a^2*c
^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d^22*f^8 - 32*B*a^2*c*d^
23*f^8 - 64*A*a*b*c*d^23*f^8 + 128*A*a^2*d^24*f^8)/(d^25*f^9))*sqrt(d*x + c
) + 15*(63*C*b^2*d^5*e^5 - 35*C*b^2*c*d^4*e^4*f - 140*C*a*b*d^5*e^4*f - 70*
B*b^2*d^5*e^4*f - 10*C*b^2*c^2*d^3*e^3*f^2 + 80*C*a*b*c*d^4*e^3*f^2 + 40*B*
b^2*c*d^4*e^3*f^2 + 80*C*a^2*d^5*e^3*f^2 + 160*B*a*b*d^5*e^3*f^2 + 80*A*b^2
*d^5*e^3*f^2 - 6*C*b^2*c^3*d^2*e^2*f^3 + 24*C*a*b*c^2*d^3*e^2*f^3 + 12*B*b^
2*c^2*d^3*e^2*f^3 - 48*C*a^2*c*d^4*e^2*f^3 - 96*B*a*b*c*d^4*e^2*f^3 - 48*A*
b^2*c*d^4*e^2*f^3 - 96*B*a^2*d^5*e^2*f^3 - 192*A*a*b*d^5*e^2*f^3 - 5*C*b^2*
c^4*d*e*f^4 + 16*C*a*b*c^3*d^2*e*f^4 + 8*B*b^2*c^3*d^2*e*f^4 - 16*C*a^2*c^2
*d^3*e*f^4 - 32*B*a*b*c^2*d^3*e*f^4 - 16*A*b^2*c^2*d^3*e*f^4 + 64*B*a^2*c*d
^4*e*f^4 + 128*A*a*b*c*d^4*e*f^4 + 128*A*a^2*d^5*e*f^4 - 7*C*b^2*c^5*f^5 +
20*C*a*b*c^4*d*f^5 + 10*B*b^2*c^4*d*f^5 - 16*C*a^2*c^3*d^2*f^5 - 32*B*a*b*c
^3*d^2*f^5 - 16*A*b^2*c^3*d^2*f^5 + 32*B*a^2*c^2*d^3*f^5 + 64*A*a*b*c^2*d^3
*f^5 - 128*A*a^2*c*d^4*f^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^4*f^5))*d/abs(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
[Out] \text{Hanged}
```

3.48 $\int \frac{(a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 540

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$
$$-\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2def^2+5}{64d^3f^4}$$
$$+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf)-C(}{96bd^3f^3}$$
$$+\frac{(de-cf)\left(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c}{64d^{7/2}f^{9/2}}$$

```
[Out] 1/64*(-c*f+d*e)*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e
*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d
*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*
(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(9/2)+1/4*C*(b*x+a)^2*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/96*(d*x+c)^(3/2)*(24*a^2*C*d^2*f^2+8*a*b*d*f*
(-6*B*d*f+3*C*c*f+5*C*d*e)+b^2*(8*d*f*(-6*A*d*f+3*B*c*f+5*B*d*e)-C*(15*c^2*
f^2+22*c*d*e*f+35*d^2*e^2))+4*b*d*f*(4*a*C*d*f+b*(-8*B*d*f+5*C*c*f+7*C*d*e)
)*x)*(f*x+e)^(1/2)/b/d^3/f^3-1/64*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-
C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2
*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))
)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1629, 152, 52, 65, 223, 212}

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf\ldots}{96bd^3f^3}$$
$$+\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2A\ldots}{64d^{7/2}f^{9/2}}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3d\ldots}{64d^3f^4}$$
$$+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -1/64*((8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e
*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*
f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))
)*Sqrt[c + d*x]*Sqrt[e + f*x])/(d^3*f^4) + (C*(a + b*x)^2*(c + d*x)^(3/2)*S
qrt[e + f*x])/(4*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2
+ 8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f -
 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f
+ b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))/(96*b*d^3*f^3) + ((d*e - c*f)*(8*a*d
*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*
(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*ArcTanh[(Sq
rt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(7/2)*f^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)\sqrt{c+dx}\left(-\frac{1}{2}b(4bcCe+3aCde+acCf-8Abdf)-\frac{1}{2}b(4aCdf+b(7Cde+5cCf-8Bdf))x\right)}{\sqrt{e+fx}}\,dx}{4b^2df}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$- \frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf}{96bd^3f^3}$$
$$- \frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2de}{64d^3f^3}$$

$$=$$
$$- \frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2de}{64d^3f^4}$$
$$+ \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$- \frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf}{96bd^3f^3}$$
$$+ \frac{((de-cf)(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e}{128d^3}$$

$$=$$
$$- \frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2de}{64d^3f^4}$$
$$+ \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$- \frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf}{96bd^3f^3}$$
$$+ \frac{((de-cf)(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e}{}$$

$$=$$
$$- \frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2de}{64d^3f^4}$$
$$+ \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$- \frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Adf}{96bd^3f^3}$$
$$+ \frac{((de-cf)(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e}{}$$

$$= -\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2a}{64d^3f^4}$$
$$+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(24a^2Cd^2f^2+8abdf(5Cde+3cCf-6Bdf)+b^2(8df(5Bde+3Bcf-6Aa}{96bd^3f^3}$$
$$+\frac{(de-cf)\,(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e}{64d^7}$$

Mathematica [A] (verified)

Time = 8.63 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(8adf(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(1}{}$$

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
[Out] (Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(8*a*d*f*(6*d*f*(4*A*d*f + B*(
-3*d*e + c*f + 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e
^2 - 10*e*f*x + 8*f^2*x^2))) + b*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f*x)
 + c*d^2*f*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x - 5
6*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) + B*(-
3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))
- 6*(d*e - c*f)*(-8*a*d*f*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^
2 + 2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f
^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f +
 c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*(Sqrt[c - (d*e)/f] -
Sqrt[c + d*x]))])/(192*d^(7/2)*f^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(508) = 1016.

Time = 1.67 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.71

method	result	size
default	Expression too large to display	2002

[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-48*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c^2*d*f^3-24*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f^2*x-12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*e*f^3-18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e^2*f^2+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*e*f^3+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^2*f^2-48*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*f^4+192*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^3*f^3*x+144*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^2*f^2-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^3*f+96*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*f^3+192*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^3*f^3*x+384*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^3*f^3-120*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^3*f-64*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f^2-64*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^2*e*f^2+50*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*e^2*f-48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*f^4+32*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^2*f^3*x+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^3*e^2*f*x-210*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^3*e^3-112*C*b*d^3*e*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-192*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e*f^3+32*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c*d^2*f^3*x+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e*f^3-20*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d*f^3*x-96*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e*f^3+192*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*f^4-96*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e*f^3+72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e^2*f^2+72*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e^2*f^2+96*C*b*d^3*f^3*x^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+128*B*b*d^3*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+128*C*a*d^3*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d*f^4-160*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^3*e*f^2*x+34*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^2*d*e*f^2+30*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*c^3*f^3+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*f^4-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^4+240*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^3*e^2*f+96*B*((d*x+c)

```
*(f*x+e))^(1/2)*(d*f)^(1/2)*a*c*d^2*f^3-48*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*b*c^2*d*f^3-60*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e^3*f-288*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)*b*d^3*e*f^2-288*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^3*e*f^2+240*
B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^3*e^2*f-160*C*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)*a*d^3*e*f^2*x+16*C*b*c*d^2*f^3*x^2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2))/f^4/((d*x+c)*(f*x+e))^(1/2)/d^3/(d*f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.92 (sec) , antiderivative size = 1114, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/768*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C
*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*
d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*
b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*s
qrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d
*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x)
+ 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^3 + 24*(C*a + B*
b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a + A*b)*
d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 + 16*(B*a
+ A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*a + B*b)*d^4)*f^
4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a + B*b)*d^4)*e*f^3
 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^4)*f^4)*x)*sqrt(
d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^
3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*
a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 +
 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3
*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*
f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f
 + c*d*f^2)*x)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^
3 + 24*(C*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 -
144*(B*a + A*b)*d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^
2*d^2 + 16*(B*a + A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*
a + B*b)*d^4)*f^4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a +
 B*b)*d^4)*e*f^3 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^
4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5)]
```

Sympy [F]

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb}{d^4f}-\frac{7\,Cbd^{13}ef^5+17\,Cbcd^{12}f^6-8\,Cad^{13}f^6-8\,Bbd^{13}f^6}{d^{16}f^7}\right)+\frac{35}{}\right.\right.\right.}{}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
[Out] 1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b/(d^4*f) - (7*C*b*d^13*e*f^5 + 17*C*b*c*d^12*f^6 - 8*C*a*d^13*f^6
 - 8*B*b*d^13*f^6)/(d^16*f^7)) + (35*C*b*d^14*e^2*f^4 + 50*C*b*c*d^13*e*f^5
 - 40*C*a*d^14*e*f^5 - 40*B*b*d^14*e*f^5 + 59*C*b*c^2*d^12*f^6 - 56*C*a*c*d
^13*f^6 - 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6)/(d^16*f^7)
```

```
) - 3*(35*C*b*d^15*e^3*f^3 + 15*C*b*c*d^14*e^2*f^4 - 40*C*a*d^15*e^2*f^4 -
40*B*b*d^15*e^2*f^4 + 9*C*b*c^2*d^13*e*f^5 - 16*C*a*c*d^14*e*f^5 - 16*B*b*c
*d^14*e*f^5 + 48*B*a*d^15*e*f^5 + 48*A*b*d^15*e*f^5 + 5*C*b*c^3*d^12*f^6 -
8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*a*c*d^14*f^6 + 16*A*b*c*d^14
*f^6 - 64*A*a*d^15*f^6)/(d^16*f^7))*sqrt(d*x + c) - 3*(35*C*b*d^4*e^4 - 20*
C*b*c*d^3*e^3*f - 40*C*a*d^4*e^3*f - 40*B*b*d^4*e^3*f - 6*C*b*c^2*d^2*e^2*f
^2 + 24*C*a*c*d^3*e^2*f^2 + 24*B*b*c*d^3*e^2*f^2 + 48*B*a*d^4*e^2*f^2 + 48*
A*b*d^4*e^2*f^2 - 4*C*b*c^3*d*e*f^3 + 8*C*a*c^2*d^2*e*f^3 + 8*B*b*c^2*d^2*e
*f^3 - 32*B*a*c*d^3*e*f^3 - 32*A*b*c*d^3*e*f^3 - 64*A*a*d^4*e*f^3 - 5*C*b*c
^4*f^4 + 8*C*a*c^3*d*f^4 + 8*B*b*c^3*d*f^4 - 16*B*a*c^2*d^2*f^4 - 16*A*b*c^
2*d^2*f^4 + 64*A*a*c*d^3*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
[Out] \text{Hanged}
```

3.49 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 246

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx &= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3}\\ &\quad-\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}\\ &\quad-\frac{(de-cf)\,(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{8d^{5/2}f^{7/2}}\end{aligned}$$

```
[Out] -1/8*(-c*f+d*e)*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*(4*A*d*f-B*(c*f+3*d*
e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(7/2)-1
/12*(-6*B*d*f+7*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^2/f^2+1/3*C*(d
*x+c)^(5/2)*(f*x+e)^(1/2)/d^2/f+1/8*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*
(4*A*d*f-B*(c*f+3*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^2/f^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used

= {965, 81, 52, 65, 223, 212}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= -\frac{(de-cf)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))}{8d^{5/2}f^{7/2}}$$
$$+\frac{\sqrt{c+dx}\sqrt{e+fx}(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))}{8d^2f^3}$$
$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(8*d^2*f^3) - ((5*C*d*e + 7*c*C*f - 6*B*d*f)*(
c + d*x)^(3/2)*Sqrt[e + f*x])/(12*d^2*f^2) + (C*(c + d*x)^(5/2)*Sqrt[e + f*
x])/(3*d^2*f) - ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(
4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
 + f*x])])/(8*d^(5/2)*f^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} + \frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}\left(-5cCde-c^2Cf+6Ad^2f\right)-\frac{1}{2}d(5Cde+7cCf-6Bdf)x\right)}{\sqrt{e+fx}}\,dx}{3d^2f}$$

$$= -\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} + \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\,dx}{8d^2f^2}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3} - \frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} - \frac{\left((de-cf)\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{16d^2f^3}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3} - \frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} - \frac{\left((de-cf)\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{\right.}{8d^3f^3}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3} - \frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} - \frac{((de-cf)(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf))))\operatorname{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+d}}{\sqrt{e+f}}\right.}{8d^3f^3}$$

$$= \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3} - \frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} - \frac{(de-cf)(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{8d^{5/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10efx}{24d^2f^3} + \frac{(de-cf)(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\left(\sqrt{c-\frac{de}{f}}-\sqrt{c+dx}\right)}\right)}{4d^{5/2}f^{7/2}}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2*d*f*x))
+ C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2
))))/(24*d^2*f^3) + ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d
*f*(4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*(S
qrt[c - (d*e)/f] - Sqrt[c + d*x]))])/(4*d^(5/2)*f^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(214) = 428.

Time = 1.67 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.10

method	result
default	$\frac{\sqrt{dx+c}\,\sqrt{fx+e}\left(16C\,d^2f^2x^2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+24A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)c\,d^2f^3-24A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}}{2\sqrt{df}}\right.\right.}{}$

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*d^2*f^2*x^2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e*f^
2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*d^3*e^2*f+24*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*d^2*f^2*x+3*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c
^3*f^3+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e^3+4*C*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)*c*d*f^2*x-20*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*d^2*
e*f*x+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2+12*B*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e
*f-6*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*c^2*f^2-8*C*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e^2)/f
^3/((d*x+c)*(f*x+e))^(1/2)/d^2/(d*f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \left[-\frac{3\,(5\,Cd^3e^3-3\,(Ccd^2+2\,Bd^3)e^2f-(Cc^2d-4\,Bcd^2-8\,Ad^3)ef^2-(Cc^3-2\,Bc^2d+8\,Acd^2)f^3)\sqrt{df}\,\mathrm{log}}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^2
 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*d^3*f^3*
x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*d - 2*B*c*d
^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*x)*sqrt(d*x
 + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3
)*e^2*f - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*
c*d^2)*f^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x
 + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8
*C*d^3*f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*
d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*
x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f}-\frac{5\,Cd^7ef^3+7\,Ccd^6f^4-6\,Bd^7f^4}{d^9f^5}\right)+\frac{3\,(5\,Cd^8e^2f^2+2\,Ccd^7ef^3-}{}\right.\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C/(d^3*f) - (5*C*d^7*e*f^3 + 7*C*c*d^6*f^4 - 6*B*d^7*f^4)/(d^9*f^5))
 + 3*(5*C*d^8*e^2*f^2 + 2*C*c*d^7*e*f^3 - 6*B*d^8*e*f^3 + C*c^2*d^6*f^4 - 2
*B*c*d^7*f^4 + 8*A*d^8*f^4)/(d^9*f^5)) + 3*(5*C*d^3*e^3 - 3*C*c*d^2*e^2*f -
 6*B*d^3*e^2*f - C*c^2*d*e*f^2 + 4*B*c*d^2*e*f^2 + 8*A*d^3*e*f^2 - C*c^3*f^
3 + 2*B*c^2*d*f^3 - 8*A*c*d^2*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(
d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

Mupad [B] (verification not implemented)

Time = 98.80 (sec) , antiderivative size = 1832, normalized size of antiderivative = 7.45

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*d^2*e + 2*A*c*d*f))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + ((2*A*c*f + 2*A*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(f^2*((
e + f*x)^(1/2) - e^(1/2))^3) - (8*A*c^(1/2)*d*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/((((c + d*x)^(1/2) - c^(1/2))
^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2
))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((
C*c^3*d^3*f^3)/4 - (5*C*d^6*e^3)/4 + (C*c^2*d^4*e*f^2)/4 + (3*C*c*d^5*e^2*f
)/4))/(f^9*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((
33*C*d^4*e^3)/2 + (19*C*c^3*d*f^3)/2 + (275*C*c^2*d^2*e*f^2)/2 + (313*C*c*d
^3*e^2*f)/2))/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(
1/2))^7*((19*C*c^3*f^3)/2 + (33*C*d^3*e^3)/2 + (313*C*c*d^2*e^2*f)/2 + (275
*C*c^2*d*e*f^2)/2))/(f^6*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2)
 - c^(1/2))^3*((17*C*c^3*d^2*f^3)/12 - (85*C*d^5*e^3)/12 + (91*C*c^2*d^3*e*
f^2)/4 + (17*C*c*d^4*e^2*f)/4))/(f^8*((e + f*x)^(1/2) - e^(1/2))^3) + (((c
```

```
+ d*x)^(1/2) - c^(1/2))^11*((C*c^3*f^3)/4 - (5*C*d^3*e^3)/4 + (3*C*c*d^2*e^
2*f)/4 + (C*c^2*d*e*f^2)/4))/(d^2*f^4*((e + f*x)^(1/2) - e^(1/2))^11) - (((
c + d*x)^(1/2) - c^(1/2))^9*((17*C*c^3*f^3)/12 - (85*C*d^3*e^3)/12 + (17*C*
c*d^2*e^2*f)/4 + (91*C*c^2*d*e*f^2)/4))/(d*f^5*((e + f*x)^(1/2) - e^(1/2))^
9) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^8*(32*C*c^2*f + 96*C*c*d*
e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^8) + (c^(1/2)*e^(1/2)*(96*C*c*d^3*e +
32*C*c^2*d^2*f)*((c + d*x)^(1/2) - c^(1/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1
/2))^4) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*d^3*e^2 + 6
4*C*c^2*d*f^2 + (704*C*c*d^2*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))^6))/
(((c + d*x)^(1/2) - c^(1/2))^12/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 -
(6*d*((c + d*x)^(1/2) - c^(1/2))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (
6*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) +
(15*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4)
- (20*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6
) + (15*d^2*((c + d*x)^(1/2) - c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))
^8)) + ((((c + d*x)^(1/2) - c^(1/2))*((B*c^2*d^2*f^2)/2 - (3*B*d^4*e^2)/2 +
 B*c*d^3*e*f))/(f^6*((e + f*x)^(1/2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1
/2))^3*((11*B*d^3*e^2)/2 + (7*B*c^2*d*f^2)/2 + 23*B*c*d^2*e*f))/(f^5*((e +
f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^5*((7*B*c^2*f^2)/2
+ (11*B*d^2*e^2)/2 + 23*B*c*d*e*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (
((c + d*x)^(1/2) - c^(1/2))^7*((B*c^2*f^2)/2 - (3*B*d^2*e^2)/2 + B*c*d*e*f)
)/(d*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (c^(1/2)*e^(1/2)*((c + d*x)^(1/2)
 - c^(1/2))^4*(32*B*d^2*e + 16*B*c*d*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4
) - (8*B*c^(3/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6)/(f^2*((e + f*x)^(1/
2) - e^(1/2))^6) - (8*B*c^(3/2)*d^2*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/
(f^4*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f
*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*(
(e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*
((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2
*((e + f*x)^(1/2) - e^(1/2))^4)) + (2*A*atanh((f^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f - d*e))/(d^(1/2)*f^(3/
2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1
/2) - e^(1/2))))*(c*f - d*e)*(c^2*f^2 + 5*d^2*e^2 + 2*c*d*e*f))/(4*d^(5/2)*
f^(7/2)) - (B*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*
x)^(1/2) - e^(1/2))))*(c*f - d*e)*(c*f + 3*d*e))/(2*d^(3/2)*f^(5/2))
```

3.50 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 290

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4b^3d^{3/2}f^{5/2}} - \frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^3\sqrt{be-af}}$$

```
[Out] 1/4*(2*b*d*f*(4*A*b*d*f-a*C*(c*f+3*d*e))+(2*a*d*f-b*c*f+b*d*e)*(4*a*C*d*f+b
*(-4*B*d*f+C*c*f+3*C*d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(
1/2))/b^3/d^(3/2)/f^(5/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d
*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))*(-a*d+b*c)^(1/2)/b^3/(-a*f+b*e)
^(1/2)+1/2*C*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-1/4*(4*a*C*d*f+b*(-4*B*d*f+C
*c*f+3*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1629, 159, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cd}{4b^3d^{3/2}f^{5/2}}$$
$$-\frac{2\sqrt{bc-ad}(Ab^2-a(bB-aC))\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^3\sqrt{be-af}}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(4aCdf+b(-4Bdf+cCf+3Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((4*a*C*d*f + b*(3*C*d*e + c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*
x])/(b^2*d*f^2) + (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + ((2*b*d*f*(
4*A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(
3*C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(4*b^3*d^(3/2)*f^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x
])])/(b^3*Sqrt[b*e - a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
```

```
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}b(4Abdf-aC(3de+cf))-\frac{1}{2}b(4aCdf+b(3Cde+cCf-4Bdf))x\right)}{(a+bx)\sqrt{e+fx}}\,dx}{2b^2df}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{\frac{1}{4}b(2bcf(4Abdf-aC(3de+cf))+a(de+cf)(4aCdf+b(3Cde+cCf-4Bdf)))+\frac{1}{4}b(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4\ldots}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2b^3df^2}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\left(\left(Ab^2-a(bB-aC)\right)(bc-ad)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^3} + \frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\int \frac{\ldots}{\sqrt{c+d\ldots}}}{8b^3df^2}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\left(2\left(Ab^2-a(bB-aC)\right)(bc-ad)\right)\text{Subst}\left(\int \frac{1}{-bc+ad-(-be+af)x^2}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^3} + \frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\text{Subst}\left(\ldots\right.}{4b^3d^2f^2}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} - \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^3\sqrt{be-af}} + \frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\text{Subst}\left(\ldots\right.}{4b^3d^2f^2}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))\tanh^{-1}\ldots}{4b^3d^{3/2}f^{5/2}} - \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^3\sqrt{be-af}}$$

Mathematica [A] (verified)

Time = 11.91 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$= \frac{\frac{8(Ab^2+a(-bB+aC))\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{4b(bCe-bBf+aCf)\sqrt{e+fx}\left(-\sqrt{f}\sqrt{de-cf}(c+dx)\sqrt{\frac{d(e+fx)}{de-cf}}+(de-cf)\sqrt{c+dx}\right.}{f^{5/2}\sqrt{de-cf}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de-cf}}}}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] ((8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (4*b*(b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*(-(Sqrt[f]*Sqrt[d*e - c*f]*
(c + d*x)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (d*e - c*f)*Sqrt[c + d*x]*ArcS
inh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(f^(5/2)*Sqrt[d*e - c*f]*Sqr
t[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (b^2*C*Sqrt[e + f*x]*(Sqrt[f]
*Sqrt[c + d*x]*(c*f + d*(e + 2*f*x)) - ((d*e - c*f)^(3/2)*ArcSinh[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)]))/(d*f^(5/
2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e)
 + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a
*f])/(4*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1821 vs. 2(252) = 504.

Time = 1.69 (sec) , antiderivative size = 1822, normalized size of antiderivative = 6.28

method	result	size
default	Expression too large to display	1822

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/8*(8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^
(1/2)*a*b^2*d^2*f^2-8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*(d*f)^(1/2)*b^3*c*d*f^2+8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*b^3*d^2*f^2-8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
```

```
/(b*x+a))*(d*f)^(1/2)*a^2*b*d^2*f^2+8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*b^2*c*d*f^2-8*B*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^2*d^2*f^2+4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*b^3*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*b^3*d^2*e*f+8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*
x+a))*(d*f)^(1/2)*a^3*d^2*f^2-8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e
+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b*c*d*f^2+4*C*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^3*d*f*x+8*C*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b*d^2*f^2-4*C*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^2*c*d*f^2+4*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)*a*b^2*d^2*e*f-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*b^3*c^2*f^2-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^
3*c*d*e*f+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^3*d^2*e^2+8*
B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*b^3*d*f-8*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^2*d*f+2*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^3*c*f-6*C*(d*f)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^3*d*e
)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/(d*f)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/b^4/d/f^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\,\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="m
axima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="g
iac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)),x)
[Out] \text{Hanged}
```

3.51 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 364

$$\begin{aligned}
&\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx \\
&= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} \\
&\quad - \frac{(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} \\
&\quad - \frac{(4aCdf+b(Cde-cCf-2Bdf))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{d}f^{3/2}} \\
&\quad + \frac{(4a^3Cdf-b^3(2Bce+Ade-Acf)+ab^2(4cCe+3Bde+Bcf)-a^2b(5Cde+3cCf+2Bdf))\operatorname{arctanh}\Big(}{b^3\sqrt{bc-ad}(be-af)^{3/2}}
\end{aligned}$$

```
[Out] -(4*a*C*d*f+b*(-2*B*d*f-C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/(f*x+e)^(1/2))/b^3/f^(3/2)/d^(1/2)+(4*a^3*C*d*f-b^3*(-A*c*f+A*d*e+2*B*c*e)
+a*b^2*(B*c*f+3*B*d*e+4*C*c*e)-a^2*b*(2*B*d*f+3*C*c*f+5*C*d*e))*arctanh((-a
*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^3/(-a*f+b*e)^
(3/2)/(-a*d+b*c)^(1/2)-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-
a*d+b*c)/(-a*f+b*e)/(b*x+a)+(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f
+C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1627, 159, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))}{b^2f(bc-ad)(be-af)}$$
$$+ \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf}{b^3\sqrt{bc-ad}(be-af)^{3/2}}$$
$$- \frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{b(a+bx)(bc-ad)(be-af)}$$
$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(4aCdf+b(-2Bdf-cCf+Cde))}{b^3\sqrt{d}f^{3/2}}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[c +
 d*x]*Sqrt[e + f*x])/(b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 - a*(b*B - a
*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e - a*f)*(a + b*x)) -
 ((4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqrt[d]*f^(3/2)) + ((4*a^3*C*d*f - b^3*(2*B
*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^2*b*(5*C*d*e
+ 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a
*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*c - a*d]*(b*e - a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{\left(Ab^2 - a(bB - aC)\right)(c + dx)^{3/2}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(2Bce+Ade-Acf)-ab(2cCe+3Bde+Bcf-2Adf)}{2b}+\left(-\frac{2a^2Cdf}{b}-b(cCe+Adf)+a(Cde+cCf+Bdf)\right)x\right)}{(a+bx)\sqrt{e+fx}}\,dx}{(bc - ad)(be - af)}$$

$$= \frac{\left(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf)\right)\sqrt{c + dx}\sqrt{e + fx}}{b^2(bc - ad)f(be - af)} - \frac{\left(Ab^2 - a(bB - aC)\right)(c + dx)^{3/2}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} - \frac{\int \frac{-\frac{(bc-ad)\left(2a^2Cf(de+cf)+b^2f(2Bce+Ade-Acf)-ab(Bf(de+cf)+Ce(de+3cf))\right)}{2b}+\frac{(bc-ad)(be-af)(4aCdf+b(Cde-cCf-2Bdf))x}{2b}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b(bc - ad)f(be - af)}$$

$$= \frac{\left(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf)\right)\sqrt{c + dx}\sqrt{e + fx}}{b^2(bc - ad)f(be - af)} - \frac{\left(Ab^2 - a(bB - aC)\right)(c + dx)^{3/2}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} - \frac{\left(4aCdf + b(Cde - cCf - 2Bdf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2b^3f} - \frac{\left(4a^3Cdf - b^3(2Bce + Ade - Acf) + ab^2(4cCe + 3Bde + Bcf) - a^2b(5Cde + 3cCf + 2Bdf)\right)}{2b^3(be - af)}$$

$$= \frac{\left(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf)\right)\sqrt{c + dx}\sqrt{e + fx}}{b^2(bc - ad)f(be - af)} - \frac{\left(Ab^2 - a(bB - aC)\right)(c + dx)^{3/2}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} - \frac{\left(4aCdf + b(Cde - cCf - 2Bdf)\right)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c + dx}\right)}{b^3df} - \frac{\left(4a^3Cdf - b^3(2Bce + Ade - Acf) + ab^2(4cCe + 3Bde + Bcf) - a^2b(5Cde + 3cCf + 2Bdf)\right)}{b^3(be - af)}$$

$$= \frac{(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf))\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{(Ab^2 - a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(4a^3Cdf - b^3(2Bce + Ade - Acf) + ab^2(4cCe + 3Bde + Bcf) - a^2b(5Cde + 3cCf + 2Bdf))\,\text{ta}}{b^3\sqrt{bc-ad}(be-af)^{3/2}} - \frac{(4aCdf + b(Cde - cCf - 2Bdf))\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^3df}$$

$$= \frac{(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf))\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{(Ab^2 - a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{(4aCdf + b(Cde - cCf - 2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{d}f^{3/2}} + \frac{(4a^3Cdf - b^3(2Bce + Ade - Acf) + ab^2(4cCe + 3Bde + Bcf) - a^2b(5Cde + 3cCf + 2Bdf))\,\text{ta}}{b^3\sqrt{bc-ad}(be-af)^{3/2}}$$

Mathematica [A] (verified)

Time = 11.28 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$= \frac{-\frac{2b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)} + \frac{4(bB-2aC)\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\text{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{2bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx} - \frac{\sqrt{de-cf}\text{arcsinh}}{\sqrt{\frac{d(e+}{de-}}}\right.}{f^{3/2}}}{2b^3}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)
*(a + b*x)) + (4*(b*B - 2*a*C)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (2*b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x] - (Sqrt[d*e - c*f]*ArcSinh
[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)])
)/f^(3/2) - (4*(b*B - 2*a*C)*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]
*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a*f] + (
2*b*(A*b^2 + a*(-(b*B) + a*C))*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*S
qrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b
*e) + a*f)^(3/2)))/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3669 vs. 2(332) = 664.

Time = 1.70 (sec) , antiderivative size = 3670, normalized size of antiderivative = 10.08

method	result	size
default	Expression too large to display	3670

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4
*d*e*f*x*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*f^2*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*
a^4*d*f^2*(d*f)^(1/2)-2*A*b^4*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*
c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f^2*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*f^2*x*(d*f)^(1/2)+B*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f^2*x*(d*f)
^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^
2*c*f^2*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a^2*b^2*c*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
*b^3*d*e^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*B*ln((-2*a*d*f*x
+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(
f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*f^2*(d*f)^(1/2)-2*B*l
n((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*f*(d*f
)^(1/2)+2*C*a*b^3*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b
*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*
c*e)/(b*x+a))*b^4*c*f^2*x*(d*f)^(1/2)-C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
```

^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d*e^2*x*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a*b^3*c*f^2*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*f^2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^4*e*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2*B*a*b^3*f*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-4*C*a^2*b^2*
f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*
b^3*c*e*f*x*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*b^4*c*e*f*x*(d*f)^(1/2)+4*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*f^2*x*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)-3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a*b^3*c*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)-3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^
2*c*f^2*x*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*a*b^3*d*e*f*(d*f)^(1/2)+2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)+3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*a^2*b^2*d*e*f*(d*f)^(1/2)+3*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*f*x*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*e*f*x*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x
+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-
a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*f*x*(d*f)^(1/2)-5*C*ln((-2*a*d*f*
x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*f*(d*f)^(1/2)+4*C*
ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*e*f*(
d*f)^(1/2)+C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*b^4*c*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*C
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*f^2*x*
(d*f)^(1/2)-2*C*a*b^3*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d
x+c)(f*x+e))^(1/2)*(d*f)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/(a*f-b*e)/f/(d*f)
^(1/2)/(b*x+a)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/b^4

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\,\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**2*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(331) = 662$.

Time = 1.40 (sec) , antiderivative size = 1354, normalized size of antiderivative = 3.72

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 5*sqrt(d*f)*C*a^
2*b*d^3*e + 3*sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*f)
*C*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f + 4*
sqrt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*B*a^2*b*d^3*f)*arctan(-1/2*(b*d^2*e + b
*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2
)*d))/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*(b^4*e*
abs(d) - a*b^3*f*abs(d))*d) - 2*(sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*
b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*e*f + 2*s
qrt(d*f)*B*a*b^2*c*d^4*e*f - 2*sqrt(d*f)*A*b^3*c*d^4*e*f + sqrt(d*f)*C*a^2*
b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2
 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*d^3*e - sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*c*d^2*f + sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a
*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*d^3*f - 2*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b*d^3*f + 2*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*
b^2*d^3*f)/((b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*b)*(b^4*e*abs(d) -
 a*b^3*f*abs(d))) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*C*abs
(d)/(b^2*d^2*f) + 1/2*(sqrt(d*f)*C*b*d*e - sqrt(d*f)*C*b*c*f + 4*sqrt(d*f)*
C*a*d*f - 2*sqrt(d*f)*B*b*d*f)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^2)/(b^3*f^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

3.52 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 484

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \frac{(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf))\sqrt{c+dx}\sqrt{e}}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{2C\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}} - \frac{(8a^4Cd^2f^2-4a^3bCdf(5de+3cf)+3a^2b^2C(5d^2e^2+10cdef+c^2f^2)-ab^3(d^2e(3Be-4Af)+c^2f(8C}{4b^3}$$

```
[Out] -1/4*(8*a^4*C*d^2*f^2-4*a^3*b*C*d*f*(3*c*f+5*d*e)+3*a^2*b^2*C*(c^2*f^2+10*c
*d*e*f+5*d^2*e^2)-a*b^3*(d^2*e*(-4*A*f+3*B*e)+c^2*f*(-B*f+8*C*e)+2*c*d*(2*A
*f^2-B*e*f+12*C*e^2))-b^4*(A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)-c^2*(3*A*f^2-4*B*
e*f+8*C*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x
+e)^(1/2))/b^3/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(5/2)+2*C*arctanh(f^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/(f*x+e)^(1/2))*d^(1/2)/b^3/f^(1/2)-1/2*(A*b^2-a*(B*b-C*a))*
(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(4*a^3*C*
d*f-a^2*b*C*(5*c*f+7*d*e)-b^3*(-3*A*c*f-A*d*e+4*B*c*e)+a*b^2*(-4*A*d*f+B*c*
f+3*B*d*e+8*C*c*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2
/(b*x+a)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1627, 154, 163, 65, 223, 212, 95, 214}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$$

$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+}{4b^2(a+bx)(bc-ad)(be-af)^2}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b^2C(c^2f^2+10cdef+5d^2e^2)-ab^3(2cd(2}{4b^3}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}+\frac{2C\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^3*(4*B*c*e - A*d*e - 3*A*c*f) +
 a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c +
 d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (2*C
*Sqrt[d]*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqr
t[f]) - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*
d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e - 4*A*f) + c^2*f*(8*C
*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^4*(A*d^2*e^2 - 2*c*d*e*
(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^3*(b*c - a*d)^(3/2)
*(b*e - a*f)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(4Bce-Ade-3Acf)-ab(4cCe+3Bde+Bcf-4Adf)}{2b}-\frac{2C(bc-ad)(be-af)x}{b}\right)}{(a+bx)^2\sqrt{e+fx}}\,dx}{2(bc-ad)(be-af)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf)\right)\sqrt{c+}}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\frac{4a^3Cdf(de+cf)-a^2bC\left(7d^2e^2+14cdef+3c^2f^2\right)+ab^2\left(d^2e(3Be-4Af)+c^2f(8Ce-Bf)+2cd\left(8Ce^2-Bef+2Af^2\right)\right)+b^3\left(Ad^2e^2-2cde(2Be-Af)-\right.}{4b}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2b(bc-ad)(be-af)^2}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf)\right)\sqrt{c+}}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(Cd)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^3} + \frac{\left(8a^4Cd^2f^2-4a^3bCdf(5de+3cf)+3a^2b^2C\left(5d^2e^2+10cdef+c^2f^2\right)-ab^3\left(d^2e(3Be-4Af)+c\right.\right.}{}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf)\right)\sqrt{c+}}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2C)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^3} + \frac{\left(8a^4Cd^2f^2-4a^3bCdf(5de+3cf)+3a^2b^2C\left(5d^2e^2+10cdef+c^2f^2\right)-ab^3\left(d^2e(3Be-4Af)+c\right.\right.}{}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde+Bcf-4Adf)\right)\sqrt{c+}}{4b^2(bc-ad)(be-af)^2(a+bx)} - \frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\left(8a^4Cd^2f^2-4a^3bCdf(5de+3cf)+3a^2b^2C\left(5d^2e^2+10cdef+c^2f^2\right)-ab^3\left(d^2e(3Be-4Af)+c\right.\right.}{} + \frac{(2C)\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^3}$$

$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3Bde + Bcf - 4Adf))\sqrt{c}}{4b^2(bc-ad)(be-af)^2(a+bx)}$$
$$- \frac{(Ab^2 - a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{2C\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$
$$- \frac{(8a^4Cd^2f^2 - 4a^3bCdf(5de+3cf) + 3a^2b^2C(5d^2e^2 + 10cdef + c^2f^2) - ab^3(d^2e(3Be-4Af) +}{}$$

Mathematica [A] (verified)

Time = 13.01 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx =$$
$$-\frac{\frac{4b(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)} + \frac{2b^2(Ab^2+a(-bB+aC))(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2} - \frac{8C\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{8C\sqrt{-bc+a}}{}}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x
)) + (2*b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c
 - a*d)*(b*e - a*f)*(a + b*x)^2) - (8*C*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqr
t[e + f*x]) + (8*C*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c +
d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a*f] - (4*b*(b*B -
 2*a*C)*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b
*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b*e) + a*f)^(3/2)) + (b*
(A*b^2 + a*(-(b*B) + a*C))*(b*d*e + 3*b*c*f - 4*a*d*f)*(Sqrt[-(b*c) + a*d]*
Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] - (d*e - c*f)*(a + b*x)*ArcT
anh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])
)/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*(a + b*x)))/b^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9099 vs. 2(446) = 892.

Time = 1.70 (sec) , antiderivative size = 9100, normalized size of antiderivative = 18.80

method	result	size
default	Expression too large to display	9100

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\,\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**3*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7922 vs. 2(445) = 890.

Time = 23.15 (sec) , antiderivative size = 7922, normalized size of antiderivative = 16.37

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/4*(8*sqrt(d*f)*C*b^4*c^2*d^2*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d^3*e^2 + 4*sq
rt(d*f)*B*b^4*c*d^3*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^4*e^2 - 3*sqrt(d*f)*B*a*
b^3*d^4*e^2 - sqrt(d*f)*A*b^4*d^4*e^2 - 8*sqrt(d*f)*C*a*b^3*c^2*d^2*e*f - 4
*sqrt(d*f)*B*b^4*c^2*d^2*e*f + 30*sqrt(d*f)*C*a^2*b^2*c*d^3*e*f + 2*sqrt(d*
f)*B*a*b^3*c*d^3*e*f - 2*sqrt(d*f)*A*b^4*c*d^3*e*f - 20*sqrt(d*f)*C*a^3*b*d
^4*e*f + 4*sqrt(d*f)*A*a*b^3*d^4*e*f + 3*sqrt(d*f)*C*a^2*b^2*c^2*d^2*f^2 +
sqrt(d*f)*B*a*b^3*c^2*d^2*f^2 + 3*sqrt(d*f)*A*b^4*c^2*d^2*f^2 - 12*sqrt(d*f
)*C*a^3*b*c*d^3*f^2 - 4*sqrt(d*f)*A*a*b^3*c*d^3*f^2 + 8*sqrt(d*f)*C*a^4*d^4
*f^2)*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e
*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e^2*abs(d) - a*b^5*d*e^2*abs(d)
 - 2*a*b^5*c*e*f*abs(d) + 2*a^2*b^4*d*e*f*abs(d) + a^2*b^4*c*f^2*abs(d) - a
^3*b^3*d*f^2*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^
2*f^2)*d) - sqrt(d*f)*C*d*log((sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2)/(b^3*f*abs(d)) + 1/2*(8*sqrt(d*f)*C*a*b^4*c*d^9*e^5 -
 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 9*sqrt(d*f)*C*a^2*b^3*d^10*e^5 + 5*sqrt(d*f)
*B*a*b^4*d^10*e^5 - sqrt(d*f)*A*b^5*d^10*e^5 - 32*sqrt(d*f)*C*a*b^4*c^2*d^8
*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^8*e^4*f + 31*sqrt(d*f)*C*a^2*b^3*c*d^9*e^
4*f - 19*sqrt(d*f)*B*a*b^4*c*d^9*e^4*f + 7*sqrt(d*f)*A*b^5*c*d^9*e^4*f + 6*
sqrt(d*f)*C*a^3*b^2*d^10*e^4*f - 2*sqrt(d*f)*B*a^2*b^3*d^10*e^4*f - 2*sqrt(
d*f)*A*a*b^4*d^10*e^4*f + 48*sqrt(d*f)*C*a*b^4*c^3*d^7*e^3*f^2 - 24*sqrt(d*
f)*B*b^5*c^3*d^7*e^3*f^2 - 34*sqrt(d*f)*C*a^2*b^3*c^2*d^8*e^3*f^2 + 26*sqrt
(d*f)*B*a*b^4*c^2*d^8*e^3*f^2 - 18*sqrt(d*f)*A*b^5*c^2*d^8*e^3*f^2 - 24*sqr
t(d*f)*C*a^3*b^2*c*d^9*e^3*f^2 + 8*sqrt(d*f)*B*a^2*b^3*c*d^9*e^3*f^2 + 8*sq
rt(d*f)*A*a*b^4*c*d^9*e^3*f^2 - 32*sqrt(d*f)*C*a*b^4*c^4*d^6*e^2*f^3 + 16*s
qrt(d*f)*B*b^5*c^4*d^6*e^2*f^3 + 6*sqrt(d*f)*C*a^2*b^3*c^3*d^7*e^2*f^3 - 14
*sqrt(d*f)*B*a*b^4*c^3*d^7*e^2*f^3 + 22*sqrt(d*f)*A*b^5*c^3*d^7*e^2*f^3 + 3
6*sqrt(d*f)*C*a^3*b^2*c^2*d^8*e^2*f^3 - 12*sqrt(d*f)*B*a^2*b^3*c^2*d^8*e^2*
f^3 - 12*sqrt(d*f)*A*a*b^4*c^2*d^8*e^2*f^3 + 8*sqrt(d*f)*C*a*b^4*c^5*d^5*e*
f^4 - 4*sqrt(d*f)*B*b^5*c^5*d^5*e*f^4 + 11*sqrt(d*f)*C*a^2*b^3*c^4*d^6*e*f^
4 + sqrt(d*f)*B*a*b^4*c^4*d^6*e*f^4 - 13*sqrt(d*f)*A*b^5*c^4*d^6*e*f^4 - 24
*sqrt(d*f)*C*a^3*b^2*c^3*d^7*e*f^4 + 8*sqrt(d*f)*B*a^2*b^3*c^3*d^7*e*f^4 +
8*sqrt(d*f)*A*a*b^4*c^3*d^7*e*f^4 - 5*sqrt(d*f)*C*a^2*b^3*c^5*d^5*f^5 + sqr
t(d*f)*B*a*b^4*c^5*d^5*f^5 + 3*sqrt(d*f)*A*b^5*c^5*d^5*f^5 + 6*sqrt(d*f)*C*
```

a^3*b^2*c^4*d^6*f^5 - 2*sqrt(d*f)*B*a^2*b^3*c^4*d^6*f^5 - 2*sqrt(d*f)*A*a*b
^4*c^4*d^6*f^5 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*C*a*b^4*c*d^7*e^4 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c*d^7*e^4 + 27*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^
3*d^8*e^4 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*B*a*b^4*d^8*e^4 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*d^8*e^4 + 24*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^2*d^6*e^
3*f - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*B*b^5*c^2*d^6*e^3*f + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c*d^7*e^3*f - 8*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c*d^
7*e^3*f - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*A*b^5*c*d^7*e^3*f - 68*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*d^8*e^3*f + 32*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^3*d
^8*e^3*f + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*A*a*b^4*d^8*e^3*f + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^3*d^5*e^2*f^2 - 12*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c
^3*d^5*e^2*f^2 - 130*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*C*a^2*b^3*c^2*d^6*e^2*f^2 + 58*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c^2*d^6*e^2*f
^2 + 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*A*b^5*c^2*d^6*e^2*f^2 + 92*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c*d^7*e^2*f^2 - 56*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^
3*c*d^7*e^2*f^2 + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*A*a*b^4*c*d^7*e^2*f^2 + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b*d^8*e^2*f^2 - 8*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*
B*a^3*b^2*d^8*e^2*f^2 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3*d^8*e^2*f^2 - 24*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^4*d^4*e*f
^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*B*b^5*c^4*d^4*e*f^3 + 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c^3*d^5*e*f^3 - 32*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c^
3*d^5*e*f^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*A*b^5*c^3*d^5*e*f^3 + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c^2*d^6*e*f^3 + 16*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*
a^2*b^3*c^2*d^6*e*f^3 - 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^2*d^6*e*f^3 - 64*sqrt(d*f)*(sqrt(d*f)

*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b*c*d^7*e*f^3
 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*B*a^3*b^2*c*d^7*e*f^3 + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3*c*d^7*e*f^3 + 15*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c^
4*d^4*f^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*B*a*b^4*c^4*d^4*f^4 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*c^4*d^4*f^4 - 44*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c
^3*d^5*f^4 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*B*a^2*b^3*c^3*d^5*f^4 + 28*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^3*d^5*f^4 + 32*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4
*b*c^2*d^6*f^4 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*B*a^3*b^2*c^2*d^6*f^4 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3*c^2*d^6*f^4 + 24*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4
*C*a*b^4*c*d^5*e^3 - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^4*B*b^5*c*d^5*e^3 - 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*d^6*e^3 + 15*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*
b^4*d^6*e^3 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*A*b^5*d^6*e^3 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^4*c^2*d^4*e^2*f - 8*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c^2*d
^4*e^2*f - 97*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^4*C*a^2*b^3*c*d^5*e^2*f + 45*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c*d^5*e^2*f + 7*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*c
*d^5*e^2*f + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*C*a^3*b^2*d^6*e^2*f - 46*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*d^6*e^2*f + 2*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^4
*d^6*e^2*f + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*C*a*b^4*c^3*d^3*e*f^2 - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c^3*d^3*e*f^2 - 101*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a
^2*b^3*c^2*d^4*e*f^2 + 49*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c^2*d^4*e*f^2 + 3*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*c^2*d^4*e*f^2 +
 188*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^4*C*a^3*b^2*c*d^5*e*f^2 - 84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c*d^5*e*f^2 - 20*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^4*c*d^5
*e*f^2 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*

f - c*d*f))^4*C*a^4*b*d^6*e*f^2 + 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^2*d^6*e*f^2 + 8*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^3*d^
6*e*f^2 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^4*C*a^2*b^3*c^3*d^3*f^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c^3*d^3*f^3 + 9*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*c^3
*d^3*f^3 + 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^4*C*a^3*b^2*c^2*d^4*f^3 - 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c^2*d^4*f^3 - 30*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*
b^4*c^2*d^4*f^3 - 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f))^4*C*a^4*b*c*d^5*f^3 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^2*c*d^5*f^3 + 40*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a
^2*b^3*c*d^5*f^3 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^4*C*a^5*d^6*f^3 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b*d^6*f^3 - 16*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^2*
d^6*f^3 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^6*C*a*b^4*c*d^3*e^2 + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^5*c*d^3*e^2 + 9*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^3*d^4*e^2
- 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^6*B*a*b^4*d^4*e^2 + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^6*A*b^5*d^4*e^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^4*c^2*d^2*e*f + 4*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^5*c
^2*d^2*e*f + 34*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^6*C*a^2*b^3*c*d^3*e*f - 18*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^4*c*d^3*e*f + 2*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5*c*d
^3*e*f - 28*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^6*C*a^3*b^2*d^4*e*f + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^3*d^4*e*f - 4*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^4*d^4*e*f
 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^6*C*a^2*b^3*c^2*d^2*f^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^4*c^2*d^2*f^2 - 3*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5*c^2*d^2*f^2 -
 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^6*C*a^3*b^2*c*d^3*f^2 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^3*c*d^3*f^2 + 4*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^4*c*d^3*f^2 +
16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)

```
)^6*C*a^4*b*d^4*f^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^6*B*a^3*b^2*d^4*f^2)/((b^6*c*e^2*abs(d) - a*b^5*d*e^
2*abs(d) - 2*a*b^5*c*e*f*abs(d) + 2*a^2*b^4*d*e*f*abs(d) + a^2*b^4*c*f^2*ab
s(d) - a^3*b^3*d*f^2*abs(d))*(b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2 - 2
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*d^2*e
- 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*c*d
*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*a*
d^2*f + (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*b
)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^3),x)
[Out] \text{Hanged}
```

3.53 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 685

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$= \frac{(4a^3Cdf - b^3(6Bce - 3Ade - 5Acf) + ab^2(12cCe + 3Bde + Bcf - 8Adf) - a^2b(9Cde + 7cCf - 2Bdf))}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$
$$- \frac{(8a^4Cd^2f^2 - 2a^3bdf(13Cde + 7cCf - 2Bdf) - b^4(3Ad^2e^2 - 2cde(3Be - 2Af) - 3c^2(8Ce^2 - 6Bef + 5}{}$$
$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$
$$- \frac{(de-cf)(b^2(Ad^2e^2 - 2cde(Be-Af) + c^2(8Ce^2 - 6Bef + 5Af^2)) + ab(d^2e(Be-4Af) - c^2f(4Ce - }{8(bc}$$

```
[Out] -1/8*(-c*f+d*e)*(b^2*(A*d^2*e^2-2*c*d*e*(-A*f+B*e)+c^2*(5*A*f^2-6*B*e*f+8*C
*e^2))+a*b*(d^2*e*(-4*A*f+B*e)-c^2*f*(-B*f+4*C*e)-2*c*d*(6*A*f^2-7*B*e*f+6*
C*e^2))-a^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)
))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(
-a*d+b*c)^(5/2)/(-a*f+b*e)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x
+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3+1/12*(4*a^3*C*d*f-b^3*(-5*A*c*f
-3*A*d*e+6*B*c*e)+a*b^2*(-8*A*d*f+B*c*f+3*B*d*e+12*C*c*e)-a^2*b*(-2*B*d*f+7
*C*c*f+9*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b
*x+a)^2-1/24*(8*a^4*C*d^2*f^2-2*a^3*b*d*f*(-2*B*d*f+7*C*c*f+13*C*d*e)-b^4*(
3*A*d^2*e^2-2*c*d*e*(-2*A*f+3*B*e)-3*c^2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(
d^2*e*(-10*A*f+3*B*e)+3*c^2*f*(-B*f+4*C*e)+2*c*d*(13*A*f^2-14*B*e*f+30*C*e^
2))-a^2*b^2*(4*d*f*(-2*A*d*f+B*c*f+4*B*d*e)-C*(3*c^2*f^2+44*c*d*e*f+33*d^2*
e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1627, 154, 156, 12, 95, 214}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx =$$

$$-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(-(a^2(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2)))+ab}{}$$

$$+\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(}{12b^2(a+bx)^2(bc-ad)(be-af)^2}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^4Cd^2f^2-2a^3bdf(-2Bdf+7cCf+13Cde)-a^2b^2(4df(-2Adf+Bcf+4Bde)-}{}$$

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{3b(a+bx)^3(bc-ad)(be-af)}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A*c*f) + a*b^2*(12*c*C*e + 3*B*d
*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*c*C*f - 2*B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(12*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) - ((8*a^4*C*d
^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4*(3*A*d^2*e^2 - 2*
c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C*e^2 - 14*B*e*f + 13
*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f) - C*(33*d^2*e^2 + 44*
c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b^2*(b*c - a*d)^2*(
b*e - a*f)^3*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e +
 f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - ((d*e - c*f)*(b^2*(A*d^2
*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2*
e*(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^2)
) - a^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f
*x])])/(8*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}-\frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(6Bce-3Ade-5Acf)-ab(6cCe+3Bde+Bcf-6Adf)}{2b}+\left(-3bcCe+3aCde+3acCf+Abdf-aBdf-\frac{2a^2Cdf}{b}\right)x\right)}{(a+bx)^3\sqrt{e+fx}}\,dx}{3(bc-ad)(be-af)}$$

$$= \frac{(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9Cde+7cCf-}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$-\int \frac{\frac{4a^3Cdf(de+cf)+ab^2\left(d^2e(3Be-8Af)+3c^2f(4Ce-Bf)+4cd\left(9Ce^2-4Bef+4Af^2\right)\right)+b^3\left(3Ad^2e^2-2cde(3Be-2Af)-3c^2\left(8Ce^2-6Bef+5Af^2\right)\right.}{4b}}{}$$

$$= \frac{(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9Cde+7cCf-}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$-\frac{(8a^4Cd^2f^2-2a^3bdf(13Cde+7cCf-2Bdf)-b^4(3Ad^2e^2-2cde(3Be-2Af)-3c^2(8Ce^2-6}{}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$+\frac{\int \frac{3b(de-cf)\left(b^2\left(Ad^2e^2-2cde(Be-Af)+c^2\left(8Ce^2-6Bef+5Af^2\right)\right)+ab\left(d^2e(Be-4Af)-c^2f(4Ce-Bf)-2cd\left(6Ce^2-7Bef+6Af^2\right)\right.\right.}{8(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{6b(bc-ad)^2(be-af)^3}$$

$$= \frac{(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9Cde+7cCf-}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$-\frac{(8a^4Cd^2f^2-2a^3bdf(13Cde+7cCf-2Bdf)-b^4(3Ad^2e^2-2cde(3Be-2Af)-3c^2(8Ce^2-6}{}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$+\frac{((de-cf)\left(b^2(Ad^2e^2-2cde(Be-Af)+c^2(8Ce^2-6Bef+5Af^2))+ab(d^2e(Be-4Af)-c^2\right.}{}$$

$$= \frac{(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2b(9Cde+7cCf-}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$-\frac{(8a^4Cd^2f^2-2a^3bdf(13Cde+7cCf-2Bdf)-b^4(3Ad^2e^2-2cde(3Be-2Af)-3c^2(8Ce^2-6}{}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$+\frac{((de-cf)\left(b^2(Ad^2e^2-2cde(Be-Af)+c^2(8Ce^2-6Bef+5Af^2))+ab(d^2e(Be-4Af)-c^2\right.}{}$$

$$= \frac{(4a^3Cdf - b^3(6Bce - 3Ade - 5Acf) + ab^2(12cCe + 3Bde + Bcf - 8Adf) - a^2b(9Cde + 7cCf -}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$- \frac{(8a^4Cd^2f^2 - 2a^3bdf(13Cde + 7cCf - 2Bdf) - b^4(3Ad^2e^2 - 2cde(3Be - 2Af) - 3c^2(8Ce^2 - 6B}{}$$

$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$- \frac{(de-cf)(b^2(Ad^2e^2 - 2cde(Be - Af) + c^2(8Ce^2 - 6Bef + 5Af^2)) + ab(d^2e(Be - 4Af) - c^2f(}{}$$

Mathematica [A] (verified)

Time = 14.91 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx =$$

$$- \frac{\frac{12C\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)} + \frac{4b(Ab^2+a(-bB+aC))(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^3} + \frac{6b(bB-2aC)(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2} - \frac{12C(-de+cf)\operatorname{arctanh}\left(\frac{\sqrt{-be+af}\sqrt{}}{\sqrt{-bc+ad}\sqrt{}}\right)}{\sqrt{-bc+ad}(-be+af)^{3/2}}}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] -1/12*((12*C*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) + (4*b*(A
*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c - a*d)*(b*e -
 a*f)*(a + b*x)^3) + (6*b*(b*B - 2*a*C)*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*
c - a*d)*(b*e - a*f)*(a + b*x)^2) - (12*C*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*
e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c)
+ a*d]*(-(b*e) + a*f)^(3/2)) + (3*(b*B - 2*a*C)*(b*d*e + 3*b*c*f - 4*a*d*f)
*(Sqrt[-(b*c) + a*d]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] - (d*e
- c*f)*(a + b*x)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) +
a*d]*Sqrt[e + f*x])]))/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*(a + b*x)
) - ((A*b^2 + a*(-(b*B) + a*C))*((2*b*(3*b*d*e + 5*b*c*f - 8*a*d*f)*(c + d*
x)^(3/2)*Sqrt[e + f*x])/(a + b*x)^2 + 3*(8*a^2*d^2*f^2 - 4*a*b*d*f*(d*e + 3
*c*f) + b^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*((Sqrt[c + d*x]*Sqrt[e + f*x
])/((-(b*e) + a*f)*(a + b*x)) + ((-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]
*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-
(b*e) + a*f)^(3/2)))))/(2*(b*c - a*d)^2*(b*e - a*f)^2))/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15989 vs. $2(653) = 1306$.

Time = 1.71 (sec) , antiderivative size = 15990, normalized size of antiderivative = 23.34

method	result	size
default	Expression too large to display	15990

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**4/(f*x+e)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25338 vs. 2(653) = 1306.

Time = 60.94 (sec) , antiderivative size = 25338, normalized size of antiderivative = 36.99

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/8*(8*sqrt(d*f)*C*b^2*c^2*d^3*e^3 - 12*sqrt(d*f)*C*a*b*c*d^4*e^3 - 2*sqrt
(d*f)*B*b^2*c*d^4*e^3 + 5*sqrt(d*f)*C*a^2*d^5*e^3 + sqrt(d*f)*B*a*b*d^5*e^3
 + sqrt(d*f)*A*b^2*d^5*e^3 - 8*sqrt(d*f)*C*b^2*c^3*d^2*e^2*f + 8*sqrt(d*f)*
C*a*b*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^2*c^2*d^3*e^2*f - 3*sqrt(d*f)*C*a^2*c
*d^4*e^2*f + 13*sqrt(d*f)*B*a*b*c*d^4*e^2*f + sqrt(d*f)*A*b^2*c*d^4*e^2*f -
 6*sqrt(d*f)*B*a^2*d^5*e^2*f - 4*sqrt(d*f)*A*a*b*d^5*e^2*f + 4*sqrt(d*f)*C*
a*b*c^3*d^2*e*f^2 + 6*sqrt(d*f)*B*b^2*c^3*d^2*e*f^2 - sqrt(d*f)*C*a^2*c^2*d
^3*e*f^2 - 13*sqrt(d*f)*B*a*b*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^2*c^2*d^3*e*f
^2 + 4*sqrt(d*f)*B*a^2*c*d^4*e*f^2 - 8*sqrt(d*f)*A*a*b*c*d^4*e*f^2 + 8*sqrt
(d*f)*A*a^2*d^5*e*f^2 - sqrt(d*f)*C*a^2*c^3*d^2*f^3 - sqrt(d*f)*B*a*b*c^3*d
^2*f^3 - 5*sqrt(d*f)*A*b^2*c^3*d^2*f^3 + 2*sqrt(d*f)*B*a^2*c^2*d^3*f^3 + 12
*sqrt(d*f)*A*a*b*c^2*d^3*f^3 - 8*sqrt(d*f)*A*a^2*c*d^4*f^3)*arctan(-1/2*(b*
d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^
2*d^2*f^2)*d))/((b^5*c^2*e^3*abs(d) - 2*a*b^4*c*d*e^3*abs(d) + a^2*b^3*d^2*
e^3*abs(d) - 3*a*b^4*c^2*e^2*f*abs(d) + 6*a^2*b^3*c*d*e^2*f*abs(d) - 3*a^3*
b^2*d^2*e^2*f*abs(d) + 3*a^2*b^3*c^2*e*f^2*abs(d) - 6*a^3*b^2*c*d*e*f^2*abs
(d) + 3*a^4*b*d^2*e*f^2*abs(d) - a^3*b^2*c^2*f^3*abs(d) + 2*a^4*b*c*d*f^3*a
bs(d) - a^5*d^2*f^3*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 -
 a^2*d^2*f^2)*d) - 1/12*(24*sqrt(d*f)*C*b^7*c^2*d^13*e^8 - 60*sqrt(d*f)*C*a
*b^6*c*d^14*e^8 + 6*sqrt(d*f)*B*b^7*c*d^14*e^8 + 33*sqrt(d*f)*C*a^2*b^5*d^1
5*e^8 - 3*sqrt(d*f)*B*a*b^6*d^15*e^8 - 3*sqrt(d*f)*A*b^7*d^15*e^8 - 144*sqr
t(d*f)*C*b^7*c^3*d^12*e^7*f + 348*sqrt(d*f)*C*a*b^6*c^2*d^13*e^7*f - 54*sqr
t(d*f)*B*b^7*c^2*d^13*e^7*f - 154*sqrt(d*f)*C*a^2*b^5*c*d^14*e^7*f + 46*sqr
t(d*f)*B*a*b^6*c*d^14*e^7*f + 14*sqrt(d*f)*A*b^7*c*d^14*e^7*f - 26*sqrt(d*f
)*C*a^3*b^4*d^15*e^7*f - 16*sqrt(d*f)*B*a^2*b^5*d^15*e^7*f + 10*sqrt(d*f)*A
*a*b^6*d^15*e^7*f + 360*sqrt(d*f)*C*b^7*c^4*d^11*e^6*f^2 - 828*sqrt(d*f)*C*
a*b^6*c^3*d^12*e^6*f^2 + 198*sqrt(d*f)*B*b^7*c^3*d^12*e^6*f^2 + 234*sqrt(d*
f)*C*a^2*b^5*c^2*d^13*e^6*f^2 - 210*sqrt(d*f)*B*a*b^6*c^2*d^13*e^6*f^2 - 6*
sqrt(d*f)*A*b^7*c^2*d^13*e^6*f^2 + 142*sqrt(d*f)*C*a^3*b^4*c*d^14*e^6*f^2 +
 92*sqrt(d*f)*B*a^2*b^5*c*d^14*e^6*f^2 - 86*sqrt(d*f)*A*a*b^6*c*d^14*e^6*f^
2 + 8*sqrt(d*f)*C*a^4*b^3*d^15*e^6*f^2 + 4*sqrt(d*f)*B*a^3*b^4*d^15*e^6*f^2
 + 8*sqrt(d*f)*A*a^2*b^5*d^15*e^6*f^2 - 480*sqrt(d*f)*C*b^7*c^5*d^10*e^5*f^
3 + 1020*sqrt(d*f)*C*a*b^6*c^4*d^11*e^5*f^3 - 390*sqrt(d*f)*B*b^7*c^4*d^11*
```

e^5*f^3 - 18*sqrt(d*f)*C*a^2*b^5*c^3*d^12*e^5*f^3 + 462*sqrt(d*f)*B*a*b^6*c
^3*d^12*e^5*f^3 - 90*sqrt(d*f)*A*b^7*c^3*d^12*e^5*f^3 - 306*sqrt(d*f)*C*a^3
*b^4*c^2*d^13*e^5*f^3 - 216*sqrt(d*f)*B*a^2*b^5*c^2*d^13*e^5*f^3 + 306*sqrt
(d*f)*A*a*b^6*c^2*d^13*e^5*f^3 - 48*sqrt(d*f)*C*a^4*b^3*c*d^14*e^5*f^3 - 24
*sqrt(d*f)*B*a^3*b^4*c*d^14*e^5*f^3 - 48*sqrt(d*f)*A*a^2*b^5*c*d^14*e^5*f^3
 + 360*sqrt(d*f)*C*b^7*c^6*d^9*e^4*f^4 - 660*sqrt(d*f)*C*a*b^6*c^5*d^10*e^4
*f^4 + 450*sqrt(d*f)*B*b^7*c^5*d^10*e^4*f^4 - 340*sqrt(d*f)*C*a^2*b^5*c^4*d
^11*e^4*f^4 - 560*sqrt(d*f)*B*a*b^6*c^4*d^11*e^4*f^4 + 260*sqrt(d*f)*A*b^7*
c^4*d^11*e^4*f^4 + 310*sqrt(d*f)*C*a^3*b^4*c^3*d^12*e^4*f^4 + 260*sqrt(d*f)
*B*a^2*b^5*c^3*d^12*e^4*f^4 - 590*sqrt(d*f)*A*a*b^6*c^3*d^12*e^4*f^4 + 120*
sqrt(d*f)*C*a^4*b^3*c^2*d^13*e^4*f^4 + 60*sqrt(d*f)*B*a^3*b^4*c^2*d^13*e^4*
f^4 + 120*sqrt(d*f)*A*a^2*b^5*c^2*d^13*e^4*f^4 - 144*sqrt(d*f)*C*b^7*c^7*d^
8*e^3*f^5 + 180*sqrt(d*f)*C*a*b^6*c^6*d^9*e^3*f^5 - 306*sqrt(d*f)*B*b^7*c^6
*d^9*e^3*f^5 + 402*sqrt(d*f)*C*a^2*b^5*c^5*d^10*e^3*f^5 + 378*sqrt(d*f)*B*a
*b^6*c^5*d^10*e^3*f^5 - 342*sqrt(d*f)*A*b^7*c^5*d^10*e^3*f^5 - 110*sqrt(d*f
)*C*a^3*b^4*c^4*d^11*e^3*f^5 - 160*sqrt(d*f)*B*a^2*b^5*c^4*d^11*e^3*f^5 + 6
70*sqrt(d*f)*A*a*b^6*c^4*d^11*e^3*f^5 - 160*sqrt(d*f)*C*a^4*b^3*c^3*d^12*e^
3*f^5 - 80*sqrt(d*f)*B*a^3*b^4*c^3*d^12*e^3*f^5 - 160*sqrt(d*f)*A*a^2*b^5*c
^3*d^12*e^3*f^5 + 24*sqrt(d*f)*C*b^7*c^8*d^7*e^2*f^6 + 12*sqrt(d*f)*C*a*b^6
*c^7*d^8*e^2*f^6 + 114*sqrt(d*f)*B*b^7*c^7*d^8*e^2*f^6 - 186*sqrt(d*f)*C*a^
2*b^5*c^6*d^9*e^2*f^6 - 126*sqrt(d*f)*B*a*b^6*c^6*d^9*e^2*f^6 + 246*sqrt(d*
f)*A*b^7*c^6*d^9*e^2*f^6 - 54*sqrt(d*f)*C*a^3*b^4*c^5*d^10*e^2*f^6 + 36*sqr
t(d*f)*B*a^2*b^5*c^5*d^10*e^2*f^6 - 450*sqrt(d*f)*A*a*b^6*c^5*d^10*e^2*f^6
 + 120*sqrt(d*f)*C*a^4*b^3*c^4*d^11*e^2*f^6 + 60*sqrt(d*f)*B*a^3*b^4*c^4*d^1
1*e^2*f^6 + 120*sqrt(d*f)*A*a^2*b^5*c^4*d^11*e^2*f^6 - 12*sqrt(d*f)*C*a*b^6
*c^8*d^7*e*f^7 - 18*sqrt(d*f)*B*b^7*c^8*d^7*e*f^7 + 26*sqrt(d*f)*C*a^2*b^5*
c^7*d^8*e*f^7 + 10*sqrt(d*f)*B*a*b^6*c^7*d^8*e*f^7 - 94*sqrt(d*f)*A*b^7*c^7
*d^8*e*f^7 + 58*sqrt(d*f)*C*a^3*b^4*c^6*d^9*e*f^7 + 8*sqrt(d*f)*B*a^2*b^5*c
^6*d^9*e*f^7 + 166*sqrt(d*f)*A*a*b^6*c^6*d^9*e*f^7 - 48*sqrt(d*f)*C*a^4*b^3
*c^5*d^10*e*f^7 - 24*sqrt(d*f)*B*a^3*b^4*c^5*d^10*e*f^7 - 48*sqrt(d*f)*A*a^
2*b^5*c^5*d^10*e*f^7 + 3*sqrt(d*f)*C*a^2*b^5*c^8*d^7*f^8 + 3*sqrt(d*f)*B*a*
b^6*c^8*d^7*f^8 + 15*sqrt(d*f)*A*b^7*c^8*d^7*f^8 - 14*sqrt(d*f)*C*a^3*b^4*c
^7*d^8*f^8 - 4*sqrt(d*f)*B*a^2*b^5*c^7*d^8*f^8 - 26*sqrt(d*f)*A*a*b^6*c^7*d
^8*f^8 + 8*sqrt(d*f)*C*a^4*b^3*c^6*d^9*f^8 + 4*sqrt(d*f)*B*a^3*b^4*c^6*d^9*
f^8 + 8*sqrt(d*f)*A*a^2*b^5*c^6*d^9*f^8 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^2*d^11*e^7 + 300*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*
a*b^6*c*d^12*e^7 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*B*b^7*c*d^12*e^7 - 165*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*d^13*e^7 + 15*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a
*b^6*d^13*e^7 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*A*b^7*d^13*e^7 + 360*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^3*d^10*e^6*f - 648*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b

^6*c^2*d^11*e^6*f + 204*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*B*b^7*c^2*d^11*e^6*f - 225*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c*d^12*e^6*f
 - 177*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*B*a*b^6*c*d^12*e^6*f - 21*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c*d^12*e^6*f + 408*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*d^13
*e^6*f + 78*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^2*B*a^2*b^5*d^13*e^6*f - 84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*d^13*e^6*f - 240*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^4
*d^9*e^5*f^2 - 348*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*C*a*b^6*c^3*d^10*e^5*f^2 - 426*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^3*d^10*e^5*f^2
+ 2271*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*C*a^2*b^5*c^2*d^11*e^5*f^2 + 195*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c^2*d^11*e^5*f^2 - 141*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A
*b^7*c^2*d^11*e^5*f^2 - 1152*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c*d^12*e^5*f^2 + 132*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c*d
^12*e^5*f^2 + 408*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*A*a*b^6*c*d^12*e^5*f^2 - 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*d^13*e^5*f^2 - 2
16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*B*a^3*b^4*d^13*e^5*f^2 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*d^13*e^5*f^2 - 240*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^5*d^
8*e^4*f^3 + 1872*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^2*C*a*b^6*c^4*d^9*e^4*f^3 + 264*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^4*d^9*e^4*f^3 - 35
49*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*C*a^2*b^5*c^3*d^10*e^4*f^3 + 675*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c^3*d^10*e^4*f^3 + 399*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7
*c^3*d^10*e^4*f^3 + 600*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^2*d^11*e^4*f^3 - 1254*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^2*d
^11*e^4*f^3 - 492*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*A*a*b^6*c^2*d^11*e^4*f^3 + 744*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c*d^12*e^4*f^3
 + 792*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*B*a^3*b^4*c*d^12*e^4*f^3 - 528*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*c*d^12*e^4*f^3 + 48*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^

5*b^2*d^13*e^4*f^3 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*d^13*e^4*f^3 + 96*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4*d^13*e^4*f^
3 + 360*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*C*b^7*c^6*d^7*e^3*f^4 - 1548*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^6*c^5*d^8*e^3*f^4 + 174*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7
*c^5*d^8*e^3*f^4 + 1761*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^4*d^9*e^3*f^4 - 1635*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c^4*d^9*
e^3*f^4 - 291*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*A*b^7*c^4*d^9*e^3*f^4 + 960*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^3*d^10*e^3*f^4 + 21
36*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*B*a^2*b^5*c^3*d^10*e^3*f^4 - 432*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*c^3*d^10*e^3*f^4 - 816*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4
*b^3*c^2*d^11*e^3*f^4 - 1008*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*c^2*d^11*e^3*f^4 + 1632*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*
c^2*d^11*e^3*f^4 - 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^2*C*a^5*b^2*c*d^12*e^3*f^4 - 192*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*c*d^12*e^
3*f^4 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^2*A*a^3*b^4*c*d^12*e^3*f^4 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^7*d^6*e^2*f^5 + 312*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C
*a*b^6*c^6*d^7*e^2*f^5 - 276*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^6*d^7*e^2*f^5 + 141*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^5*d^
8*e^2*f^5 + 1245*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^2*B*a*b^6*c^5*d^8*e^2*f^5 - 111*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c^5*d^8*e^2*f^5 - 10
80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*C*a^3*b^4*c^4*d^9*e^2*f^5 - 1374*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^4*d^9*e^2*f^5 + 1428*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a
*b^6*c^4*d^9*e^2*f^5 + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c^3*d^10*e^2*f^5 + 432*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*c^3
*d^10*e^2*f^5 - 2208*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f))^2*A*a^2*b^5*c^3*d^10*e^2*f^5 + 288*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^5*b^2*c^2*d^11*
e^2*f^5 + 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*B*a^4*b^3*c^2*d^11*e^2*f^5 + 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d

*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4*c^2*d^11*e^2*f^5
 + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*C*a*b^6*c^7*d^6*e*f^6 + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^7*d^6*e*f^6 - 219*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^6
*d^7*e*f^6 - 303*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^2*B*a*b^6*c^6*d^7*e*f^6 + 225*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c^6*d^7*e*f^6 + 192*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C
*a^3*b^4*c^5*d^8*e*f^6 + 228*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^5*d^8*e*f^6 - 1128*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*c^5*d
^8*e*f^6 + 264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*C*a^4*b^3*c^4*d^9*e*f^6 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*c^4*d^9*e*f^6 + 139
2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^2*A*a^2*b^5*c^4*d^9*e*f^6 - 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^5*b^2*c^3*d^10*e*f^6 - 192*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3
*c^3*d^10*e*f^6 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*A*a^3*b^4*c^3*d^10*e*f^6 - 15*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^7*d^6*f^7
 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*B*a*b^6*c^7*d^6*f^7 - 75*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c^7*d^6*f^7 + 72*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^6*d^7*
f^7 + 54*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*B*a^2*b^5*c^6*d^7*f^7 + 300*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*c^6*d^7*f^7 - 120*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3
*c^5*d^8*f^7 - 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*B*a^3*b^4*c^5*d^8*f^7 - 336*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*c^5*d^8*f^7 + 48*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*C*a^5*b^2*c^4*d^9*f^7 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*c^4*d^9*f^7 + 96*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4*c^4*d^9*
f^7 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^4*C*b^7*c^2*d^9*e^6 - 600*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c*d^10*e^6 + 60*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c*d^10*e
^6 + 330*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^4*C*a^2*b^5*d^11*e^6 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*d^11*e^6 - 30*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*d^11*e^6 -

192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*C*b^7*c^3*d^8*e^5*f - 312*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^2*d^9*e^5*f - 276*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^2*d^9*
e^5*f + 1896*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^4*C*a^2*b^5*c*d^10*e^5*f + 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c*d^10*e^5*f - 24*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^
7*c*d^10*e^5*f - 1212*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^4*C*a^3*b^4*d^11*e^5*f - 120*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*d^11*e^5*f + 2
04*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^4*A*a*b^6*d^11*e^5*f - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^4*C*b^7*c^4*d^7*e^4*f^2 + 1008*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^3*d^8*
e^4*f^2 + 168*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^4*B*b^7*c^3*d^8*e^4*f^2 - 1302*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^2*d^9*e^4*f^2 + 57
0*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^4*B*a*b^6*c^2*d^9*e^4*f^2 + 306*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c^2*d^9*e^4*f^2 - 1716*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c
*d^10*e^4*f^2 - 864*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^4*B*a^2*b^5*c*d^10*e^4*f^2 - 492*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c*d^10*e^4*f^2
 + 1656*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^4*C*a^4*b^3*d^11*e^4*f^2 + 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*d^11*e^4*f^2 - 264*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*
b^5*d^11*e^4*f^2 - 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^4*C*b^7*c^5*d^6*e^3*f^3 + 1200*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^4*d^7*e^3*f
^3 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^4*B*b^7*c^4*d^7*e^3*f^3 - 3696*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^3*d^8*e^3*f^3 - 720*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a
*b^6*c^3*d^8*e^3*f^3 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^4*A*b^7*c^3*d^8*e^3*f^3 + 5304*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^2*d^9
*e^3*f^3 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^4*B*a^2*b^5*c^2*d^9*e^3*f^3 - 504*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^2*d^9*e^3*f^3 -
1200*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^4*C*a^4*b^3*c*d^10*e^3*f^3 + 528*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c*d^10*e^3*f^3 + 1488*sqrt(

d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^
2*b^5*c*d^10*e^3*f^3 - 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*d^11*e^3*f^3 - 672*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*d^11*e^
3*f^3 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^4*A*a^3*b^4*d^11*e^3*f^3 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*b^7*c^6*d^5*e^2*f^4 - 1176*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*
a*b^6*c^5*d^6*e^2*f^4 + 156*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^5*d^6*e^2*f^4 + 2094*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^4*d^
7*e^2*f^4 - 954*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*B*a*b^6*c^4*d^7*e^2*f^4 - 42*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c^4*d^7*e^2*f^4 - 792*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4
*C*a^3*b^4*c^3*d^8*e^2*f^4 + 2544*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*c^3*d^8*e^2*f^4 + 888*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6
*c^3*d^8*e^2*f^4 - 2112*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*c^2*d^9*e^2*f^4 - 2544*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^2*d^
9*e^2*f^4 - 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*A*a^2*b^5*c^2*d^9*e^2*f^4 + 1104*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c*d^10*e^2*f^4
 + 1152*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^4*B*a^4*b^3*c*d^10*e^2*f^4 - 1104*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*c*d^10*e^2*f^4 + 192*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C
*a^6*b*d^11*e^2*f^4 + 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^4*B*a^5*b^2*d^11*e^2*f^4 + 384*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*d^11*e^2*
f^4 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^4*C*a*b^6*c^6*d^5*e*f^5 - 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^6*d^5*e*f^5 + 648*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^
5*c^5*d^6*e*f^5 + 888*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^4*B*a*b^6*c^5*d^6*e*f^5 - 120*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c^5*d^6*e*f^5 - 1
404*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*C*a^3*b^4*c^4*d^7*e*f^5 - 1608*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*c^4*d^7*e*f^5 + 684*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6
*c^4*d^7*e*f^5 + 1200*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^4*C*a^4*b^3*c^3*d^8*e*f^5 + 1200*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^3*d^8*e*

f^5 - 2256*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*c^3*d^8*e*f^5 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c^2*d^9*e*f^5 - 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*c^2*d^9*e*f^5 + 2640*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*c^2*d^9*e*f^5 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^6*b*c*d^10*e*f^5 - 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^5*b^2*c*d^10*e*f^5 - 768*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*c*d^10*e*f^5 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^6*d^5*f^6 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c^6*d^5*f^6 + 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c^6*d^5*f^6 - 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^5*d^6*f^6 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*c^5*d^6*f^6 - 780*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^5*d^6*f^6 + 456*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*c^4*d^7*f^6 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^4*d^7*f^6 + 1608*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*c^4*d^7*f^6 - 528*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c^3*d^8*f^6 - 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*c^3*d^8*f^6 - 1392*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*c^3*d^8*f^6 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^6*b*c^2*d^9*f^6 + 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^5*b^2*c^2*d^9*f^6 + 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*c^2*d^9*f^6 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^2*d^7*e^5 + 600*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c*d^8*e^5 - 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c*d^8*e^5 - 330*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*d^9*e^5 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*d^9*e^5 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*d^9*e^5 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^3*d^6*e^4*f + 1344*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^2*d^7*e^4*f + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c^2*d^7*e^4*f - 2798*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c*d^8*e^4*f - 46*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6

*B*a*b^6*c*d^8*e^4*f + 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c*d^8*e^4*f + 1448*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*d^9*e^4*f + 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*d^9*e^4*f - 208*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*d^9*e^4*f - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^4*d^5*e^3*f^2 + 1008*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^3*d^6*e^3*f^2 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c^3*d^6*e^3*f^2 - 3516*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^2*d^7*e^3*f^2 - 684*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^2*d^7*e^3*f^2 - 108*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^2*d^7*e^3*f^2 + 5552*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c*d^8*e^3*f^2 + 784*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c*d^8*e^3*f^2 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c*d^8*e^3*f^2 - 2600*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*d^9*e^3*f^2 - 472*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*d^9*e^3*f^2 + 424*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^5*d^9*e^3*f^2 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^5*d^4*e^2*f^3 + 1536*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^4*d^5*e^2*f^3 + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c^4*d^5*e^2*f^3 - 4212*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^3*d^6*e^2*f^3 - 372*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^3*d^6*e^2*f^3 - 36*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^3*d^6*e^2*f^3 + 6816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^2*d^7*e^2*f^3 + 1368*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c^2*d^7*e^2*f^3 + 432*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c^2*d^7*e^2*f^3 - 6792*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*c*d^8*e^2*f^3 - 1560*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*c*d^8*e^2*f^3 - 408*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^5*c*d^8*e^2*f^3 + 2592*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^5*b^2*d^9*e^2*f^3 + 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^4*b^3*d^9*e^2*f^3 - 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^3*b^4*d^9*e^2*f^3 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^5*d^4*e* f^4 + 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c^5*d^4*e*f^4 - 890*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^4*d^5*e*f^4 - 946*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^4*d^5*e*f^4 - 50*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^4*d^5*e*f^4 + 2576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^3*d^6*e*f^4 + 2128*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c^3*d^6*e*f^4 + 272*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c^3*d^6*e*f^4 - 3960*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*c^2*d^7*e*f^4 - 2760*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*c^2*d^7*e*f^4 - 840*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^5*c^2*d^7*e*f^4 + 3520*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^5*b^2*c*d^8*e*f^4 + 1952*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^4*b^3*c*d^8*e*f^4 + 832*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^3*b^4*c*d^8*e*f^4 - 1216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^6*b*d^9*e*f^4 - 704*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^5*b^2*d^9*e*f^4 - 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^4*b^3*d^9*e*f^4 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^5*d^4*f^5 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^5*d^4*f^5 - 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^5*d^4*f^5 + 248*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^4*d^5*f^5 + 148*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c^4*d^5*f^5 + 800*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c^4*d^5*f^5 - 728*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*c^3*d^6*f^5 - 328*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*c^3*d^6*f^5 - 1736*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^5*c^3*d^6*f^5 + 1056*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^5*b^2*c^2*d^7*f^5 + 432*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^4*b^3*c^2*d^7*f^5 + 2016*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^3*b^4*c^2*d^7*f^5 - 832*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^6*b*c*d^8*f^5 - 320*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^5*b^2*c*d^8*f^5 - 1216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^4*b^3*c*d^8*f^5 + 256*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^7*d^9*f^5 + 128*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x

+ c)*d*f - c*d*f))^6*B*a^6*b*d^9*f^5 + 256*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^5*b^2*d^9*f^5 + 120*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b^7
*c^2*d^5*e^4 - 300*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^8*C*a*b^6*c*d^6*e^4 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*b^7*c*d^6*e^4 + 165*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^2*b^
5*d^7*e^4 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^8*B*a*b^6*d^7*e^4 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*d^7*e^4 + 144*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b^7*c^3*d^4*e^
3*f - 900*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*C*a*b^6*c^2*d^5*e^3*f - 6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*b^7*c^2*d^5*e^3*f + 1578*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^2*b^5
*c*d^6*e^3*f - 78*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^8*B*a*b^6*c*d^6*e^3*f - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*c*d^6*e^3*f - 762*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^3
*b^4*d^7*e^3*f + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^8*B*a^2*b^5*d^7*e^3*f + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a*b^6*d^7*e^3*f + 120*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b
^7*c^4*d^3*e^2*f^2 - 852*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^8*C*a*b^6*c^3*d^4*e^2*f^2 - 30*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*b^7*c^3*d^4*e^2*f
^2 + 2460*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*C*a^2*b^5*c^2*d^5*e^2*f^2 + 168*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a*b^6*c^2*d^5*e^2*f^2 - 60*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A
*b^7*c^2*d^5*e^2*f^2 - 3114*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^8*C*a^3*b^4*c*d^6*e^2*f^2 - 156*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^2*b^5*c*d^6
*e^2*f^2 + 210*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^8*A*a*b^6*c*d^6*e^2*f^2 + 1296*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^4*b^3*d^7*e^2*f^2 + 108*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*
B*a^3*b^4*d^7*e^2*f^2 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^8*A*a^2*b^5*d^7*e^2*f^2 - 60*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a*b^6*c^4*d^3*e*
f^3 - 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*B*b^7*c^4*d^3*e*f^3 + 582*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^2*b^5*c^3*d^4*e*f^3 + 390*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a*b^6
*c^3*d^4*e*f^3 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x

+ c)*d*f - c*d*f))^8*A*b^7*c^3*d^4*e*f^3 - 1710*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^3*b^4*c^2*d^5*e*f^3 -
768*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^8*B*a^2*b^5*c^2*d^5*e*f^3 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a*b^6*c^2*d^5*e*f^3 + 2064*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^4*b^3*
c*d^6*e*f^3 + 696*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^8*B*a^3*b^4*c*d^6*e*f^3 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^2*b^5*c*d^6*e*f^3 - 816
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
8*C*a^5*b^2*d^7*e*f^3 - 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^8*B*a^4*b^3*d^7*e*f^3 + 240*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^3*b^4*d^7*e*f^3
 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^8*C*a^2*b^5*c^4*d^3*f^4 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a*b^6*c^4*d^3*f^4 + 75*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*c^4*d^3*
f^4 - 174*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^8*C*a^3*b^4*c^3*d^4*f^4 - 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^2*b^5*c^3*d^4*f^4 - 330*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a*b^6
*c^3*d^4*f^4 + 480*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^8*C*a^4*b^3*c^2*d^5*f^4 + 156*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^3*b^4*c^2*d^5*f^4 + 48
0*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^8*A*a^2*b^5*c^2*d^5*f^4 - 528*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^8*C*a^5*b^2*c*d^6*f^4 - 192*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^4*b^3*c*d^6*
f^4 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^8*A*a^3*b^4*c*d^6*f^4 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^6*b*d^7*f^4 + 96*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^5*b^2*d^7*f
^4 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^10*C*b^7*c^2*d^3*e^3 + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a*b^6*c*d^4*e^3 - 6*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c*d^4*e^3
 - 33*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^10*C*a^2*b^5*d^5*e^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^10*B*a*b^6*d^5*e^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*d^5*e^3 - 24*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C
*b^7*c^3*d^2*e^2*f + 168*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^10*C*a*b^6*c^2*d^3*e^2*f - 12*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c^2*d^3*e^2*f
 - 297*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*

d*f))^10*C*a^2*b^5*c*d^4*e^2*f + 39*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*a*b^6*c*d^4*e^2*f + 3*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c*d^4
*e^2*f + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^10*C*a^3*b^4*d^5*e^2*f - 18*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*a^2*b^5*d^5*e^2*f - 12*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*b^
6*d^5*e^2*f + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^10*C*a*b^6*c^3*d^2*e*f^2 + 18*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c^3*d^2*e*f^2 - 147*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10
*C*a^2*b^5*c^2*d^3*e*f^2 - 39*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^10*B*a*b^6*c^2*d^3*e*f^2 + 9*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c^2*d^3*e
*f^2 + 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^10*C*a^3*b^4*c*d^4*e*f^2 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*a^2*b^5*c*d^4*e*f^2 - 24*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*
b^6*c*d^4*e*f^2 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^10*C*a^4*b^3*d^5*e*f^2 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a^2*b^5*d^5*e*f^2 - 3*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^1
0*C*a^2*b^5*c^3*d^2*f^3 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^10*B*a*b^6*c^3*d^2*f^3 - 15*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c^3*d^2*f^3
+ 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^10*C*a^3*b^4*c^2*d^3*f^3 + 6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^10*B*a^2*b^5*c^2*d^3*f^3 + 36*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*b^6*c^2
*d^3*f^3 - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^10*C*a^4*b^3*c*d^4*f^3 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a^2*b^5*c*d^4*f^3 + 48*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a^5
*b^2*d^5*f^3)/((b^8*c^2*e^3*abs(d) - 2*a*b^7*c*d*e^3*abs(d) + a^2*b^6*d^2*e
^3*abs(d) - 3*a*b^7*c^2*e^2*f*abs(d) + 6*a^2*b^6*c*d*e^2*f*abs(d) - 3*a^3*b
^5*d^2*e^2*f*abs(d) + 3*a^2*b^6*c^2*e*f^2*abs(d) - 6*a^3*b^5*c*d*e*f^2*abs(
d) + 3*a^4*b^4*d^2*e*f^2*abs(d) - a^3*b^5*c^2*f^3*abs(d) + 2*a^4*b^4*c*d*f^
3*abs(d) - a^5*b^3*d^2*f^3*abs(d))*(b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f
^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*
d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*b)^3)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^4),x)
[Out] \text{Hanged}
```

3.54 $\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 718

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= -\frac{(2aCdf-b(8Bdf-7C(de+cf)))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2}$$
$$+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cdef+15c^2}{}$$
$$+\frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2f+3c^2de}{}$$

```
[Out] 1/64*(16*a^2*d^2*f^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c
*f+d*e)))-16*a*b*d*f*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2
*d*f*(4*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)))+b^2*(C*(35*c^4*
f^4+20*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+20*c*d^3*e^3*f+35*d^4*e^4)+8*d*f*(2*A
*d*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)-B*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2
*f+5*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(9/
2)/f^(9/2)-1/24*(2*a*C*d*f-b*(8*B*d*f-7*C*(c*f+d*e)))*(b*x+a)^2*(d*x+c)^(1/
2)*(f*x+e)^(1/2)/b/d^2/f^2+1/4*C*(b*x+a)^3*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/
f-1/192*(32*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2*(16*B*d*f-11*C*(c*f+d*e))-16*a*b^
2*d*f*(C*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c*f+d*e)))+
b^3*(5*C*(21*c^3*f^3+19*c^2*d*e*f^2+19*c*d^2*e^2*f+21*d^3*e^3)+8*d*f*(18*A*
d*f*(c*f+d*e)-B*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)))+2*b*d*f*(6*b*d*f*(-8*A
*b*d*f+C*a*c*f+C*a*d*e+6*C*b*c*e)+(4*a*d*f-5*b*(c*f+d*e))*(2*a*C*d*f-b*(8*B
*d*f-7*C*(c*f+d*e))))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^4/f^4
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1629, 158, 152, 65, 223, 212}

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(16a^2d^2f^2(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))-16abdf(2df(4Adj}{}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df(6df(4Adf-3B(cf+de)}{}$$

$$+\frac{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(-2aCdf+8bBdf-7bC(cf+de))}{24bd^2f^2}$$

$$+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$

```
[In] Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((8*b*B*d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt
[e + f*x])/(24*b*d^2*f^2) + (C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*
b*d*f) - (Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(
16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f + 1
5*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 + 19
*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*f) -
 B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*C*e +
 a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*d*f -
2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(192*b*d^4*f^4) + ((16*a^2*d^2*f^2*(C*(
3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) - 16*
a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*
(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) + b^2*(C*(35
*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^
4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*
c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(9/2)*f^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)^2\left(-\frac{1}{2}b(6bcCe+aCde+acCf-8Abdf)+\frac{1}{2}b(8bBdf-2aCdf-7bC(de+cf))x\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{4b^2df}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)\left(-\frac{1}{4}b\left(4a^2Cdf(de+cf)+4b^2ce(8Bdf-7C(de+cf))-ab\left(7C(de-cf)^2+8df(6Adf-B(de+cf))\right)\right)-\frac{1}{4}b(6bdf(6bcCe+aCde+\cdots}{\sqrt{c+dx}\sqrt{e+fx}}}{12b^2d^2f^2}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cd\cdots}{} + \frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2\cdots}{}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cd\cdots}{} + \frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2\cdots}{}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cd\cdots}{} + \frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2\cdots}{}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{24bd^2f^2}$$
$$+ \frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$
$$- \frac{\sqrt{c+dx}\sqrt{e+fx}(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(de+cf))-16ab^2df(C(15d^2e^2+14cde}{}$$
$$+ \frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2f}{}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 632, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(48a^2d^2f^2(4Bdf+C(-3de-3cf+2dfx))+16abdf(6df(4Adf+B(-3de-3cf}{}$$

```
[In] Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(48*a^2*d^2*f^2*(4*B*d*f + C*(
-3*d*e - 3*c*f + 2*d*f*x)) + 16*a*b*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f
 + 2*d*f*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f
*x + 8*f^2*x^2))) + b^2*(-(C*(105*c^3*f^3 + 5*c^2*d*f^2*(19*e - 14*f*x) + c
*d^2*f*(95*e^2 - 68*e*f*x + 56*f^2*x^2) + d^3*(105*e^3 - 70*e^2*f*x + 56*e*
f^2*x^2 - 48*f^3*x^3))) + 8*d*f*(6*A*d*f*(-3*d*e - 3*c*f + 2*d*f*x) + B*(15
*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))
+ 3*(16*a^2*d^2*f^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
 - B*(d*e + c*f))) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f
^2 + 5*c^3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3
*c^2*f^2))) + b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20
*c^3*d*e*f^3 + 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*
f^2) - B*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh
[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(192*d^(9/2)*f^(9/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. 2(686) = 1372.

Time = 1.68 (sec) , antiderivative size = 2528, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	2528

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/384*(-112*C*b^2*d^3*e*f^2*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-72*B*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b
^2*c^2*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+60*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^3*f+54*C*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^
2*d^2*e^2*f^2-320*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*c*d^2*f^3*x-320
*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*b*d^3*e*f^2*x+136*C*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)*b^2*c*d^2*e*f^2*x+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^2*f^2-192*B*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c
*d^3*f^4+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*b^2*c^2*d^2*f^4-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
^2*c*d^2*f^3+448*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^2*e*f^2+192*
C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a^2*d^3*f^3*x+96*C*ln(1/2*(2*d*f*x+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^3*e*f^3+1
05*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*b^2*c^4*f^4+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^4-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*b^2*d^3*e*f^2+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^3*f^3-210
*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^3*f^3-210*C*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*b^2*d^3*e^3+96*C*b^2*d^3*f^3*x^3*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)+128*B*b^2*d^3*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-288*C
*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c*d^2*f^3-288*C*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*a^2*d^3*e*f^2+480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a
*b*c^2*d*f^3+480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*e^2*f-190*C*
(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d*e*f^2-190*C*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b^2*c*d^2*e^2*f+384*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)*a*b*d^3*f^3*x-192*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a^2*d^4*e*f^3-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d*f^4-120*B*ln(1/2*(2*d
*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d^4*e^
3*f+144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a^2*c^2*d^2*f^4+144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
```

```
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^4*e^2*f^2-384*A*ln(1/2*(2*d*f*x+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*f^4-384*
A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a*b*d^4*e*f^3+288*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^2*f^4+288*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e^2*f^2+240*B*(d*f)^
(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d*f^3+240*B*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b^2*d^3*e^2*f+140*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^3*
e^2*f*x-144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*a*b*c^2*d^2*e*f^3-144*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e^2*f^2+60*C*ln(1/2*(2*d*
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d*e
*f^3+192*A*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*d^3*f^3*x+96*A*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^
3*e*f^3+768*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*f^3+384*A*ln(1/2*
(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^
4*f^4-240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*a*b*c^3*d*f^4-240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*d^4*e^3*f-576*B*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*a*b*c*d^2*f^3-576*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*
d^3*e*f^2+224*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^2*e*f^2-160*B*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b^2*c*d^2*f^3*x-160*B*((d*x+c)*(f*x+e))^
(1/2)*(d*f)^(1/2)*b^2*d^3*e*f^2*x+192*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e*f^3+140*C*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)*b^2*c^2*d*f^3*x+256*C*a*b*d^3*f^3*x^2*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)-112*C*b^2*c*d^2*f^3*x^2*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^4/d^4/(d*f)^(1/2)/((d*x+c)*(f*x+e))^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 1436, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/768*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^
3*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b +
 A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(
C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^
```

```
2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b +
A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x
^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt
(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*b^2*d^4*f^4*x^
3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2
*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a
*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 +
 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*
C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(3
5*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 +
 (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b
^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1/384*(3*(35*C*b^
2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^3*f + 6*(3*C*b^2*c
^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e^2*f
^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(C*a^2 + 2*B*a*b +
A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^2*c^4 + 128*A*a^2*
d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - 6
4*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)
*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f +
 c*d*f^2)*x)) - 2*(48*C*b^2*d^4*f^4*x^3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2
*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a
*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*
c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 -
 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*
(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(35*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d
^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 + (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*
b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f
*x + e))/(d^5*f^5)]
```

Sympy [F]

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)**2*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\left(2(dx+c)\left(4(dx+c)\left(\frac{6(dx+c)Cb^2}{d^5f}-\frac{7Cb^2d^{20}ef^5+25Cb^2cd^{19}f^6-16Cabd^{20}f^6-8Bb^2d^{20}f^6}{d^{24}f^7}\right)\right.\right.\right.}{}$$

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b^2/(d^5*f) - (7*C*b^2*d^20*e*f^5 + 25*C*b^2*c*d^19*f^6 - 16*C*a*b
*d^20*f^6 - 8*B*b^2*d^20*f^6)/(d^24*f^7)) + (35*C*b^2*d^21*e^2*f^4 + 90*C*b
^2*c*d^20*e*f^5 - 80*C*a*b*d^21*e*f^5 - 40*B*b^2*d^21*e*f^5 + 163*C*b^2*c^2
*d^19*f^6 - 208*C*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^21*f^6
 + 96*B*a*b*d^21*f^6 + 48*A*b^2*d^21*f^6)/(d^24*f^7)) - 3*(35*C*b^2*d^22*e^
3*f^3 + 55*C*b^2*c*d^21*e^2*f^4 - 80*C*a*b*d^22*e^2*f^4 - 40*B*b^2*d^22*e^2
*f^4 + 73*C*b^2*c^2*d^20*e*f^5 - 128*C*a*b*c*d^21*e*f^5 - 64*B*b^2*c*d^21*e
*f^5 + 48*C*a^2*d^22*e*f^5 + 96*B*a*b*d^22*e*f^5 + 48*A*b^2*d^22*e*f^5 + 93
*C*b^2*c^3*d^19*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*f^6 + 80*C
*a^2*c*d^21*f^6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 64*B*a^2*d^2
2*f^6 - 128*A*a*b*d^22*f^6)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C*b^2*d^4*e^4
 + 20*C*b^2*c*d^3*e^3*f - 80*C*a*b*d^4*e^3*f - 40*B*b^2*d^4*e^3*f + 18*C*b^
2*c^2*d^2*e^2*f^2 - 48*C*a*b*c*d^3*e^2*f^2 - 24*B*b^2*c*d^3*e^2*f^2 + 48*C*
a^2*d^4*e^2*f^2 + 96*B*a*b*d^4*e^2*f^2 + 48*A*b^2*d^4*e^2*f^2 + 20*C*b^2*c^
```

```
3*d*e*f^3 - 48*C*a*b*c^2*d^2*e*f^3 - 24*B*b^2*c^2*d^2*e*f^3 + 32*C*a^2*c*d^
3*e*f^3 + 64*B*a*b*c*d^3*e*f^3 + 32*A*b^2*c*d^3*e*f^3 - 64*B*a^2*d^4*e*f^3
- 128*A*a*b*d^4*e*f^3 + 35*C*b^2*c^4*f^4 - 80*C*a*b*c^3*d*f^4 - 40*B*b^2*c^
3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*b*c^2*d^2*f^4 + 48*A*b^2*c^2*d^2*f^
4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3*f^4 + 128*A*a^2*d^4*f^4)*log(abs(-
sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*
d^4*f^4))*d/abs(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int(((a + b*x)^2*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.55 $\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 371

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$
$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-6abdf(4Bdf-3C(de+cf))-b^2(C(15d^2e^2+14cdef+15c^2f^2)+6df(4A\cdots}{24bd^3f^3}$$
$$+\frac{(2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3\cdots}{8d^{7/2}f^{7/2}}$$

```
[Out] 1/8*(2*a*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e))
)-b*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2*d*f*(4*A*d*f*(c*
f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)+1/3*C*(b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)
^(1/2)/b/d/f-1/24*(8*a^2*C*d^2*f^2-6*a*b*d*f*(4*B*d*f-3*C*(c*f+d*e))-b^2*(C
*(15*c^2*f^2+14*c*d*e*f+15*d^2*e^2)+6*d*f*(4*A*d*f-3*B*(c*f+d*e)))+2*b*d*f*
(2*a*C*d*f-b*(6*B*d*f-5*C*(c*f+d*e)))*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^3/
f^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used

$= \{1629, 152, 65, 223, 212\}$

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de)}{24bd^3f^3}$$

$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2adf(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))-b(2df(4Adf(cf+d\right.}{8d^{7/2}f^{7/2}}$$

$$+\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

```
[In] Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (Sqrt[c + d*x]*Sqrt
[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2*(C
*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))
) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(24*b*d^3*f^3)
+ ((2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*
e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) +
 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcT
anh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*d^(7/2)*f^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{\int \frac{(a+bx)\left(-\frac{1}{2}b(4bcCe+aCde+acCf-6Abdf)+\frac{1}{2}b(6bBdf-2aCdf-5bC(de+cf))x\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3b^2df}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2 - 6abdf(4Bdf-3C(de+cf)) - b^2(C(15d^2e^2+14cdef+15c^2f^2) -}{24bd^3f^3} + \frac{(2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf))) - b(C(5d^3e^3+3cd^2e^2f+3c^2def^2}{16d^3f^3}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2 - 6abdf(4Bdf-3C(de+cf)) - b^2(C(15d^2e^2+14cdef+15c^2f^2) -}{24bd^3f^3} + \frac{(2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf))) - b(C(5d^3e^3+3cd^2e^2f+3c^2def^2}{8}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2 - 6abdf(4Bdf - 3C(de+cf)) - b^2(C(15d^2e^2 + 14cdef + 15c^2f^2)}{24bd^3f^3} + \frac{(2adf(C(3d^2e^2 + 2cdef + 3c^2f^2) + 4df(2Adf - B(de+cf))) - b(C(5d^3e^3 + 3cd^2e^2f + 3c^2def}{8d^4}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2 - 6abdf(4Bdf - 3C(de+cf)) - b^2(C(15d^2e^2 + 14cdef + 15c^2f^2)}{24bd^3f^3} + \frac{(2adf(C(3d^2e^2 + 2cdef + 3c^2f^2) + 4df(2Adf - B(de+cf))) - b(C(5d^3e^3 + 3cd^2e^2f + 3c^2def}{8d^{7/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(6adf(4Bdf + C(-3de - 3cf + 2dfx)) + b(6df(4Adf + B(-3de - 3cf + 2dfx)) + C(15}{24d^3f^3} - \frac{(-2adf(C(3d^2e^2 + 2cdef + 3c^2f^2) + 4df(2Adf - B(de+cf))) + b(C(5d^3e^3 + 3cd^2e^2f + 3c^2def^2 + 5c}{8d^{7/2}f^{7/2}}$$

```
[In] Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(6*a*d*f*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*
x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f*x)) + C*(15*c^2*f^2 + 2
*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))/(24*d^3*f^3)
 - ((-2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(
d*e + c*f))) + b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3)
 + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*Ar
cTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(8*d^(7/2)*f^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. 2(345) = 690.

Time = 1.68 (sec) , antiderivative size = 1199, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1199

```
[In] int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/48*(-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*b*c*d^2*e^2*f+12*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^2*e*f^2-24*A*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*f^3-24*A*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3
*e*f^2+24*B*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*b*d^2*f^2*x-20*C*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)*b*d^2*e*f*x-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*a*d^2*e*f-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b*c^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^3+12*B*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e*f^2+24*C*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*a*d^2*f^2*x+18*C*ln(1/2*(2*d*f*x+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*e^2*f+48*A*(d*f)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*f^2+48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1
/2)*a*d^2*f^2+48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*a*d^3*f^3+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^
2*f^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e^2-24*B*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^2*f^3-
24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*a*d^3*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*f^3+18*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^2*f+18*C*ln(1/2*(2*d*f*x+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d*f^3+28*
C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d*e*f+16*C*b*d^2*f^2*x^2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2)-9*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*e*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e*f-3
6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d*f^2-20*C*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)*b*c*d*f^2*x)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^3/d^3/(d*f)^(1/2
)/((d*x+c)*(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \left[-\frac{3\,(5\,Cbd^3e^3+3\,(Cbcd^2-2\,(Ca+Bb)d^3)e^2f+(3\,Cbc^2d-4\,(Ca+Bb)cd^2+8\,(Ba+Ab)d^3)ef^2+(5\right.$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/96*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2*f + (3*C*b
*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b*c^3 - 16*A
*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*b*d^3*f^
3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5
*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*
f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e)
)/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2
*f + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b
*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(-d
*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)
/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8*C*b*d^3*f^3*x^2 +
15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5*C*b*c^2
*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*f^2 + (5
*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f
^4)]
```

Sympy [F]

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)Cb}{d^4f}-\frac{5\,Cbd^{12}ef^3+13\,Cbcd^{11}f^4-6\,Cad^{12}f^4-6\,Bbd^{12}f^4}{d^{15}f^5}\right)+\frac{3\,(5\,C\cdots}{}\right.\right.}{}$$

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C*b/(d^4*f) - (5*C*b*d^12*e*f^3 + 13*C*b*c*d^11*f^4 - 6*C*a*d^12*f^4
 - 6*B*b*d^12*f^4)/(d^15*f^5)) + 3*(5*C*b*d^13*e^2*f^2 + 8*C*b*c*d^12*e*f^3
 - 6*C*a*d^13*e*f^3 - 6*B*b*d^13*e*f^3 + 11*C*b*c^2*d^11*f^4 - 10*C*a*c*d^1
2*f^4 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4)/(d^15*f^5)) +
3*(5*C*b*d^3*e^3 + 3*C*b*c*d^2*e^2*f - 6*C*a*d^3*e^2*f - 6*B*b*d^3*e^2*f +
3*C*b*c^2*d*e*f^2 - 4*C*a*c*d^2*e*f^2 - 4*B*b*c*d^2*e*f^2 + 8*B*a*d^3*e*f^2
 + 8*A*b*d^3*e*f^2 + 5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*f^3 + 8*
B*a*c*d^2*f^3 + 8*A*b*c*d^2*f^3 - 16*A*a*d^3*f^3)*log(abs(-sqrt(d*f)*sqrt(d
*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^3))*d/abs(
d)
```

Mupad [B] (verification not implemented)

Time = 140.16 (sec) , antiderivative size = 2621, normalized size of antiderivative = 7.06

$$\int \frac{(a+bx)\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

[In] int(((a + b*x)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*b*c*f + 2*A*b*d*e))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*A*b*c*f + 2*A*b*d*e))/(d*
f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*b*c^(1/2)*e^(1/2)*((c + d*x)^(1/2
) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^
(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) -
c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/
2))*((3*C*a*d^3*e^2)/2 + (3*C*a*c^2*d*f^2)/2 + C*a*c*d^2*e*f))/(f^6*((e + f
*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*a*c^2*f^2)/2
+ (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3)
 + (((c + d*x)^(1/2) - c^(1/2))^7*((3*C*a*c^2*f^2)/2 + (3*C*a*d^2*e^2)/2 +
C*a*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) -
 c^(1/2))^5*((11*C*a*c^2*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(d*
f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^4*(32*C*a*c*f + 32*C*a*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d
*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*
((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2
*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) - ((((
c + d*x)^(1/2) - c^(1/2))^3*((85*C*b*d^4*e^3)/12 + (85*C*b*c^3*d*f^3)/12 +
(17*C*b*c*d^3*e^2*f)/4 + (17*C*b*c^2*d^2*e*f^2)/4))/(f^8*((e + f*x)^(1/2) -
 e^(1/2))^3) - (((c + d*x)^(1/2) - c^(1/2))*((5*C*b*d^5*e^3)/4 + (5*C*b*c^3
*d^2*f^3)/4 + (3*C*b*c*d^4*e^2*f)/4 + (3*C*b*c^2*d^3*e*f^2)/4))/(f^9*((e +
f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((33*C*b*c^3*f^3)/2
 + (33*C*b*d^3*e^3)/2 + (327*C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))
/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(1/2))^11*((5*
C*b*c^3*f^3)/4 + (5*C*b*d^3*e^3)/4 + (3*C*b*c*d^2*e^2*f)/4 + (3*C*b*c^2*d*e
*f^2)/4))/(d^3*f^4*((e + f*x)^(1/2) - e^(1/2))^11) + (((c + d*x)^(1/2) - c^
(1/2))^9*((85*C*b*c^3*f^3)/12 + (85*C*b*d^3*e^3)/12 + (17*C*b*c*d^2*e^2*f)/
4 + (17*C*b*c^2*d*e*f^2)/4))/(d^2*f^5*((e + f*x)^(1/2) - e^(1/2))^9) - (((c
 + d*x)^(1/2) - c^(1/2))^7*((33*C*b*c^3*f^3)/2 + (33*C*b*d^3*e^3)/2 + (327*
C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))/(d*f^6*((e + f*x)^(1/2) - e^
(1/2))^7) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*b*c^2*f^2
 + 128*C*b*d^2*e^2 + (896*C*b*c*d*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))
^6) + (64*C*b*c^(3/2)*e^(3/2)*((c + d*x)^(1/2) - c^(1/2))^8)/(f^4*((e + f*x
)^(1/2) - e^(1/2))^8) + (64*C*b*c^(3/2)*d^2*e^(3/2)*((c + d*x)^(1/2) - c^(1
/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^1
```

2/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 - (6*d*((c + d*x)^(1/2) - c^(1/2))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (6*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) + (15*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4) - (20*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6) + (15*d^2*((c + d*x)^(1/2) - c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))^8)) - ((((c + d*x)^(1/2) - c^(1/2))*((3*B*b*d^3*e^2)/2 + (3*B*b*c^2*d*f^2)/2 + B*b*c*d^2*e*f))/(f^6*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*B*b*c^2*f^2)/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^7*((3*B*b*c^2*f^2)/2 + (3*B*b*d^2*e^2)/2 + B*b*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*B*b*c^2*f^2)/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^4*(32*B*b*c*f + 32*B*b*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) + ((((c + d*x)^(1/2) - c^(1/2))*(2*B*a*c*f + 2*B*a*d*e))/(f^3*((e + f*x)^(1/2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*B*a*c*f + 2*B*a*d*e))/(d*f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*B*a*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - (4*A*a*atan((d*((e + f*x)^(1/2) - e^(1/2)))/((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2) + (B*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) + (C*a*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) - (2*A*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f + d*e))/(d^(3/2)*f^(3/2)) - (2*B*a*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f + d*e))/(d^(3/2)*f^(3/2)) - (C*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f + d*e)*(5*c^2*f^2 + 5*d^2*e^2 - 2*c*d*e*f))/(4*d^(7/2)*f^(7/2))

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}+\frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4d^{5/2}f^{5/2}}$$

```
[Out] 1/4*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh
(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(5/2)+1/2*C*(d*x+c)
^(3/2)*(f*x+e)^(1/2)/d^2/f-1/4*(-4*B*d*f+5*C*c*f+3*C*d*e)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/d^2/f^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {965, 81, 65, 223, 212}

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((3*C*d*e + 5*c*C*f - 4*B*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(d^2*f^2)
+ (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) + ((C*(3*d^2*e^2 + 2*c*d*e*f
+ 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 965

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\text{integral} = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\int \frac{\frac{1}{2}(-3cCde-c^2Cf+4Ad^2f)-\frac{1}{2}d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2d^2f}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{8d^2f^2}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{4d^3f^2}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{4d^3f^2}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4d^{5/2}f^{5/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(4Bdf+C(-3de-3cf+2dfx))}{4d^2f^2} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{4d^{5/2}f^{5/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
[Out] (Sqrt[c + d*x]*Sqrt[e + f*x]*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x)))/(4*d
^2*f^2) + ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e
 + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*d^(5
/2)*f^(5/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(138) = 276.

Time = 5.70 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\frac{\left(8A\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)d^2f^2-4B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)cd\,f^2-4B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}+}{2\sqrt{df}}\right.\right.}{}$

```
[In] int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(8*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e*f+4*C*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)*d*f*x+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d*e*f+3*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e^2+8*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*d*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*f-6*C*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2
)/f^2/d^2/((d*x+c)*(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \left[\frac{(3\,Cd^2e^2+2\,(Ccd-2\,Bd^2)ef+(3\,Cc^2-4\,Bcd+8\,Ad^2)f^2)\sqrt{df}\log\left(8\,d^2f^2x^2+d^2e^2+6\,cdef+c^2f^2+\right.}{}\right.$$
$$-\frac{(3\,Cd^2e^2+2\,(Ccd-2\,Bd^2)ef+(3\,Cc^2-4\,Bcd+8\,Ad^2)f^2)\sqrt{-df}\arctan\left(\frac{(2\,dfx+de+cf)\sqrt{-df}\sqrt{dx+c}\sqrt{fx+e}}{2\,(d^2f^2x^2+cdef+(d^2ef+cdf^2)x)}\right)}{8\,d^3f^3}$$

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^
2)*f^2)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*
d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d
*f^2)*x) + 4*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^2)*f^2)*sqrt(d
*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*
```

```
e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d
*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (
d^2*e*f + c*d*f^2)*x)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^
2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(\frac{2\,(dx+c)C}{d^3f}-\frac{3\,Cd^6ef+5\,Ccd^5f^2-4\,Bd^6f^2}{d^8f^3}\right)-\frac{(3\,Cd^2e^2+2\,Ccdef-4\,Bd^2ef+3\,Cc^2f^2-4}{}\right.}{4\,|d|}$$

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
[Out] 1/4*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3*
f) - (3*C*d^6*e*f + 5*C*c*d^5*f^2 - 4*B*d^6*f^2)/(d^8*f^3)) - (3*C*d^2*e^2
+ 2*C*c*d*e*f - 4*B*d^2*e*f + 3*C*c^2*f^2 - 4*B*c*d*f^2 + 8*A*d^2*f^2)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(
d*f)*d^2*f^2))*d/abs(d)
```

Mupad [B] (verification not implemented)

Time = 50.86 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$=\frac{\frac{(2\,B\,c\,f+2\,B\,d\,e)\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{f^3\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}+\frac{(2\,B\,c\,f+2\,B\,d\,e)\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{d\,f^2\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^3}-\frac{8\,B\,\sqrt{c}\,\sqrt{e}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{f^2\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^2}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{e+f\,x}-\sqrt{e}\right)^4}+\frac{d^2}{f^2}-\frac{2\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{f\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^2}}$$
$$-\frac{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(\frac{3\,C\,c^2\,d\,f^2}{2}+C\,c\,d^2\,e\,f+\frac{3\,C\,d^3\,e^2}{2}\right)}{f^6\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}-\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3\,\left(\frac{11\,C\,c^2\,f^2}{2}+25\,C\,c\,d\,e\,f+\frac{11\,C\,d^2\,e^2}{2}\right)}{f^5\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^3}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7\,\left(\frac{3\,C\,c^2\,f^2}{2}+C\,c\,d\,e\,f+\frac{3\,C\,d^2\,e^2}{2}\right)}{d^2\,f^3\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^7}-\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5\,\left(\frac{11\,C\,c^2\,f^2}{2}+25\,C\,c\,d\,e\,f+\frac{11\,C\,d^2\,e^2}{2}\right)}{d\,f^4\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^5}+\frac{\sqrt{c}\,\sqrt{e}\,(32\,C\,c\,f+32\,C\,d\,e)\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{f^4\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{e+f\,x}-\sqrt{e}\right)^8}+\frac{d^4}{f^4}-\frac{4\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{f\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^6}-\frac{4\,d^3\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{f^3\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^2}+\frac{6\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{f^2\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)^4}}$$
$$-\frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}{\sqrt{-d\,f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d\,f}}-\frac{2\,B\,\mathrm{atanh}\left(\frac{\sqrt{f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{d}\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}\right)\,(c\,f+d\,e)}{d^{3/2}\,f^{3/2}}$$
$$+\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{d}\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}\right)\,(3\,c^2\,f^2+2\,c\,d\,e\,f+3\,d^2\,e^2)}{2\,d^{5/2}\,f^{5/2}}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(f^3*((e + f*x)^(1/2) -
e^(1/2))) + ((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(d*f^2*((e
+ f*x)^(1/2) - e^(1/2))^3) - (8*B*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2
))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/((((c + d*x)^(1/2) - c^(1/2))^4/(
(e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2))^2
)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((3*C*
d^3*e^2)/2 + (3*C*c^2*d*f^2)/2 + C*c*d^2*e*f))/(f^6*((e + f*x)^(1/2) - e^(1
/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*c^2*f^2)/2 + (11*C*d^2*e^2)/2
 + 25*C*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) -
 c^(1/2))^7*((3*C*c^2*f^2)/2 + (3*C*d^2*e^2)/2 + C*c*d*e*f))/(d^2*f^3*((e +
 f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*C*c^2*f^2)/
2 + (11*C*d^2*e^2)/2 + 25*C*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5)
 + (c^(1/2)*e^(1/2)*(32*C*c*f + 32*C*d*e)*((c + d*x)^(1/2) - c^(1/2))^4)/(f
^4*((e + f*x)^(1/2) - e^(1/2))^4))/((((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x
)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e
 + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((
e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*(
(e + f*x)^(1/2) - e^(1/2))^4)) - (4*A*atan((d*((e + f*x)^(1/2) - e^(1/2)))/
((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2) - (2*B*atanh((f^(
1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c
*f + d*e))/(d^(3/2)*f^(3/2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)
))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*
f))/(2*d^(5/2)*f^(5/2))
```

3.57 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}} - \frac{2(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}$$

```
[Out] -(2*a*C*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/(f*x+e)^(1/2))/b^2/d^(3/2)/f^(3/2)-2*(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e
)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/
(-a*f+b*e)^(1/2)+C*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1629, 163, 65, 223, 212, 95, 214}

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}} - \frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}} + \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f -
2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^2*d^
(3/2)*f^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[b*c - a*d]*Sqrt[b*e -
a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \frac{\int \frac{\frac{1}{2}b(2Abdf-aC(de+cf))-\frac{1}{2}b(2aCdf+b(Cde+cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^2df} \\
&= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \left(A-\frac{a(bB-aC)}{b^2}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx \\
&\quad + \frac{(-2aCdf-b(Cde+cCf-2Bdf))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2b^2df} \\
&= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} \\
&\quad + \left(2\left(A-\frac{a(bB-aC)}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{-bc+ad-(-be+af)x^2}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right) \\
&\quad + \frac{(-2aCdf-b(Cde+cCf-2Bdf))\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2d^2f} \\
&= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{\sqrt{bc-ad}\sqrt{be-af}} \\
&\quad + \frac{(-2aCdf-b(Cde+cCf-2Bdf))\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^2d^2f} \\
&= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}} \\
&\quad - \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{\sqrt{bc-ad}\sqrt{be-af}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\frac{bC\sqrt{c+dx}\sqrt{e+fx}}{df}+\frac{2(Ab^2+a(-bB+aC))\arctan\left(\frac{\sqrt{bc-ad}\sqrt{e+fx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{bc-ad}\sqrt{-be+af}}-\frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{d^{3/2}f^{3/2}}}{b^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((b*C*Sqrt[c + d*x]*Sqrt[e + f*x])/(d*f) + (2*(A*b^2 + a*(-(b*B) + a*C))*Ar
cTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(
Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*
d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2)*f^
(3/2)))/b^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(160) = 320.

Time = 1.69 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(2A\sqrt{df}\,\ln\left(\frac{-2adfx+bcfx+bdex+2\sqrt{\frac{a^2df-acfb-abde+b^2ce}{b^2}}\,\sqrt{(dx+c)(fx+e)}\,b-acf-ade+2bce}{bx+a}\right)b^2df-2B\ln\left(\frac{2dfx+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right.}{}$

```
[In] int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(2*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*b^2*d*f-2*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2*d*f-2
*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*
a*b*d*f+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b*d*f+C*ln(1/2
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2*c*f+C*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*b^2*d*e+2*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e
*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*
```

```
b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*d*f-2*C*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(
1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^2)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/d/(d*f)^(1/2)/b^3/((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)/f
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

3.58 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 254

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)}+\frac{2C\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}} + \frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade-Acf)+ab^2(4cCe+Bde+Bcf-2Adf))\operatorname{arctanh}\left(\frac{\sqrt{be}\cdots}{\sqrt{bc}\cdots}\right)}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}$$

```
[Out] (2*a^3*C*d*f-3*a^2*b*C*(c*f+d*e)-b^3*(-A*c*f-A*d*e+2*B*c*e)+a*b^2*(-2*A*d*f
+B*c*f+B*d*e+4*C*c*e))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1
/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)+2*C*arctanh(f^(1/2
)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(1/2)/f^(1/2)-(A*b^2-a*(B*b-C*
a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used

= {1627, 163, 65, 223, 212, 95, 214}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{b(a+bx)(bc-ad)(be-af)}+\frac{2C\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) + (2*C*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(b^2*Sqrt[d]*Sqrt[f]) + ((2*a^3*C*d*f - 3*a^2*b*C*(d*e + c*f) - b
^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e + B*d*e + B*c*f - 2*A*d*f))*A
rcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b
^2*(b*c - a*d)^(3/2)*(b*e - a*f)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{-\frac{a^2C(de+cf)+b^2(2Bce-Ade-Acf)-ab(2cCe+Bde+Bcf-2Adf)}{2b}-\frac{C(bc-ad)(be-af)x}{b}}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{(bc-ad)(be-af)}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{C\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^2} - \frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade-Acf)+ab^2(4cCe+Bde+Bcf-2Adf))\int \frac{}{(a+}}{2b^2(bc-ad)(be-af)}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(2C)\text{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2d} - \frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade-Acf)+ab^2(4cCe+Bde+Bcf-2Adf))\text{Subs}}{b^2(bc-ad)(be-af)}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} + \frac{(2a^3Cdf - 3a^2bC(de + cf) - b^3(2Bce - Ade - Acf) + ab^2(4cCe + Bde + Bcf - 2Adf))\tanh^-}{b^2(bc - ad)^{3/2}(be - af)^{3/2}} + \frac{(2C)\text{Subst}\left(\int \frac{1}{1-\frac{fx^2}{d}}\,dx, x, \frac{\sqrt{c+dx}}{\sqrt{e+fx}}\right)}{b^2d}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{b(bc - ad)(be - af)(a + bx)} + \frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}} + \frac{(2a^3Cdf - 3a^2bC(de + cf) - b^3(2Bce - Ade - Acf) + ab^2(4cCe + Bde + Bcf - 2Adf))\tanh^-}{b^2(bc - ad)^{3/2}(be - af)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

$$\int \frac{A + Bx + Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}}\,dx = -\frac{\frac{b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)} + \frac{(-2a^3Cdf+3a^2bC(de+cf)-ab^2(4cCe+Bde+Bcf-2Adf)+b^3(2Bce-A(de+cf)))\arctan\left(\frac{\sqrt{bc-ad}\sqrt{e}}{\sqrt{-be+af}\sqrt{}}\right.}{(bc-ad)^{3/2}(-be+af)^{3/2}}}{b^2}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*
(b*e - a*f)*(a + b*x)) + ((-2*a^3*C*d*f + 3*a^2*b*C*(d*e + c*f) - a*b^2*(4*
c*C*e + B*d*e + B*c*f - 2*A*d*f) + b^3*(2*B*c*e - A*(d*e + c*f)))*ArcTan[(S
qrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])]])/((b*c -
a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) - (2*C*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sq
rt[f]*Sqrt[c + d*x])])/(Sqrt[d]*Sqrt[f]))/b^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. 2(226) = 452.

Time = 1.69 (sec) , antiderivative size = 2973, normalized size of antiderivative = 11.70

method	result	size
default	Expression too large to display	2973

```
[In] int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*(f*x+e)^(1/2)*(d*x+c)^(1/2)*(-2*B*a*b^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*a^2*b^2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-A*ln
((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*f*x*(d*f)^(
1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*d*e*
x*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a
))*b^4*c*e*x*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*c*e*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*
a^2*b^2*d*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e
)/(b*x+a))*a^2*b^2*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a
^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^3*b*d*f*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d
*e+2*b*c*e)/(b*x+a))*a^4*d*f*(d*f)^(1/2)+2*A*b^4*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*A*ln((-2*a*d*f*x+
b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*f*x*(d*f)^(1/2)-B*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*x*(d*f)^(
1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*
e*x*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^
(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a*b^3*c*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln
(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a
*b^3*d*e*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*
x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*c*f*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*e*(d*f)^(1/2)-A*ln((-2*a*d*f*x+
b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*(d*f)^(1/2)-A*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d*e*(d*f)^(1/2)
-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*f*
```

```
(d*f)^(1/2)+2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*a^2*b^2*d*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-
2*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*a*b^3*c*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*C*ln((-2*a*d
*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*c*f*(d*f)^(1/2)+3*C
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*(d*f
)^(1/2)-2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3
*b*d*f*x*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*a^2*b^2*c*f*x*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f
-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*x*(d*f)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*x*(d*f)^(1/2))/((d*x+c)*(
f*x+e))^(1/2)/(a*d-b*c)/(a*f-b*e)/(b*x+a)/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)/b^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. 2(225) = 450.

Time = 1.10 (sec) , antiderivative size = 1319, normalized size of antiderivative = 5.19

$$\int \frac{A + Bx + Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 3*sqrt(d*f)*C*a^
2*b*d^3*e + sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*f)*C
*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f + 2*sq
rt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*A*a*b^2*d^3*f)*arctan(-1/2*(b*d^2*e + b*c
*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*
d))/((b^4*c*e*abs(d) - a*b^3*d*e*abs(d) - a*b^3*c*f*abs(d) + a^2*b^2*d*f*ab
s(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d) - 2*(
sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5
*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*e*f + 2*sqrt(d*f)*B*a*b^2*c*d^4*e*f - 2*sq
rt(d*f)*A*b^3*c*d^4*e*f + sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2
*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d^3*
e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^2*A*b^3*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*c*d^2*f + 2
```

```
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*C*a^3*d^3*f - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*B*a^2*b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^2*d^3*f)/((b*d^4*e^2 - 2*b*c*d^
3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^4*b)*(b^4*c*e*abs(d) - a*b^3*d*e*abs(d) - a*b^3*c*f*a
bs(d) + a^2*b^2*d*f*abs(d))) - sqrt(d*f)*C*log((sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2)/(b^2*f*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.59 $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 424

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2}$$
$$+\frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(de+cf))+a^2b(2Bdf-5C(de+cf)))\sqrt{}}{4b(bc-ad)^2(be-af)^2(a+bx)}$$
$$-\frac{(b^2(3Ad^2e^2-2cde(2Be-Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)-c^2f(8Ce-Bf)-}{4(bc-}$$

```
[Out] -1/4*(b^2*(3*A*d^2*e^2-2*c*d*e*(-A*f+2*B*e)+c^2*(3*A*f^2-4*B*e*f+8*C*e^2))+
a*b*(d^2*e*(-8*A*f+B*e)-c^2*f*(-B*f+8*C*e)-2*c*d*(4*A*f^2-7*B*e*f+4*C*e^2))
+a^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e))))*arcta
nh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c
)^(5/2)/(-a*f+b*e)^(5/2)-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2
)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+1/4*(2*a^3*C*d*f+a*b^2*(-6*A*d*f+B*c*f+
B*d*e+8*C*c*e)-b^3*(4*B*c*e-3*A*(c*f+d*e))+a^2*b*(2*B*d*f-5*C*(c*f+d*e)))*(
d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used

= {1627, 156, 12, 95, 214}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))+ab(-2cd(4Af^2-7\right.}{4(bc-}$$

$$+\frac{\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-}{4b(a+bx)(bc-ad)^2(be-af)^2}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^2) + ((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f -
6*A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e + c*
f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x
)) - ((b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f +
3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^2
 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f
*(2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
b*c - a*d]*Sqrt[e + f*x])])/(4*(b*c - a*d)^(5/2)*(b*e - a*f)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(4cCe+Bde+Bcf-4Adf)+b^2(4Bce-3A(de+cf))}{2b}+\left(-2bcCe+2aCde+2acCf+Abdf-aBdf-\frac{a^2Cdf}{b}\right)x}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2(bc-ad)(be-af)}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(de+cf))+a^2b(2Bdf-5C(de+ \cdots}{4b(bc-ad)^2(be-af)^2(a+bx)} + \frac{\int \frac{b^2(3Ad^2e^2-2cde(2Be-Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)-c^2f(8Ce-Bf)-2cd(4Ce^2-7Bef+4Af^2))+a^2(C(\cdots}{4(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2(bc-ad)^2(be-af)^2}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(de+cf))+a^2b(2Bdf-5C(de+ \cdots}{4b(bc-ad)^2(be-af)^2(a+bx)} + \frac{(b^2(3Ad^2e^2-2cde(2Be-Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)-c^2f(8Ce- \cdots}{\cdots}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{2b(bc - ad)(be - af)(a + bx)^2} + \frac{(2a^3Cdf + ab^2(8cCe + Bde + Bcf - 6Adf) - b^3(4Bce - 3A(de + cf)) + a^2b(2Bdf - 5C(de + c}{4b(bc - ad)^2(be - af)^2(a + bx)} + \frac{(b^2(3Ad^2e^2 - 2cde(2Be - Af) + c^2(8Ce^2 - 4Bef + 3Af^2)) + ab(d^2e(Be - 8Af) - c^2f(8Ce - }{}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{2b(bc - ad)(be - af)(a + bx)^2} + \frac{(2a^3Cdf + ab^2(8cCe + Bde + Bcf - 6Adf) - b^3(4Bce - 3A(de + cf)) + a^2b(2Bdf - 5C(de + c}{4b(bc - ad)^2(be - af)^2(a + bx)} - \frac{(b^2(3Ad^2e^2 - 2cde(2Be - Af) + c^2(8Ce^2 - 4Bef + 3Af^2)) + ab(d^2e(Be - 8Af) - c^2f(8Ce - }{4}$$

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.99

$$\int \frac{A + Bx + Cx^2}{(a + bx)^3\sqrt{c + dx}\sqrt{e + fx}}\,dx = \frac{1}{4}\left(-\frac{\sqrt{c + dx}\sqrt{e + fx}(4b^3Bcex - ab^2(8cCex + B(-2ce + dex + cfx)) + a^2b(5Cdex + Bd(e - 2fx) + c(}{(b} + \frac{(b^2(3Ad^2e^2 + 2cde(-2Be + Af) + c^2(8Ce^2 - 4Bef + 3Af^2)) + ab(d^2e(Be - 8Af) + c^2f(-8Ce + Bf)}{(bc - a}\right.$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-((Sqrt[c + d*x]*Sqrt[e + f*x]*(4*b^3*B*c*e*x - a*b^2*(8*c*C*e*x + B*(-2*c
*e + d*e*x + c*f*x)) + a^2*b*(5*C*d*e*x + B*d*(e - 2*f*x) + c*(-6*C*e + B*f
 + 5*C*f*x)) + a^3*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + A*b*(8*a^2*d*
f + b^2*(2*c*e - 3*d*e*x - 3*c*f*x) + a*b*(-5*d*e - 5*c*f + 6*d*f*x))))/((b
*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^2)) + ((b^2*(3*A*d^2*e^2 + 2*c*d*e*(-2*
B*e + A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f)
+ c^2*f*(-8*C*e + B*f) - 2*c*d*(4*C*e^2 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d
^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))))*ArcTan[
(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b*c
- a*d)^(5/2)*(-(b*e) + a*f)^(5/2)))/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7118 vs. $2(398) = 796$.

Time = 1.69 (sec) , antiderivative size = 7119, normalized size of antiderivative = 16.79

method	result	size
default	Expression too large to display	7119

```
[In] int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. $2(397) = 794$.

Time = 164.61 (sec) , antiderivative size = 4058, normalized size of antiderivative = 9.57

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/16*(((8*C*a^2*b^2*c^2 - 4*(2*C*a^3*b + B*a^2*b^2)*c*d + (3*C*a^4 + B*a^3
*b + 3*A*a^2*b^2)*d^2)*e^2 - 2*(2*(2*C*a^3*b + B*a^2*b^2)*c^2 - (C*a^4 + 7*
B*a^3*b + A*a^2*b^2)*c*d + 2*(B*a^4 + 2*A*a^3*b)*d^2)*e*f + (8*A*a^4*d^2 +
(3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*c^2 - 4*(B*a^4 + 2*A*a^3*b)*c*d)*f^2 + ((
8*C*b^4*c^2 - 4*(2*C*a*b^3 + B*b^4)*c*d + (3*C*a^2*b^2 + B*a*b^3 + 3*A*b^4)
*d^2)*e^2 - 2*(2*(2*C*a*b^3 + B*b^4)*c^2 - (C*a^2*b^2 + 7*B*a*b^3 + A*b^4)*
c*d + 2*(B*a^2*b^2 + 2*A*a*b^3)*d^2)*e*f + (8*A*a^2*b^2*d^2 + (3*C*a^2*b^2
 + B*a*b^3 + 3*A*b^4)*c^2 - 4*(B*a^2*b^2 + 2*A*a*b^3)*c*d)*f^2)*x^2 + 2*((8*
C*a*b^3*c^2 - 4*(2*C*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^3*b + B*a^2*b^2 + 3*A*
a*b^3)*d^2)*e^2 - 2*(2*(2*C*a^2*b^2 + B*a*b^3)*c^2 - (C*a^3*b + 7*B*a^2*b^2
 + A*a*b^3)*c*d + 2*(B*a^3*b + 2*A*a^2*b^2)*d^2)*e*f + (8*A*a^3*b*d^2 + (3*
C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*c^2 - 4*(B*a^3*b + 2*A*a^2*b^2)*c*d)*f^2)*
x)*sqrt((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f)*log((a^2*c^2*f^2 + (8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2)*e^2 - 2*(4*a*b*c^2 - 3*a^2*c*d)*e*f + (b^2*d^2*e^2
 + 2*(3*b^2*c*d - 4*a*b*d^2)*e*f + (b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*f^2)*x
^2 + 4*(a*c*f - (2*b*c - a*d)*e - (b*d*e + (b*c - 2*a*d)*f)*x)*sqrt((b^2*c
- a*b*d)*e - (a*b*c - a^2*d)*f)*sqrt(d*x + c)*sqrt(f*x + e) + 2*((4*b^2*c*d
 - 3*a*b*d^2)*e^2 + 2*(2*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*e*f - (3*a*b*c^2
- 4*a^2*c*d)*f^2)*x)/(b^2*x^2 + 2*a*b*x + a^2)) + 4*((2*(3*C*a^2*b^3 - B*a*
b^4 - A*b^5)*c^2 - (9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c*d + (3*C*a^4*b +
 B*a^3*b^2 - 5*A*a^2*b^3)*d^2)*e^2 - ((9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)
```

```
*c^2 - 4*(3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c*d + (3*C*a^5 + 5*B*a^4*b -
 13*A*a^3*b^2)*d^2)*e*f + ((3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c^2 - (3*C
*a^5 + 5*B*a^4*b - 13*A*a^3*b^2)*c*d + 4*(B*a^5 - 2*A*a^4*b)*d^2)*f^2 + ((4
*(2*C*a*b^4 - B*b^5)*c^2 - (13*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c*d + (5*C*
a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*d^2)*e^2 - ((13*C*a^2*b^3 - 5*B*a*b^4 - 3*
A*b^5)*c^2 - 4*(5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c*d + (7*C*a^4*b + B*a
^3*b^2 - 9*A*a^2*b^3)*d^2)*e*f + ((5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c^2
 - (7*C*a^4*b + B*a^3*b^2 - 9*A*a^2*b^3)*c*d + 2*(C*a^5 + B*a^4*b - 3*A*a^3
*b^2)*d^2)*f^2)*x)*sqrt(d*x + c)*sqrt(f*x + e))/((a^2*b^6*c^3 - 3*a^3*b^5*c
^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*e^3 - 3*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*
d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*e^2*f + 3*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d
 + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*e*f^2 - (a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*
a^7*b*c*d^2 - a^8*d^3)*f^3 + ((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
a^3*b^5*d^3)*e^3 - 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b
^4*d^3)*e^2*f + 3*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^
3*d^3)*e*f^2 - (a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*f^3)*x^2 + 2*((a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*
d^3)*e^3 - 3*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3
)*e^2*f + 3*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)
*e*f^2 - (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*f^3)
*x), -1/8*(((8*C*a^2*b^2*c^2 - 4*(2*C*a^3*b + B*a^2*b^2)*c*d + (3*C*a^4 + B
*a^3*b + 3*A*a^2*b^2)*d^2)*e^2 - 2*(2*(2*C*a^3*b + B*a^2*b^2)*c^2 - (C*a^4
+ 7*B*a^3*b + A*a^2*b^2)*c*d + 2*(B*a^4 + 2*A*a^3*b)*d^2)*e*f + (8*A*a^4*d^
2 + (3*C*a^4 + B*a^3*b + 3*A*a^2*b^2)*c^2 - 4*(B*a^4 + 2*A*a^3*b)*c*d)*f^2
+ ((8*C*b^4*c^2 - 4*(2*C*a*b^3 + B*b^4)*c*d + (3*C*a^2*b^2 + B*a*b^3 + 3*A*
b^4)*d^2)*e^2 - 2*(2*(2*C*a*b^3 + B*b^4)*c^2 - (C*a^2*b^2 + 7*B*a*b^3 + A*b
^4)*c*d + 2*(B*a^2*b^2 + 2*A*a*b^3)*d^2)*e*f + (8*A*a^2*b^2*d^2 + (3*C*a^2*
b^2 + B*a*b^3 + 3*A*b^4)*c^2 - 4*(B*a^2*b^2 + 2*A*a*b^3)*c*d)*f^2)*x^2 + 2*
((8*C*a*b^3*c^2 - 4*(2*C*a^2*b^2 + B*a*b^3)*c*d + (3*C*a^3*b + B*a^2*b^2 +
3*A*a*b^3)*d^2)*e^2 - 2*(2*(2*C*a^2*b^2 + B*a*b^3)*c^2 - (C*a^3*b + 7*B*a^2
*b^2 + A*a*b^3)*c*d + 2*(B*a^3*b + 2*A*a^2*b^2)*d^2)*e*f + (8*A*a^3*b*d^2 +
 (3*C*a^3*b + B*a^2*b^2 + 3*A*a*b^3)*c^2 - 4*(B*a^3*b + 2*A*a^2*b^2)*c*d)*f
^2)*x)*sqrt(-(b^2*c - a*b*d)*e + (a*b*c - a^2*d)*f)*arctan(1/2*(a*c*f - (2*
b*c - a*d)*e - (b*d*e + (b*c - 2*a*d)*f)*x)*sqrt(-(b^2*c - a*b*d)*e + (a*b*
c - a^2*d)*f)*sqrt(d*x + c)*sqrt(f*x + e)/((b^2*c^2 - a*b*c*d)*e^2 - (a*b*c
^2 - a^2*c*d)*e*f + ((b^2*c*d - a*b*d^2)*e*f - (a*b*c*d - a^2*d^2)*f^2)*x^2
 + ((b^2*c*d - a*b*d^2)*e^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*e*f - (a*b*c^
2 - a^2*c*d)*f^2)*x)) - 2*((2*(3*C*a^2*b^3 - B*a*b^4 - A*b^5)*c^2 - (9*C*a^
3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c*d + (3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*
d^2)*e^2 - ((9*C*a^3*b^2 - B*a^2*b^3 - 7*A*a*b^4)*c^2 - 4*(3*C*a^4*b + B*a^
3*b^2 - 5*A*a^2*b^3)*c*d + (3*C*a^5 + 5*B*a^4*b - 13*A*a^3*b^2)*d^2)*e*f +
((3*C*a^4*b + B*a^3*b^2 - 5*A*a^2*b^3)*c^2 - (3*C*a^5 + 5*B*a^4*b - 13*A*a^
3*b^2)*c*d + 4*(B*a^5 - 2*A*a^4*b)*d^2)*f^2 + ((4*(2*C*a*b^4 - B*b^5)*c^2 -
 (13*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c*d + (5*C*a^3*b^2 - B*a^2*b^3 - 3*A*
a*b^4)*d^2)*e^2 - ((13*C*a^2*b^3 - 5*B*a*b^4 - 3*A*b^5)*c^2 - 4*(5*C*a^3*b^
```

```
2 - B*a^2*b^3 - 3*A*a*b^4)*c*d + (7*C*a^4*b + B*a^3*b^2 - 9*A*a^2*b^3)*d^2)
*e*f + ((5*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*c^2 - (7*C*a^4*b + B*a^3*b^2
- 9*A*a^2*b^3)*c*d + 2*(C*a^5 + B*a^4*b - 3*A*a^3*b^2)*d^2)*f^2)*x)*sqrt(d*
x + c)*sqrt(f*x + e))/((a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a
^5*b^3*d^3)*e^3 - 3*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*
b^2*d^3)*e^2*f + 3*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b
*d^3)*e*f^2 - (a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3)*f^3
 + ((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*e^3 - 3*(a*b^
7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*e^2*f + 3*(a^2*b^6
*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*e*f^2 - (a^3*b^5*c^
3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*f^3)*x^2 + 2*((a*b^7*c
^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*e^3 - 3*(a^2*b^6*c^3
- 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*e^2*f + 3*(a^3*b^5*c^3 -
 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*e*f^2 - (a^4*b^4*c^3 - 3*
a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*f^3)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7939 vs. $2(397) = 794$.

Time = 38.67 (sec) , antiderivative size = 7939, normalized size of antiderivative = 18.72

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/4*(8*sqrt(d*f)*C*b^2*c^2*d^2*e^2 - 8*sqrt(d*f)*C*a*b*c*d^3*e^2 - 4*sqrt(
d*f)*B*b^2*c*d^3*e^2 + 3*sqrt(d*f)*C*a^2*d^4*e^2 + sqrt(d*f)*B*a*b*d^4*e^2
+ 3*sqrt(d*f)*A*b^2*d^4*e^2 - 8*sqrt(d*f)*C*a*b*c^2*d^2*e*f - 4*sqrt(d*f)*B
*b^2*c^2*d^2*e*f + 2*sqrt(d*f)*C*a^2*c*d^3*e*f + 14*sqrt(d*f)*B*a*b*c*d^3*e
*f + 2*sqrt(d*f)*A*b^2*c*d^3*e*f - 4*sqrt(d*f)*B*a^2*d^4*e*f - 8*sqrt(d*f)*
A*a*b*d^4*e*f + 3*sqrt(d*f)*C*a^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b*c^2*d^2*f^2
 + 3*sqrt(d*f)*A*b^2*c^2*d^2*f^2 - 4*sqrt(d*f)*B*a^2*c*d^3*f^2 - 8*sqrt(d*f
)*A*a*b*c*d^3*f^2 + 8*sqrt(d*f)*A*a^2*d^4*f^2)*arctan(-1/2*(b*d^2*e + b*c*d
*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d)
)/((b^4*c^2*e^2*abs(d) - 2*a*b^3*c*d*e^2*abs(d) + a^2*b^2*d^2*e^2*abs(d) -
2*a*b^3*c^2*e*f*abs(d) + 4*a^2*b^2*c*d*e*f*abs(d) - 2*a^3*b*d^2*e*f*abs(d)
+ a^2*b^2*c^2*f^2*abs(d) - 2*a^3*b*c*d*f^2*abs(d) + a^4*d^2*f^2*abs(d))*sqr
t(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d) + 1/2*(8*sqrt(
d*f)*C*a*b^4*c*d^9*e^5 - 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 5*sqrt(d*f)*C*a^2*b^
3*d^10*e^5 + sqrt(d*f)*B*a*b^4*d^10*e^5 + 3*sqrt(d*f)*A*b^5*d^10*e^5 - 32*s
qrt(d*f)*C*a*b^4*c^2*d^8*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^8*e^4*f + 15*sqrt
(d*f)*C*a^2*b^3*c*d^9*e^4*f - 3*sqrt(d*f)*B*a*b^4*c*d^9*e^4*f - 9*sqrt(d*f)
*A*b^5*c*d^9*e^4*f + 2*sqrt(d*f)*C*a^3*b^2*d^10*e^4*f + 2*sqrt(d*f)*B*a^2*b
^3*d^10*e^4*f - 6*sqrt(d*f)*A*a*b^4*d^10*e^4*f + 48*sqrt(d*f)*C*a*b^4*c^3*d
^7*e^3*f^2 - 24*sqrt(d*f)*B*b^5*c^3*d^7*e^3*f^2 - 10*sqrt(d*f)*C*a^2*b^3*c^
2*d^8*e^3*f^2 + 2*sqrt(d*f)*B*a*b^4*c^2*d^8*e^3*f^2 + 6*sqrt(d*f)*A*b^5*c^2
*d^8*e^3*f^2 - 8*sqrt(d*f)*C*a^3*b^2*c*d^9*e^3*f^2 - 8*sqrt(d*f)*B*a^2*b^3*
c*d^9*e^3*f^2 + 24*sqrt(d*f)*A*a*b^4*c*d^9*e^3*f^2 - 32*sqrt(d*f)*C*a*b^4*c
^4*d^6*e^2*f^3 + 16*sqrt(d*f)*B*b^5*c^4*d^6*e^2*f^3 - 10*sqrt(d*f)*C*a^2*b^
3*c^3*d^7*e^2*f^3 + 2*sqrt(d*f)*B*a*b^4*c^3*d^7*e^2*f^3 + 6*sqrt(d*f)*A*b^5
*c^3*d^7*e^2*f^3 + 12*sqrt(d*f)*C*a^3*b^2*c^2*d^8*e^2*f^3 + 12*sqrt(d*f)*B*
a^2*b^3*c^2*d^8*e^2*f^3 - 36*sqrt(d*f)*A*a*b^4*c^2*d^8*e^2*f^3 + 8*sqrt(d*f
)*C*a*b^4*c^5*d^5*e*f^4 - 4*sqrt(d*f)*B*b^5*c^5*d^5*e*f^4 + 15*sqrt(d*f)*C*
a^2*b^3*c^4*d^6*e*f^4 - 3*sqrt(d*f)*B*a*b^4*c^4*d^6*e*f^4 - 9*sqrt(d*f)*A*b
^5*c^4*d^6*e*f^4 - 8*sqrt(d*f)*C*a^3*b^2*c^3*d^7*e*f^4 - 8*sqrt(d*f)*B*a^2*
b^3*c^3*d^7*e*f^4 + 24*sqrt(d*f)*A*a*b^4*c^3*d^7*e*f^4 - 5*sqrt(d*f)*C*a^2*
b^3*c^5*d^5*f^5 + sqrt(d*f)*B*a*b^4*c^5*d^5*f^5 + 3*sqrt(d*f)*A*b^5*c^5*d^5
*f^5 + 2*sqrt(d*f)*C*a^3*b^2*c^4*d^6*f^5 + 2*sqrt(d*f)*B*a^2*b^3*c^4*d^6*f^
```

5 - 6*sqrt(d*f)*A*a*b^4*c^4*d^6*f^5 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c*d^7*e^4 + 12*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c*d
^7*e^4 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^2*C*a^2*b^3*d^8*e^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*d^8*e^4 - 9*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*d^8*e^4 + 24
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*C*a*b^4*c^2*d^6*e^3*f - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^2*B*b^5*c^2*d^6*e^3*f + 44*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c*d^7*e^3
*f - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*B*a*b^4*c*d^7*e^3*f - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*c*d^7*e^3*f - 32*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*d^8*e^3
*f - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*B*a^2*b^3*d^8*e^3*f + 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*d^8*e^3*f + 24*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^4*c^3*d^5*e
^2*f^2 - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^2*B*b^5*c^3*d^5*e^2*f^2 - 118*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c^2*d^6*e^2*f^2 + 46*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B
*a*b^4*c^2*d^6*e^2*f^2 + 26*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*A*b^5*c^2*d^6*e^2*f^2 + 32*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c*d^7*e^
2*f^2 + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^2*B*a^2*b^3*c*d^7*e^2*f^2 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c*d^7*e^2*f^2 + 8*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b
*d^8*e^2*f^2 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*B*a^3*b^2*d^8*e^2*f^2 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3*d^8*e^2*f^2 - 24*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*
C*a*b^4*c^4*d^4*e*f^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*B*b^5*c^4*d^4*e*f^3 + 44*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^3*c^3*d^5*e*f
^3 - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*B*a*b^4*c^3*d^5*e*f^3 - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*c^3*d^5*e*f^3 + 32*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^2*c^2
*d^6*e*f^3 + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*B*a^2*b^3*c^2*d^6*e*f^3 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^2*d^6*e*f^3 - 16*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C

*a^4*b*c*d^7*e*f^3 - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*B*a^3*b^2*c*d^7*e*f^3 + 80*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3*c*d^7*e*f^3
+ 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^2*C*a^2*b^3*c^4*d^4*f^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^4*c^4*d^4*f^4 - 9*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^5*c^4*d^4*f^4
 - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*C*a^3*b^2*c^3*d^5*f^4 - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^3*c^3*d^5*f^4 + 40*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^4*c^3*d
^5*f^4 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^2*C*a^4*b*c^2*d^6*f^4 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^2*c^2*d^6*f^4 - 40*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^3
*c^2*d^6*f^4 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^4*C*a*b^4*c*d^5*e^3 - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c*d^5*e^3 - 15*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*
d^6*e^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
 - c*d*f))^4*B*a*b^4*d^6*e^3 + 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^5*d^6*e^3 + 16*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^4*c^2*d^4*e^2*f
 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^4*B*b^5*c^2*d^4*e^2*f - 89*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*c*d^5*e^2*f + 37*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c*d^5*e
^2*f + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^4*A*b^5*c*d^5*e^2*f + 46*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^2*d^6*e^2*f - 2*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*d^6*e
^2*f - 42*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
 c*d*f))^4*A*a*b^4*d^6*e^2*f + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^4*c^3*d^3*e*f^2 - 12*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^5*c^3*d^
3*e*f^2 - 89*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^4*C*a^2*b^3*c^2*d^4*e*f^2 + 37*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c^2*d^4*e*f^2 + 15*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b
^5*c^2*d^4*e*f^2 + 148*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^4*C*a^3*b^2*c*d^5*e*f^2 - 44*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c*d^5*e*f^2 -
 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*A*a*b^4*c*d^5*e*f^2 - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b*d^6*e*f^2 - 8*sqrt(d*f)*(sqrt(d*f)*s

qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^2*d^6*e*f^2 +
 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*A*a^2*b^3*d^6*e*f^2 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^3*c^3*d^3*f^3 + 3*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^4*c^3*d^3*f
^3 + 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^4*A*b^5*c^3*d^3*f^3 + 46*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^2*c^2*d^4*f^3 - 2*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^3*c^2*d^
4*f^3 - 42*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^4*A*a*b^4*c^2*d^4*f^3 - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b*c*d^5*f^3 - 8*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^2*c*d^
5*f^3 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^4*A*a^2*b^3*c*d^5*f^3 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*d^6*f^3 + 16*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b*d^6*f^3 -
 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^4*A*a^3*b^2*d^6*f^3 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^6*C*a*b^4*c*d^3*e^2 + 4*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^5*c*d^3*e^2 + 5*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a
^2*b^3*d^4*e^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^6*B*a*b^4*d^4*e^2 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5*d^4*e^2 - 8*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^4*c^2*d^2*e
*f + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^6*B*b^5*c^2*d^2*e*f + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^3*c*d^3*e*f - 14*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^4*c*d^3*e*f
 - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^6*A*b^5*c*d^3*e*f - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^6*C*a^3*b^2*d^4*e*f + 4*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^3*d^4*e*f + 8*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A
*a*b^4*d^4*e*f + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^6*C*a^2*b^3*c^2*d^2*f^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^4*c^2*d^2*f^2 - 3*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^5
*c^2*d^2*f^2 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^6*C*a^3*b^2*c*d^3*f^2 + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^3*c*d^3*f^2 + 8*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^
4*c*d^3*f^2 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^6*C*a^4*b*d^4*f^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s

```
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^3*d^4*f^2)/((b^6*c^2*e^2*abs(
d) - 2*a*b^5*c*d*e^2*abs(d) + a^2*b^4*d^2*e^2*abs(d) - 2*a*b^5*c^2*e*f*abs(
d) + 4*a^2*b^4*c*d*e*f*abs(d) - 2*a^3*b^3*d^2*e*f*abs(d) + a^2*b^4*c^2*f^2*
abs(d) - 2*a^3*b^3*c*d*f^2*abs(d) + a^4*b^2*d^2*f^2*abs(d))*(b*d^4*e^2 - 2*
b*c*d^3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^4*b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 826

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$$+\frac{(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A(de+cf))+a^2b(4Bdf-7C(de+cf))}{12b(bc-ad)^2(be-af)^2(a+bx)^2}$$

$$+\frac{(4a^4Cd^2f^2+8a^3bdf(Bdf-2C(de+cf))-b^4(15Ad^2e^2-2cde(9Be-7Af)+3c^2(8Ce^2-6Bef+5A}{}$$

$$+\frac{(b^3(5Ad^3e^3-3cd^2e^2(2Be-Af)+c^2de(8Ce^2-4Bef+3Af^2)+c^3f(8Ce^2-6Bef+5Af^2))+ab^2(d}{}$$

```
[Out] 1/8*(b^3*(5*A*d^3*e^3-3*c*d^2*e^2*(-A*f+2*B*e)+c^2*d*e*(3*A*f^2-4*B*e*f+8*C
*e^2)+c^3*f*(5*A*f^2-6*B*e*f+8*C*e^2))+a*b^2*(d^3*e^2*(-18*A*f+B*e)-c^3*f^2
*(-B*f+4*C*e)-c*d^2*e*(12*A*f^2-23*B*e*f+4*C*e^2)-c^2*d*f*(18*A*f^2-23*B*e*
f+40*C*e^2))-2*a^3*d*f*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*
(c*f+d*e)))+a^2*b*(C*(c^3*f^3+23*c^2*d*e*f^2+23*c*d^2*e^2*f+d^3*e^3)+4*d*f*
(6*A*d*f*(c*f+d*e)-B*(c^2*f^2+10*c*d*e*f+d^2*e^2))))*arctanh((-a*f+b*e)^(1/
2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c)^(7/2)/(-a*f+b*e
)^(7/2)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-
a*f+b*e)/(b*x+a)^3+1/12*(2*a^3*C*d*f+a*b^2*(-10*A*d*f+B*c*f+B*d*e+12*C*c*e)
-b^3*(6*B*c*e-5*A*(c*f+d*e))+a^2*b*(4*B*d*f-7*C*(c*f+d*e)))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2+1/24*(4*a^4*C*d^2*f^2+8*
a^3*b*d*f*(B*d*f-2*C*(c*f+d*e))-b^4*(15*A*d^2*e^2-2*c*d*e*(-7*A*f+9*B*e)+3*
c^2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(d^2*e*(-44*A*f+3*B*e)-3*c^2*f*(-B*f+4
*C*e)-2*c*d*(22*A*f^2-29*B*e*f+6*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-34*c*d*e*f+3
*d^2*e^2)+2*d*f*(22*A*d*f-5*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-
a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1627, 156, 12, 95, 214}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{3b(bc-ad)(be-af)(a+bx)^3}$$
$$+\frac{(-2df(C(3d^2e^2+2cdfe+3c^2f^2)+4df(2Adf-B(de+cf)))\,a^3+b(C(d^3e^3+23cd^2fe^2+23c^2df^2e+c^3\cdots}{}$$
$$+\frac{(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf))a^3-b^2(C(3d^2e^2-34cdfe+3c^2f^2)+2df(22Adf-5B(de+cf)\cdots}{}$$
$$+\frac{(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))\cdots}{12b(bc-ad)^2(be-af)^2(a+bx)^2}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/3*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^3) + ((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*f -
 10*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*e +
c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(12*b*(b*c - a*d)^2*(b*e - a*f)^2*(a +
b*x)^2) + ((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(
15*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2
)) - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2
 - 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^2)
+ 2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b*(
b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)) + ((b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2
*B*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*
e*f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) - c
*d^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f + 18*
A*f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
 - B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 +
c^3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))
))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])]
)/(8*(b*c - a*d)^(7/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1627

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1]
```

Rubi steps

$$\text{integral} = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(6cCe+Bde+Bcf-6Adf)+b^2(6Bce-5A(de+cf))}{2b}+\left(-3bcCe+3aCde+3acCf+2Abdf-2aBdf-\frac{a^2Cdf}{b}\right)x}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3(bc-ad)(be-af)}$$

$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A(de+cf))+a^2b(4Bdf-7C(de\ldots}{12b(bc-ad)^2(be-af)^2(a+bx)^2} + \frac{\int \frac{\frac{2a^3Cdf(de+cf)+b^3\left(15Ad^2e^2-2cde(9Be-7Af)+3c^2\left(8Ce^2-6Bef+5Af^2\right)\right)+ab^2\left(d^2e(3Be-34Af)-3c^2f(4Ce-Bf)-2cd\left(6Ce^2-23Bef+17A\ldots\right.\right.}{4b}}{(a\ldots}}{6(bc\ldots}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf + ab^2(12cCe + Bde + Bcf - 10Adf) - b^3(6Bce - 5A(de+cf)) + a^2b(4Bdf - 7C(de \cdots}{12b(bc-ad)^2(be-af)^2(a+bx)^2} + \frac{(4a^4Cd^2f^2 + 8a^3bdf(Bdf - 2C(de+cf)) - b^4(15Ad^2e^2 - 2cde(9Be-7Af) + 3c^2(8Ce^2 - 6Be \cdots}{} - \frac{\int \frac{3(b^3(5Ad^3e^3 - 3cd^2e^2(2Be-Af) + c^2de(8Ce^2 - 4Bef + 3Af^2) + c^3f(8Ce^2 - 6Bef + 5Af^2)) + ab^2(d^3e^2(Be-18Af) - c^3f^2(4Ce-Bf \cdots}{}}{}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf + ab^2(12cCe + Bde + Bcf - 10Adf) - b^3(6Bce - 5A(de+cf)) + a^2b(4Bdf - 7C(de \cdots}{12b(bc-ad)^2(be-af)^2(a+bx)^2} + \frac{(4a^4Cd^2f^2 + 8a^3bdf(Bdf - 2C(de+cf)) - b^4(15Ad^2e^2 - 2cde(9Be-7Af) + 3c^2(8Ce^2 - 6Be \cdots}{} - \frac{(b^3(5Ad^3e^3 - 3cd^2e^2(2Be-Af) + c^2de(8Ce^2 - 4Bef + 3Af^2) + c^3f(8Ce^2 - 6Bef + 5Af^2)) + \cdots}{}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf + ab^2(12cCe + Bde + Bcf - 10Adf) - b^3(6Bce - 5A(de+cf)) + a^2b(4Bdf - 7C(de \cdots}{12b(bc-ad)^2(be-af)^2(a+bx)^2} + \frac{(4a^4Cd^2f^2 + 8a^3bdf(Bdf - 2C(de+cf)) - b^4(15Ad^2e^2 - 2cde(9Be-7Af) + 3c^2(8Ce^2 - 6Be \cdots}{} - \frac{(b^3(5Ad^3e^3 - 3cd^2e^2(2Be-Af) + c^2de(8Ce^2 - 4Bef + 3Af^2) + c^3f(8Ce^2 - 6Bef + 5Af^2)) + \cdots}{}$$

$$= -\frac{(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{(2a^3Cdf + ab^2(12cCe + Bde + Bcf - 10Adf) - b^3(6Bce - 5A(de+cf)) + a^2b(4Bdf - 7C(de \cdots}{12b(bc-ad)^2(be-af)^2(a+bx)^2} + \frac{(4a^4Cd^2f^2 + 8a^3bdf(Bdf - 2C(de+cf)) - b^4(15Ad^2e^2 - 2cde(9Be-7Af) + 3c^2(8Ce^2 - 6Be \cdots}{} + \frac{(b^3(5Ad^3e^3 - 3cd^2e^2(2Be-Af) + c^2de(8Ce^2 - 4Bef + 3Af^2) + c^3f(8Ce^2 - 6Bef + 5Af^2)) + \cdots}{}$$

Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\sqrt{c+dx}\sqrt{e+fx}(6b^5cex(4cCex+B(2ce-3dex-3cfx))+6a^5df(-4Bdf+C(3de+3cf-2dfx))+}{} + \frac{(ab^2(d^3e^2(Be-18Af)+c^3f^2(-4Ce+Bf)+c^2df(-40Ce^2+23Bef-18Af^2)+cd^2e(-4Ce^2+23Be}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*b^5*c*e*x*(4*c*C*e*x + B*(2*c*e - 3*d
*e*x - 3*c*f*x)) + 6*a^5*d*f*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + a*b
^4*(-12*c*C*e*x*(-2*c*e + d*e*x + c*f*x) + B*(3*d^2*e^2*x^2 + 2*c*d*e*x*(-2
5*e + 29*f*x) + c^2*(4*e^2 - 50*e*f*x + 3*f^2*x^2))) + a^4*b*(12*B*d*f*(c*f
 + d*(e - 2*f*x)) - C*(3*c^2*f^2 + 2*c*d*f*(29*e - 25*f*x) + d^2*(3*e^2 - 5
0*e*f*x + 4*f^2*x^2))) + a^2*b^3*(d^2*e*x*(8*B*e + 3*C*e*x - 10*B*f*x) + c^
2*(8*B*f*(-2*e + f*x) + C*(8*e^2 + 14*e*f*x + 3*f^2*x^2)) - 2*c*d*(C*e*x*(-
7*e + 17*f*x) + B*(8*e^2 - 62*e*f*x + 5*f^2*x^2))) + a^3*b^2*(c^2*f*(10*C*e
 - 3*B*f - 8*C*f*x) + 2*c*d*(B*f*(17*e - 7*f*x) + C*(5*e^2 - 62*e*f*x + 8*f
^2*x^2)) - d^2*(8*C*e*x*(e - 2*f*x) + B*(3*e^2 + 14*e*f*x + 8*f^2*x^2))) +
A*b*(72*a^4*d^2*f^2 + 18*a^3*b*d*f*(-5*d*e - 5*c*f + 6*d*f*x) + b^4*(15*d^2
*e^2*x^2 + 2*c*d*e*x*(-5*e + 7*f*x) + c^2*(8*e^2 - 10*e*f*x + 15*f^2*x^2))
- 2*a*b^3*(c^2*f*(13*e - 20*f*x) + 2*d^2*e*x*(-10*e + 11*f*x) + c*d*(13*e^2
 - 34*e*f*x + 22*f^2*x^2)) + a^2*b^2*(33*c^2*f^2 + 2*c*d*f*(43*e - 59*f*x)
+ d^2*(33*e^2 - 118*e*f*x + 44*f^2*x^2)))))/((b*c - a*d)^3*(b*e - a*f)^3*(a
 + b*x)^3) + ((a*b^2*(d^3*e^2*(B*e - 18*A*f) + c^3*f^2*(-4*C*e + B*f) + c^2
*d*f*(-40*C*e^2 + 23*B*e*f - 18*A*f^2) + c*d^2*e*(-4*C*e^2 + 23*B*e*f - 12*
A*f^2)) + b^3*(5*A*d^3*e^3 + 3*c*d^2*e^2*(-2*B*e + A*f) + c^2*d*e*(8*C*e^2
- 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - 2*a^3*d*f*(C*
(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) + a^
2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 + c^3*f^3) + 4*d*f*(6*A*d
*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))))*ArcTan[(Sqrt[b*c - a
*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/(8*(b*c - a*d)^(7/2
)*(-(b*e) + a*f)^(7/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. 2(794) = 1588.

Time = 1.70 (sec) , antiderivative size = 18802, normalized size of antiderivative = 22.76

method	result	size
default	Expression too large to display	18802

```
[In] int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25632 vs. 2(796) = 1592.

Time = 58.08 (sec) , antiderivative size = 25632, normalized size of antiderivative = 31.03

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/8*(8*sqrt(d*f)*C*b^3*c^2*d^3*e^3 - 4*sqrt(d*f)*C*a*b^2*c*d^4*e^3 - 6*sqrt
(d*f)*B*b^3*c*d^4*e^3 + sqrt(d*f)*C*a^2*b*d^5*e^3 + sqrt(d*f)*B*a*b^2*d^5*e
^3 + 5*sqrt(d*f)*A*b^3*d^5*e^3 + 8*sqrt(d*f)*C*b^3*c^3*d^2*e^2*f - 40*sqrt(
d*f)*C*a*b^2*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^3*c^2*d^3*e^2*f + 23*sqrt(d*f)
*C*a^2*b*c*d^4*e^2*f + 23*sqrt(d*f)*B*a*b^2*c*d^4*e^2*f + 3*sqrt(d*f)*A*b^3
*c*d^4*e^2*f - 6*sqrt(d*f)*C*a^3*d^5*e^2*f - 4*sqrt(d*f)*B*a^2*b*d^5*e^2*f
 - 18*sqrt(d*f)*A*a*b^2*d^5*e^2*f - 4*sqrt(d*f)*C*a*b^2*c^3*d^2*e*f^2 - 6*sq
rt(d*f)*B*b^3*c^3*d^2*e*f^2 + 23*sqrt(d*f)*C*a^2*b*c^2*d^3*e*f^2 + 23*sqrt(
d*f)*B*a*b^2*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^3*c^2*d^3*e*f^2 - 4*sqrt(d*f)*
C*a^3*c*d^4*e*f^2 - 40*sqrt(d*f)*B*a^2*b*c*d^4*e*f^2 - 12*sqrt(d*f)*A*a*b^2
*c*d^4*e*f^2 + 8*sqrt(d*f)*B*a^3*d^5*e*f^2 + 24*sqrt(d*f)*A*a^2*b*d^5*e*f^2
 + sqrt(d*f)*C*a^2*b*c^3*d^2*f^3 + sqrt(d*f)*B*a*b^2*c^3*d^2*f^3 + 5*sqrt(d
*f)*A*b^3*c^3*d^2*f^3 - 6*sqrt(d*f)*C*a^3*c^2*d^3*f^3 - 4*sqrt(d*f)*B*a^2*b
*c^2*d^3*f^3 - 18*sqrt(d*f)*A*a*b^2*c^2*d^3*f^3 + 8*sqrt(d*f)*B*a^3*c*d^4*f
^3 + 24*sqrt(d*f)*A*a^2*b*c*d^4*f^3 - 16*sqrt(d*f)*A*a^3*d^5*f^3)*arctan(-1
/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^
2 - a^2*d^2*f^2)*d))/((b^6*c^3*e^3*abs(d) - 3*a*b^5*c^2*d*e^3*abs(d) + 3*a^
2*b^4*c*d^2*e^3*abs(d) - a^3*b^3*d^3*e^3*abs(d) - 3*a*b^5*c^3*e^2*f*abs(d)
+ 9*a^2*b^4*c^2*d*e^2*f*abs(d) - 9*a^3*b^3*c*d^2*e^2*f*abs(d) + 3*a^4*b^2*d
^3*e^2*f*abs(d) + 3*a^2*b^4*c^3*e*f^2*abs(d) - 9*a^3*b^3*c^2*d*e*f^2*abs(d)
 + 9*a^4*b^2*c*d^2*e*f^2*abs(d) - 3*a^5*b*d^3*e*f^2*abs(d) - a^3*b^3*c^3*f^
3*abs(d) + 3*a^4*b^2*c^2*d*f^3*abs(d) - 3*a^5*b*c*d^2*f^3*abs(d) + a^6*d^3*
f^3*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d)
 - 1/12*(24*sqrt(d*f)*C*b^7*c^2*d^13*e^8 - 12*sqrt(d*f)*C*a*b^6*c*d^14*e^8
- 18*sqrt(d*f)*B*b^7*c*d^14*e^8 + 3*sqrt(d*f)*C*a^2*b^5*d^15*e^8 + 3*sqrt(d
*f)*B*a*b^6*d^15*e^8 + 15*sqrt(d*f)*A*b^7*d^15*e^8 - 144*sqrt(d*f)*C*b^7*c^
3*d^12*e^7*f + 60*sqrt(d*f)*C*a*b^6*c^2*d^13*e^7*f + 90*sqrt(d*f)*B*b^7*c^2
*d^13*e^7*f - 52*sqrt(d*f)*C*a^2*b^5*c*d^14*e^7*f + 40*sqrt(d*f)*B*a*b^6*c*
d^14*e^7*f - 76*sqrt(d*f)*A*b^7*c*d^14*e^7*f + 16*sqrt(d*f)*C*a^3*b^4*d^15*
e^7*f - 10*sqrt(d*f)*B*a^2*b^5*d^15*e^7*f - 44*sqrt(d*f)*A*a*b^6*d^15*e^7*f
 + 360*sqrt(d*f)*C*b^7*c^4*d^11*e^6*f^2 - 108*sqrt(d*f)*C*a*b^6*c^3*d^12*e^
6*f^2 - 162*sqrt(d*f)*B*b^7*c^3*d^12*e^6*f^2 + 252*sqrt(d*f)*C*a^2*b^5*c^2*
d^13*e^6*f^2 - 300*sqrt(d*f)*B*a*b^6*c^2*d^13*e^6*f^2 + 156*sqrt(d*f)*A*b^7
```

*c^2*d^13*e^6*f^2 - 80*sqrt(d*f)*C*a^3*b^4*c*d^14*e^6*f^2 + 50*sqrt(d*f)*B*a^2*b^5*c*d^14*e^6*f^2 + 220*sqrt(d*f)*A*a*b^6*c*d^14*e^6*f^2 - 4*sqrt(d*f)*C*a^4*b^3*d^15*e^6*f^2 - 8*sqrt(d*f)*B*a^3*b^4*d^15*e^6*f^2 + 44*sqrt(d*f)*A*a^2*b^5*d^15*e^6*f^2 - 480*sqrt(d*f)*C*b^7*c^5*d^10*e^5*f^3 + 60*sqrt(d*f)*C*a*b^6*c^4*d^11*e^5*f^3 + 90*sqrt(d*f)*B*b^7*c^4*d^11*e^5*f^3 - 588*sqrt(d*f)*C*a^2*b^5*c^3*d^12*e^5*f^3 + 792*sqrt(d*f)*B*a*b^6*c^3*d^12*e^5*f^3 - 180*sqrt(d*f)*A*b^7*c^3*d^12*e^5*f^3 + 144*sqrt(d*f)*C*a^3*b^4*c^2*d^13*e^5*f^3 - 90*sqrt(d*f)*B*a^2*b^5*c^2*d^13*e^5*f^3 - 396*sqrt(d*f)*A*a*b^6*c^2*d^13*e^5*f^3 + 24*sqrt(d*f)*C*a^4*b^3*c*d^14*e^5*f^3 + 48*sqrt(d*f)*B*a^3*b^4*c*d^14*e^5*f^3 - 264*sqrt(d*f)*A*a^2*b^5*c*d^14*e^5*f^3 + 360*sqrt(d*f)*C*b^7*c^6*d^9*e^4*f^4 + 60*sqrt(d*f)*C*a*b^6*c^5*d^10*e^4*f^4 + 90*sqrt(d*f)*B*b^7*c^5*d^10*e^4*f^4 + 770*sqrt(d*f)*C*a^2*b^5*c^4*d^11*e^4*f^4 - 1070*sqrt(d*f)*B*a*b^6*c^4*d^11*e^4*f^4 + 170*sqrt(d*f)*A*b^7*c^4*d^11*e^4*f^4 - 80*sqrt(d*f)*C*a^3*b^4*c^3*d^12*e^4*f^4 + 50*sqrt(d*f)*B*a^2*b^5*c^3*d^12*e^4*f^4 + 220*sqrt(d*f)*A*a*b^6*c^3*d^12*e^4*f^4 - 60*sqrt(d*f)*C*a^4*b^3*c^2*d^13*e^4*f^4 - 120*sqrt(d*f)*B*a^3*b^4*c^2*d^13*e^4*f^4 + 660*sqrt(d*f)*A*a^2*b^5*c^2*d^13*e^4*f^4 - 144*sqrt(d*f)*C*b^7*c^7*d^8*e^3*f^5 - 108*sqrt(d*f)*C*a*b^6*c^6*d^9*e^3*f^5 - 162*sqrt(d*f)*B*b^7*c^6*d^9*e^3*f^5 - 588*sqrt(d*f)*C*a^2*b^5*c^5*d^10*e^3*f^5 + 792*sqrt(d*f)*B*a*b^6*c^5*d^10*e^3*f^5 - 180*sqrt(d*f)*A*b^7*c^5*d^10*e^3*f^5 - 80*sqrt(d*f)*C*a^3*b^4*c^4*d^11*e^3*f^5 + 50*sqrt(d*f)*B*a^2*b^5*c^4*d^11*e^3*f^5 + 220*sqrt(d*f)*A*a*b^6*c^4*d^11*e^3*f^5 + 80*sqrt(d*f)*C*a^4*b^3*c^3*d^12*e^3*f^5 + 160*sqrt(d*f)*B*a^3*b^4*c^3*d^12*e^3*f^5 - 880*sqrt(d*f)*A*a^2*b^5*c^3*d^12*e^3*f^5 + 24*sqrt(d*f)*C*b^7*c^8*d^7*e^2*f^6 + 60*sqrt(d*f)*C*a*b^6*c^7*d^8*e^2*f^6 + 90*sqrt(d*f)*B*b^7*c^7*d^8*e^2*f^6 + 252*sqrt(d*f)*C*a^2*b^5*c^6*d^9*e^2*f^6 - 300*sqrt(d*f)*B*a*b^6*c^6*d^9*e^2*f^6 + 156*sqrt(d*f)*A*b^7*c^6*d^9*e^2*f^6 + 144*sqrt(d*f)*C*a^3*b^4*c^5*d^10*e^2*f^6 - 90*sqrt(d*f)*B*a^2*b^5*c^5*d^10*e^2*f^6 - 396*sqrt(d*f)*A*a*b^6*c^5*d^10*e^2*f^6 - 60*sqrt(d*f)*C*a^4*b^3*c^4*d^11*e^2*f^6 - 120*sqrt(d*f)*B*a^3*b^4*c^4*d^11*e^2*f^6 + 660*sqrt(d*f)*A*a^2*b^5*c^4*d^11*e^2*f^6 - 12*sqrt(d*f)*C*a*b^6*c^8*d^7*e*f^7 - 18*sqrt(d*f)*B*b^7*c^8*d^7*e*f^7 - 52*sqrt(d*f)*C*a^2*b^5*c^7*d^8*e*f^7 + 40*sqrt(d*f)*B*a*b^6*c^7*d^8*e*f^7 - 76*sqrt(d*f)*A*b^7*c^7*d^8*e*f^7 - 80*sqrt(d*f)*C*a^3*b^4*c^6*d^9*e*f^7 + 50*sqrt(d*f)*B*a^2*b^5*c^6*d^9*e*f^7 + 220*sqrt(d*f)*A*a*b^6*c^6*d^9*e*f^7 + 24*sqrt(d*f)*C*a^4*b^3*c^5*d^10*e*f^7 + 48*sqrt(d*f)*B*a^3*b^4*c^5*d^10*e*f^7 - 264*sqrt(d*f)*A*a^2*b^5*c^5*d^10*e*f^7 + 3*sqrt(d*f)*C*a^2*b^5*c^8*d^7*f^8 + 3*sqrt(d*f)*B*a*b^6*c^8*d^7*f^8 + 15*sqrt(d*f)*A*b^7*c^8*d^7*f^8 + 16*sqrt(d*f)*C*a^3*b^4*c^7*d^8*f^8 - 10*sqrt(d*f)*B*a^2*b^5*c^7*d^8*f^8 - 44*sqrt(d*f)*A*a*b^6*c^7*d^8*f^8 - 4*sqrt(d*f)*C*a^4*b^3*c^6*d^9*f^8 - 8*sqrt(d*f)*B*a^3*b^4*c^6*d^9*f^8 + 44*sqrt(d*f)*A*a^2*b^5*c^6*d^9*f^8 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^2*d^11*e^7 + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^6*c*d^12*e^7 + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c*d^12*e^7 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*d^13*e^7 - 15*sqrt(d*f)*(sqrt(d*f)*s

qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*d^13*e^7 - 75
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*A*b^7*d^13*e^7 + 360*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^2*C*b^7*c^3*d^10*e^6*f + 72*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^6*c^2*d^11*e^6*f -
156*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^2*B*b^7*c^2*d^11*e^6*f + 171*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c*d^12*e^6*f - 453*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c*d
^12*e^6*f + 135*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*A*b^7*c*d^12*e^6*f - 78*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*d^13*e^6*f + 84*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*
b^5*d^13*e^6*f + 390*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*A*a*b^6*d^13*e^6*f - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^4*d^9*e^5*f^2 - 828
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*C*a*b^6*c^3*d^10*e^5*f^2 - 186*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^3*d^10*e^5*f^2 - 423*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c
^2*d^11*e^5*f^2 + 1449*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^2*B*a*b^6*c^2*d^11*e^5*f^2 + 45*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c^2*d^11*e^5*f
^2 - 300*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^2*C*a^3*b^4*c*d^12*e^5*f^2 + 360*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c*d^12*e^5*f^2 - 900*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A
*a*b^6*c*d^12*e^5*f^2 + 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*d^13*e^5*f^2 - 48*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*d^13*e^
5*f^2 - 720*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^2*A*a^2*b^5*d^13*e^5*f^2 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^5*d^8*e^4*f^3 + 1392*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*
a*b^6*c^4*d^9*e^4*f^3 + 504*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^2*B*b^7*c^4*d^9*e^4*f^3 + 267*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^3*d^1
0*e^4*f^3 - 981*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*B*a*b^6*c^3*d^10*e^4*f^3 - 105*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c^3*d^10*e^4*f^3 + 1
902*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^2*C*a^3*b^4*c^2*d^11*e^4*f^3 - 2196*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^2*d^11*e^4*f^3 + 90*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*
a*b^6*c^2*d^11*e^4*f^3 - 648*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*

e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c*d^12*e^4*f^3 + 144*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*c*d
^12*e^4*f^3 + 2160*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*A*a^2*b^5*c*d^12*e^4*f^3 - 48*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^5*b^2*d^13*e^4*f^3 -
96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
))^2*B*a^4*b^3*d^13*e^4*f^3 + 480*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4*d^13*e^4*f^3 + 360*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*b^7*c^6*
d^7*e^3*f^4 - 828*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^2*C*a*b^6*c^5*d^8*e^3*f^4 - 186*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*b^7*c^5*d^8*e^3*f^4 + 2
67*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*C*a^2*b^5*c^4*d^9*e^3*f^4 - 981*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c^4*d^9*e^3*f^4 - 105*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^7*c
^4*d^9*e^3*f^4 - 3048*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^3*d^10*e^3*f^4 + 3504*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^3*d^1
0*e^3*f^4 + 840*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*A*a*b^6*c^3*d^10*e^3*f^4 + 432*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c^2*d^11*e^3*f^4
- 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*B*a^3*b^4*c^2*d^11*e^3*f^4 - 1440*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*c^2*d^11*e^3*f^4 + 192*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*C*a^5*b^2*c*d^12*e^3*f^4 + 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*c*d^12*e^3*f^4 - 1920*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4
*c*d^12*e^3*f^4 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*C*b^7*c^7*d^6*e^2*f^5 + 72*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^6*c^6*d^7*e^2*f^5
- 156*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^2*B*b^7*c^6*d^7*e^2*f^5 - 423*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^5*d^8*e^2*f^5 + 1449*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b
^6*c^5*d^8*e^2*f^5 + 45*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^2*A*b^7*c^5*d^8*e^2*f^5 + 1902*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^4*d^9*e^
2*f^5 - 2196*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*B*a^2*b^5*c^4*d^9*e^2*f^5 + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*c^4*d^9*e^2*f^5 + 432
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*C*a^4*b^3*c^3*d^10*e^2*f^5 - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*b^4*c^3*d^10*e^2*f^5 - 1440*sqrt(d

```
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2
*b^5*c^3*d^10*e^2*f^5 - 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*C*a^5*b^2*c^2*d^11*e^2*f^5 - 576*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*c^
2*d^11*e^2*f^5 + 2880*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*A*a^3*b^4*c^2*d^11*e^2*f^5 + 60*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a*b^6*c^7*d^6*e*f
^6 + 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^2*B*b^7*c^7*d^6*e*f^6 + 171*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^6*d^7*e*f^6 - 453*sqrt(d*f)
*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*
c^6*d^7*e*f^6 + 135*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^2*A*b^7*c^6*d^7*e*f^6 - 300*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^5*d^8*e*f^6 + 3
60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*B*a^2*b^5*c^5*d^8*e*f^6 - 900*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a*b^6*c^5*d^8*e*f^6 - 648*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c
^4*d^9*e*f^6 + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*B*a^3*b^4*c^4*d^9*e*f^6 + 2160*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^2*b^5*c^4*d^9*e*f^6
 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*C*a^5*b^2*c^3*d^10*e*f^6 + 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^4*b^3*c^3*d^10*e*f^6 - 1920*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*
a^3*b^4*c^3*d^10*e*f^6 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*C*a^2*b^5*c^7*d^6*f^7 - 15*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^6*c^7*d^6*f^
7 - 75*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^2*A*b^7*c^7*d^6*f^7 - 78*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*b^4*c^6*d^7*f^7 + 84*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^2*b^5*c^6*d
^7*f^7 + 390*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*A*a*b^6*c^6*d^7*f^7 + 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^4*b^3*c^5*d^8*f^7 - 48*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a^3*
b^4*c^5*d^8*f^7 - 720*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*
x + c)*d*f - c*d*f))^2*A*a^2*b^5*c^5*d^8*f^7 - 48*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^5*b^2*c^4*d^9*f^7 -
96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^2*B*a^4*b^3*c^4*d^9*f^7 + 480*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^2*A*a^3*b^4*c^4*d^9*f^7 + 240*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*b^7*c^2*d^9
*e^6 - 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^4*C*a*b^6*c*d^10*e^6 - 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
```

qrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c*d^10*e^6 + 30*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*d^11
*e^6 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^4*B*a*b^6*d^11*e^6 + 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*d^11*e^6 - 192*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*b^7*c^3*d^8*e^5*
f - 696*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^4*C*a*b^6*c^2*d^9*e^5*f - 84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^2*d^9*e^5*f - 132*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c
*d^10*e^5*f + 1188*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^4*B*a*b^6*c*d^10*e^5*f + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c*d^10*e^5*f + 120*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*
a^3*b^4*d^11*e^5*f - 204*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*d^11*e^5*f - 960*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*d^11*e^5*f -
96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^4*C*b^7*c^4*d^7*e^4*f^2 + 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^3*d^8*e^4*f^2 + 264*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^3*d
^8*e^4*f^2 + 930*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^4*C*a^2*b^5*c^2*d^9*e^4*f^2 - 414*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c^2*d^9*e^4*f^2
+ 42*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^4*A*b^7*c^2*d^9*e^4*f^2 + 1176*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c*d^10*e^4*f^2 - 2364*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*
b^5*c*d^10*e^4*f^2 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^4*A*a*b^6*c*d^10*e^4*f^2 - 576*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*d^11*e^4*
f^2 + 264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^4*B*a^3*b^4*d^11*e^4*f^2 + 2592*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*d^11*e^4*f^2 - 192*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*b
^7*c^5*d^6*e^3*f^3 + 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^4*d^7*e^3*f^3 + 264*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*b^7*c^4*d^7*e^3*
f^3 - 1656*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^4*C*a^2*b^5*c^3*d^8*e^3*f^3 - 1608*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c^3*d^8*e^3*f^3 - 504
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
4*A*b^7*c^3*d^8*e^3*f^3 - 1296*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^2*d^9*e^3*f^3 + 2568*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5

*c^2*d^9*e^3*f^3 + 1344*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^2*d^9*e^3*f^3 - 1344*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*c*d^10*e
^3*f^3 + 1632*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^4*B*a^3*b^4*c*d^10*e^3*f^3 - 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*c*d^10*e^3*f^3 + 6
72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^4*C*a^5*b^2*d^11*e^3*f^3 + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*d^11*e^3*f^3 - 3264*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*
d^11*e^3*f^3 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^4*C*b^7*c^6*d^5*e^2*f^4 - 696*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^5*d^6*e^2*f^4 -
84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f)
)^4*B*b^7*c^5*d^6*e^2*f^4 + 930*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^4*d^7*e^2*f^4 - 414*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c
^4*d^7*e^2*f^4 + 42*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^4*A*b^7*c^4*d^7*e^2*f^4 - 1296*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^3*d^8*e^2*f^
4 + 2568*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^4*B*a^2*b^5*c^3*d^8*e^2*f^4 + 1344*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^3*d^8*e^2*f^4 + 3840*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4
*C*a^4*b^3*c^2*d^9*e^2*f^4 - 3792*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^2*d^9*e^2*f^4 - 4032*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*
b^5*c^2*d^9*e^2*f^4 - 672*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c*d^10*e^2*f^4 - 144*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*c*d^10
*e^2*f^4 + 3264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^4*A*a^3*b^4*c*d^10*e^2*f^4 - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^6*b*d^11*e^2*f^4 - 384*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4
*B*a^5*b^2*d^11*e^2*f^4 + 1632*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*d^11*e^2*f^4 - 120*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a*b^6*c^6*d
^5*e*f^5 - 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^4*B*b^7*c^6*d^5*e*f^5 - 132*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^5*d^6*e*f^5 + 1188*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*
B*a*b^6*c^5*d^6*e*f^5 + 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^4*A*b^7*c^5*d^6*e*f^5 + 1176*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^4*d^7*e
*f^5 - 2364*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f

- c*d*f))^4*B*a^2*b^5*c^4*d^7*e*f^5 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^4*d^7*e*f^5 - 1344*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*c^3*d^8*e*f^5 + 1632*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^3*d^8*e*f^5 - 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*c^3*d^8*e*f^5 - 672*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c^2*d^9*e*f^5 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*c^2*d^9*e*f^5 + 3264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*c^2*d^9*e*f^5 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^6*b*c*d^10*e*f^5 + 768*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^5*b^2*c*d^10*e*f^5 - 3264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*c*d^10*e*f^5 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^2*b^5*c^6*d^5*f^6 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a*b^6*c^6*d^5*f^6 + 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*b^7*c^6*d^5*f^6 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^3*b^4*c^5*d^6*f^6 - 204*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^2*b^5*c^5*d^6*f^6 - 960*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a*b^6*c^5*d^6*f^6 - 576*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^4*b^3*c^4*d^7*f^6 + 264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^3*b^4*c^4*d^7*f^6 + 2592*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^2*b^5*c^4*d^7*f^6 + 672*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^5*b^2*c^3*d^8*f^6 + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^4*b^3*c^3*d^8*f^6 - 3264*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^3*b^4*c^3*d^8*f^6 - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*C*a^6*b*c^2*d^9*f^6 - 384*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*B*a^5*b^2*c^2*d^9*f^6 + 1632*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*A*a^4*b^3*c^2*d^9*f^6 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^2*d^7*e^5 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c*d^8*e^5 + 180*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*b^7*c*d^8*e^5 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*d^9*e^5 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*d^9*e^5 - 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*d^9*e^5 - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^3*d^6*e^4*f + 1056*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +

c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^2*d^7*e^4*f + 288*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*
b^7*c^2*d^7*e^4*f - 110*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c*d^8*e^4*f - 1246*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c*d^8*e^4*f
- 230*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^6*A*b^7*c*d^8*e^4*f - 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*d^9*e^4*f + 208*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*d^9*e^4
*f + 980*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^6*A*a*b^6*d^9*e^4*f - 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*b^7*c^4*d^5*e^3*f^2 + 720*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^3
*d^6*e^3*f^2 + 216*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^6*B*b^7*c^3*d^6*e^3*f^2 - 1716*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^2*d^7*e^3*f^2
- 1476*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^6*B*a*b^6*c^2*d^7*e^3*f^2 - 324*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^2*d^7*e^3*f^2 - 832*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*
b^4*c*d^8*e^3*f^2 + 3184*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c*d^8*e^3*f^2 + 1568*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c*d^8*e^3
*f^2 + 472*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f
- c*d*f))^6*C*a^4*b^3*d^9*e^3*f^2 - 424*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*d^9*e^3*f^2 - 2744*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^
2*b^5*d^9*e^3*f^2 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^6*C*b^7*c^5*d^4*e^2*f^3 + 1056*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^4*d^5*e^2*
f^3 + 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^6*B*b^7*c^4*d^5*e^2*f^3 - 1716*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^3*d^6*e^2*f^3 - 1476*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*
B*a*b^6*c^3*d^6*e^2*f^3 - 324*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^6*A*b^7*c^3*d^6*e^2*f^3 + 1128*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^2*
d^7*e^2*f^3 + 3456*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^6*B*a^2*b^5*c^2*d^7*e^2*f^3 + 1944*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a*b^6*c^2*d^7*e^2*f
^3 + 2088*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^6*C*a^4*b^3*c*d^8*e^2*f^3 - 4056*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*c*d^8*e^2*f^3 - 4296*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*
A*a^2*b^5*c*d^8*e^2*f^3 - 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2

*e + (d*x + c)*d*f - c*d*f))^6*C*a^5*b^2*d^9*e^2*f^3 + 288*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^4*b^3*d^9*e
^2*f^3 + 4176*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^6*A*a^3*b^4*d^9*e^2*f^3 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a*b^6*c^5*d^4*e*f^4 + 180*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*
b^7*c^5*d^4*e*f^4 - 110*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^6*C*a^2*b^5*c^4*d^5*e*f^4 - 1246*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^4*d^5*e*
f^4 - 230*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^6*A*b^7*c^4*d^5*e*f^4 - 832*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^3*d^6*e*f^4 + 3184*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2
*b^5*c^3*d^6*e*f^4 + 1568*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^6*A*a*b^6*c^3*d^6*e*f^4 + 2088*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*c^2*d^7*
e*f^4 - 4056*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^6*B*a^3*b^4*c^2*d^7*e*f^4 - 4296*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^2*b^5*c^2*d^7*e*f^4 - 272
0*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^6*C*a^5*b^2*c*d^8*e*f^4 + 2624*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d
^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^4*b^3*c*d^8*e*f^4 + 5728*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^3*b^4*c*
d^8*e*f^4 + 704*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^6*C*a^6*b*d^9*e*f^4 + 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^5*b^2*d^9*e*f^4 - 3520*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^4*
b^3*d^9*e*f^4 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^6*C*a^2*b^5*c^5*d^4*f^5 - 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a*b^6*c^5*d^4*f^5 - 150*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*
A*b^7*c^5*d^4*f^5 - 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^6*C*a^3*b^4*c^4*d^5*f^5 + 208*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^2*b^5*c^4*d^5*f^5
+ 980*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^6*A*a*b^6*c^4*d^5*f^5 + 472*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^4*b^3*c^3*d^6*f^5 - 424*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*B*a^3*b^4*c^
3*d^6*f^5 - 2744*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^6*A*a^2*b^5*c^3*d^6*f^5 - 816*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^5*b^2*c^2*d^7*f^5 + 288*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6
*B*a^4*b^3*c^2*d^7*f^5 + 4176*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2
*e + (d*x + c)*d*f - c*d*f))^6*A*a^3*b^4*c^2*d^7*f^5 + 704*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^6*b*c*d^8*f

^5 + 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^6*B*a^5*b^2*c*d^8*f^5 - 3520*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^4*b^3*c*d^8*f^5 - 128*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^6*C*a^7*d^9*f
^5 - 256*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^6*B*a^6*b*d^9*f^5 + 1408*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^6*A*a^5*b^2*d^9*f^5 + 120*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b^7*c^2*d^5*e^
4 - 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^8*C*a*b^6*c*d^6*e^4 - 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^
2*e + (d*x + c)*d*f - c*d*f))^8*B*b^7*c*d^6*e^4 + 15*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^2*b^5*d^7*e^4 + 1
5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^8*B*a*b^6*d^7*e^4 + 75*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^8*A*b^7*d^7*e^4 + 144*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b^7*c^3*d^4*e^3*f - 612*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*
a*b^6*c^2*d^5*e^3*f - 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^8*B*b^7*c^2*d^5*e^3*f + 192*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^2*b^5*c*d^6*e^3*f
 + 540*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*
d*f))^8*B*a*b^6*c*d^6*e^3*f + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*c*d^6*e^3*f - 24*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^3*b^4*d^7*e^
3*f - 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*B*a^2*b^5*d^7*e^3*f - 420*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a*b^6*d^7*e^3*f + 120*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*b^7*c^4*d^3
*e^2*f^2 - 612*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^8*C*a*b^6*c^3*d^4*e^2*f^2 - 150*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*b^7*c^3*d^4*e^2*f^2 + 1026
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
8*C*a^2*b^5*c^2*d^5*e^2*f^2 + 810*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a*b^6*c^2*d^5*e^2*f^2 + 90*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*c^2*
d^5*e^2*f^2 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^8*C*a^3*b^4*c*d^6*e^2*f^2 - 1350*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^2*b^5*c*d^6*e^2*f^2 -
 540*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*
f))^8*A*a*b^6*c*d^6*e^2*f^2 - 108*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^4*b^3*d^7*e^2*f^2 + 240*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^3*b^4*d
^7*e^2*f^2 + 900*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c
)*d*f - c*d*f))^8*A*a^2*b^5*d^7*e^2*f^2 - 60*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a*b^6*c^4*d^3*e*f^3 - 90*sq

rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B
*b^7*c^4*d^3*e*f^3 + 192*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^8*C*a^2*b^5*c^3*d^4*e*f^3 + 540*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a*b^6*c^3*d^4*e*
f^3 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*A*b^7*c^3*d^4*e*f^3 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^3*b^4*c^2*d^5*e*f^3 - 1350*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^2*
b^5*c^2*d^5*e*f^3 - 540*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (
d*x + c)*d*f - c*d*f))^8*A*a*b^6*c^2*d^5*e*f^3 - 744*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^4*b^3*c*d^6*e*f^3
+ 1440*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c
*d*f))^8*B*a^3*b^4*c*d^6*e*f^3 + 1080*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^2*b^5*c*d^6*e*f^3 + 288*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*a^5*b
^2*d^7*e*f^3 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^8*B*a^4*b^3*d^7*e*f^3 - 960*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^3*b^4*d^7*e*f^3 + 15*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*C*
a^2*b^5*c^4*d^3*f^4 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^8*B*a*b^6*c^4*d^3*f^4 + 75*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*b^7*c^4*d^3*f^4 - 24*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8
*C*a^3*b^4*c^3*d^4*f^4 - 90*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
+ (d*x + c)*d*f - c*d*f))^8*B*a^2*b^5*c^3*d^4*f^4 - 420*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a*b^6*c^3*d^4*f
^4 - 108*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^8*C*a^4*b^3*c^2*d^5*f^4 + 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^3*b^4*c^2*d^5*f^4 + 900*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^2*b
^5*c^2*d^5*f^4 + 288*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
+ c)*d*f - c*d*f))^8*C*a^5*b^2*c*d^6*f^4 - 240*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*B*a^4*b^3*c*d^6*f^4 - 960*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8
*A*a^3*b^4*c*d^6*f^4 - 96*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
(d*x + c)*d*f - c*d*f))^8*C*a^6*b*d^7*f^4 + 480*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^8*A*a^4*b^3*d^7*f^4 - 24*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*
C*b^7*c^2*d^3*e^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d
*x + c)*d*f - c*d*f))^10*C*a*b^6*c*d^4*e^3 + 18*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c*d^4*e^3 - 3*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a
^2*b^5*d^5*e^3 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^10*B*a*b^6*d^5*e^3 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*d^5*e^3 - 24*sqrt(d*f)*(s

qrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*b^7*c^3*
d^2*e^2*f + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^10*C*a*b^6*c^2*d^3*e^2*f + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c^2*d^3*e^2*f - 69*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C
*a^2*b^5*c*d^4*e^2*f - 69*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f))^10*B*a*b^6*c*d^4*e^2*f - 9*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c*d^4*e^2*f + 1
8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^10*C*a^3*b^4*d^5*e^2*f + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^10*B*a^2*b^5*d^5*e^2*f + 54*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*b^6*d^5*e^2*f
 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f))^10*C*a*b^6*c^3*d^2*e*f^2 + 18*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^10*B*b^7*c^3*d^2*e*f^2 - 69*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a^2*b^5*c
^2*d^3*e*f^2 - 69*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
c)*d*f - c*d*f))^10*B*a*b^6*c^2*d^3*e*f^2 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c^2*d^3*e*f^2 + 12*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*
C*a^3*b^4*c*d^4*e*f^2 + 120*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^10*B*a^2*b^5*c*d^4*e*f^2 + 36*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*b^6*c*d^4*e*
f^2 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f -
c*d*f))^10*B*a^3*b^4*d^5*e*f^2 - 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a^2*b^5*d^5*e*f^2 - 3*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a^2*b^5*c
^3*d^2*f^3 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^10*B*a*b^6*c^3*d^2*f^3 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*b^7*c^3*d^2*f^3 + 18*sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*C*a^3*
b^4*c^2*d^3*f^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x
 + c)*d*f - c*d*f))^10*B*a^2*b^5*c^2*d^3*f^3 + 54*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a*b^6*c^2*d^3*f^3 - 2
4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))
^10*B*a^3*b^4*c*d^4*f^3 - 72*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*
e + (d*x + c)*d*f - c*d*f))^10*A*a^2*b^5*c*d^4*f^3 + 48*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^10*A*a^3*b^4*d^5*f^3
)/((b^8*c^3*e^3*abs(d) - 3*a*b^7*c^2*d*e^3*abs(d) + 3*a^2*b^6*c*d^2*e^3*abs
(d) - a^3*b^5*d^3*e^3*abs(d) - 3*a*b^7*c^3*e^2*f*abs(d) + 9*a^2*b^6*c^2*d*e
^2*f*abs(d) - 9*a^3*b^5*c*d^2*e^2*f*abs(d) + 3*a^4*b^4*d^3*e^2*f*abs(d) + 3
*a^2*b^6*c^3*e*f^2*abs(d) - 9*a^3*b^5*c^2*d*e*f^2*abs(d) + 9*a^4*b^4*c*d^2*
e*f^2*abs(d) - 3*a^5*b^3*d^3*e*f^2*abs(d) - a^3*b^5*c^3*f^3*abs(d) + 3*a^4*
b^4*c^2*d*f^3*abs(d) - 3*a^5*b^3*c*d^2*f^3*abs(d) + a^6*b^2*d^3*f^3*abs(d))
*(b*d^4*e^2 - 2*b*c*d^3*e*f + b*c^2*d^2*f^2 - 2*(sqrt(d*f)*sqrt(d*x + c) -

```
sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*d^2*e - 2*(sqrt(d*f)*sqrt(d*x + c)
 - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*a*d^2*f + (sqrt(d*f)*sqrt(d*x
+ c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^4*b)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^(1/2)),x)
[Out] \text{Hanged}
```

3.61 $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1182

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$

$$= \frac{2(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f+Bd(de-2cf))-b^3(C(16d^3e^3}{315b^3d^3}$$

$$-\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)+(adf-4b(de+cf))(2aCdf-b(3Bdf-2C(de+cf))))\sqrt{a}}{105b^2d^2f^3}$$

$$-\frac{2(2aCdf-b(3Bdf-2C(de+cf)))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2}$$

$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}$$

$$-\frac{2\sqrt{-bc+ad}(16a^4Cd^4f^4-8a^3bd^3f^3(Cde+cCf+3Bdf)+3a^2b^2d^2f^2(df(5Bde+5Bcf+14Adf)-2}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f\right.}{}$$

```
[Out] 2/9*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-2/21*(2*a*C*d*f-b*(3*
B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d^2/f^2-2
/105*(7*b*d*f*(-3*A*b*d*f+C*a*c*f+C*a*d*e+C*b*c*e)+(a*d*f-4*b*(c*f+d*e))*(2
*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e))))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1
/2)/b^2/d^2/f^3+2/315*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*
e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3
*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+
8*d^2*e^2))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^3/f^3-2/315*(
16*a^4*C*d^4*f^4-8*a^3*b*d^3*f^3*(3*B*d*f+C*c*f+C*d*e)+3*a^2*b^2*d^2*f^2*(d
```

```
*f*(14*A*d*f+5*B*c*f+5*B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2))-a*b^3*d*f*(C*(
8*c^3*f^3-6*c^2*d*e*f^2-6*c*d^2*e^2*f+8*d^3*e^3)+3*d*f*(14*A*d*f*(c*f+d*e)-
B*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)))+b^4*(2*C*(8*c^4*f^4-4*c^3*d*e*f^3-3*c^2
*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4)+3*d*f*(14*A*d*f*(c^2*f^2-c*d*e*f+d^2*
e^2)-B*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))))*EllipticE(d^(1/
2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*
c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(7/2)/f^4/(d*x+c)
^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/315*(-a*f+b*e)*(-c*f+d*e)*(8*a^3*C*d^
3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2
)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7
*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2))))*EllipticF(d^(1/2)*(
b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(
1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(7/2)/
f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 1154, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}$$
$$+\frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2}$$
$$-\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+b}}{105b^2d^2f^3}$$
$$-\frac{2\sqrt{ad-bc}((2C(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-4c^3df^3e+8c^4f^4)+3df(14Adf(d^2e^2-cdfe+c^2f^2)-B}{}$$
$$+\frac{2\left(\frac{8Cdfa^3}{b}+3(Cde-cCf-4Bdf)a^2-3b\left(\frac{Cfc^2}{d}-2Bfc+Bde-7Adf\right)a+b^2\left(\frac{8Cfc^3}{d^2}+\frac{3Cec^2}{d}+21Afc\right.\right.}{315b^2df}$$
$$-\frac{2\sqrt{ad-bc}(be-af)(de-cf)\left(-(((C(16d^3e^3-3c^2df^2e-8c^3f^3)+3df(7Adf(2de-cf)-B(8d^2e^2-cdf\right.}{}$$

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*((8*a^3*C*d*f)/b - 3*a*b*(B*d*e - 2*B*c*f + (c^2*C*f)/d - 7*A*d*f) + 3*a
^2*(C*d*e - c*C*f - 4*B*d*f) + b^2*((3*c^2*C*e)/d - 42*A*d*e - (16*C*d*e^3)
/f^2 + 21*A*c*f + (8*c^3*C*f)/d^2 - B*(3*c*e - (24*d*e^2)/f + (12*c^2*f)/d)
))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(315*b^2*d*f) - (2*(7*b*d*f*(
b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*d*f) - (a*d*f - 4*b*(d*e + c*f))*(3*b*B
```

```
*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x
)^(3/2))/(105*b^2*d^2*f^3) + (2*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))
*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(21*b*d^2*f^2) + (2*C*(a +
b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) - (2*Sqrt[-(b*c) + a*
d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^
2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^
2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + 8*c^3*f
^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*d*e*f + 5*c^2*f^2)))
 + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3
+ 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) - B*(8*d^3*e^3
 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c -
a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) +
a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e
- c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b
^2*d*f^2*((c^2*C - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3
*c^2*d*e*f^2 - 8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c
*d*e*f - 4*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(
b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], (
(b*c - a*d)*f)/(d*(b*e - a*f))])/(315*b^4*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e
+ f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
```

```
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{2\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3}{2}b(bcCe+aCde+acCf-3Abdf)+\frac{3}{2}b(3bBdf-2aCdf-2bC(de+\ldots}{9b^2df}$$

$$= \frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{4\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3}{4}b(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-\frac{3}{4}b(7bdf(bcCe+a\ldots}{\sqrt{a+bx}}}{63b^2d^2f^2}$$

$$= -\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf\ldots}{105b^2d^2f^3} + \frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{8\int \frac{\sqrt{e+fx}\left(-\frac{3}{8}b(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-(bce+ade+3acf)(7\ldots}{\ldots}}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3+3a^2bd^2f^2(Cde-cCf-4Bdf)-3ab^2df^2((c^2C-7Ad^2)f+Bd(de-2cf))-b^3(C\ldots}{\ldots} - \frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf\ldots}{105b^2d^2f^3} + \frac{2(3bBdf-2aCdf-2bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{21bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} + \frac{16\int \frac{-\frac{3}{16}b(8a^4Cd^3f^3(de+cf)-a^3bd^2f^2(12Bdf(de+cf)+C(3d^2e^2+10cdef+3c^2f^2))-3a^2b^2df(C(d^3e^3+c^3f^3)-df(7Adf(de+cf\ldots}{\ldots}}{\ldots}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + Bd(de - 2cf)) - b^3(C(}{3}$$

$$- \frac{2(7bdf(bcCe + aCde + acCf - 3Abdf) - (adf - 4b(de + cf))(3bBdf - 2aCdf - 2bC(de + cf))}{105b^2d^2f^3}$$

$$+ \frac{2(3bBdf - 2aCdf - 2bC(de + cf))\sqrt{a + bx}(c + dx)^{3/2}(e + fx)^{3/2}}{21bd^2f^2}$$

$$+ \frac{2C(a + bx)^{3/2}(c + dx)^{3/2}(e + fx)^{3/2}}{9bdf}$$

$$- \frac{((be - af)(de - cf)(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + B}{}$$

$$- \frac{(16a^4Cd^4f^4 - 8a^3bd^3f^3(Cde + cCf + 3Bdf) + 3a^2b^2d^2f^2(df(5Bde + 5Bcf + 14Adf) - 2C(d^2e}{}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + Bd(de - 2cf)) - b^3(C(}{3}$$

$$- \frac{2(7bdf(bcCe + aCde + acCf - 3Abdf) - (adf - 4b(de + cf))(3bBdf - 2aCdf - 2bC(de + cf))}{105b^2d^2f^3}$$

$$+ \frac{2(3bBdf - 2aCdf - 2bC(de + cf))\sqrt{a + bx}(c + dx)^{3/2}(e + fx)^{3/2}}{21bd^2f^2}$$

$$+ \frac{2C(a + bx)^{3/2}(c + dx)^{3/2}(e + fx)^{3/2}}{9bdf}$$

$$- \frac{\Big((be - af)(de - cf)(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + B}{}$$

$$- \frac{\Big((16a^4Cd^4f^4 - 8a^3bd^3f^3(Cde + cCf + 3Bdf) + 3a^2b^2d^2f^2(df(5Bde + 5Bcf + 14Adf) - 2C(d}{}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + Bd(de - 2cf)) - b^3(C}{}$$

$$- \frac{2(7bdf(bcCe + aCde + acCf - 3Abdf) - (adf - 4b(de + cf))(3bBdf - 2aCdf - 2bC(de + cf}{105b^2d^2f^3}$$

$$+ \frac{2(3bBdf - 2aCdf - 2bC(de + cf))\sqrt{a + bx}(c + dx)^{3/2}(e + fx)^{3/2}}{21bd^2f^2}$$

$$+ \frac{2C(a + bx)^{3/2}(c + dx)^{3/2}(e + fx)^{3/2}}{9bdf}$$

$$- \frac{2\sqrt{-bc + ad}(16a^4Cd^4f^4 - 8a^3bd^3f^3(Cde + cCf + 3Bdf) + 3a^2b^2d^2f^2(df(5Bde + 5Bcf + 14A}{}$$

$$- \frac{\Big((be - af)(de - cf)\,(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f +}{}$$

$$= \frac{2(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C - 7Ad^2)f + Bd(de - 2cf)) - b^3(C}{}$$

$$- \frac{2(7bdf(bcCe + aCde + acCf - 3Abdf) - (adf - 4b(de + cf))(3bBdf - 2aCdf - 2bC(de + cf}{105b^2d^2f^3}$$

$$+ \frac{2(3bBdf - 2aCdf - 2bC(de + cf))\sqrt{a + bx}(c + dx)^{3/2}(e + fx)^{3/2}}{21bd^2f^2}$$

$$+ \frac{2C(a + bx)^{3/2}(c + dx)^{3/2}(e + fx)^{3/2}}{9bdf}$$

$$- \frac{2\sqrt{-bc + ad}(16a^4Cd^4f^4 - 8a^3bd^3f^3(Cde + cCf + 3Bdf) + 3a^2b^2d^2f^2(df(5Bde + 5Bcf + 14A}{}$$

$$- \frac{2\sqrt{-bc + ad}(be - af)(de - cf)\,(8a^3Cd^3f^3 + 3a^2bd^2f^2(Cde - cCf - 4Bdf) - 3ab^2df^2((c^2C -}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.21 (sec) , antiderivative size = 1422, normalized size of antiderivative = 1.20

$$\int \sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\left(A + Bx + Cx^2\right)\,dx$$

$$= \frac{2\Big(-b^2\sqrt{-a + \frac{bc}{d}}(16a^4Cd^4f^4 - 8a^3bd^3f^3(Cde + cCf + 3Bdf) + 3a^2b^2d^2f^2(df(5Bde + 5Bcf + 14Adf) -}{}$$

[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]

[Out] (2*(-(b^2*Sqrt[-a + (b*c)/d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*d^3*e^3 + 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e + c*f) + B*(-5*d^2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8*c^3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(8*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2*(c*C*f + 4*B*d*f + C*d*(e + 2*f*x)) + a*b^2*d*f*(3*d*f*(7*A*d*f + B*(2*d*e + 2*c*f + 3*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 5*f^2*x^2))) + b^3*(C*(8*c^3*f^3 - 3*c^2*d*f^2*(e + 2*f*x) + c*d^2*f*(-3*e^2 + 2*e*f*x + 5*f^2*x^2) + d^3*(8*e^3 - 6*e^2*f*x + 5*e*f^2*x^2 + 35*f^3*x^3)) + 3*d*f*(7*A*d*f*(c*f + d*(e + 3*f*x)) + B*(-4*c^2*f^2 + c*d*f*(2*e + 3*f*x) + d^2*(-4*e^2 + 3*e*f*x + 15*f^2*x^2))))) - I*(b*c - a*d)*f*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*d^3*e^3 + 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e + c*f) + B*(-5*d^2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8*c^3*f^3))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(d*e - c*f)*(8*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2*(C*d*e - c*C*f + 4*B*d*f) - 3*a*b^2*d^2*f*(C*d*e^2 + f*(-2*B*d*e + B*c*f - 7*A*d*f)) + b^3*(C*(8*d^3*e^3 + 3*c*d^2*e^2*f - 16*c^3*f^3) - 3*d*f*(-7*A*d*f*(d*e - 2*c*f) + B*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(315*b^5*Sqrt[-a + (b*c)/d]*d^4*f^4*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 2077, normalized size of antiderivative = 1.76

method	result	size
elliptic	Expression too large to display	2077
default	Expression too large to display	14766

[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(

```
2/9*C*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*
d*e)*C)/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+
b*c*e*x+a*c*e)^(1/2)+2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C
*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C
*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/
d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)+2/3*(A*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*C*a*c*e-2
/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*
(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*
f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*
f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3
*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*
x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c*e-2/5*(A*b*d*f+
B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+
7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*
e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*a*d*f+A*b*c*f+A*b
*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*C*a*c*e-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d
*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2
/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*
f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*
b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2
*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d
))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a
*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF
(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*a*c*f+A*a*d*
e+A*b*c*e+B*a*c*e-4/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f
+4*b*d*e)*C)/b/d/f*a*c*e-2/5*(A*b*d*f+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d
*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c
*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e)
)/b/d/f*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(A*a*d*f+A*b*c*f+A*b*d*e+B*a*c*
f+B*a*d*e+B*b*c*e+1/3*C*a*c*e-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a
*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b*d*f
+B*a*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e
+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d
*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/
d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f
+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(
e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-
c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1916, normalized size of antiderivative = 1.62

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/945*(3*(35*C*b^5*d^5*f^5*x^3 + 8*C*b^5*d^5*e^3*f^2 - 3*(C*b^5*c*d^4 + (C*
a*b^4 + 4*B*b^5)*d^5)*e^2*f^3 - (3*C*b^5*c^2*d^3 - 2*(C*a*b^4 + 3*B*b^5)*c*
d^4 + 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*d^5)*e*f^4 + (8*C*b^5*c^3*d^2 - 3
*(C*a*b^4 + 4*B*b^5)*c^2*d^3 - 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*c*d^4 +
(8*C*a^3*b^2 - 12*B*a^2*b^3 + 21*A*a*b^4)*d^5)*f^5 + 5*(C*b^5*d^5*e*f^4 + (
C*b^5*c*d^4 + (C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 - (6*C*b^5*d^5*e^2*f^3 - (2
*C*b^5*c*d^4 + (2*C*a*b^4 + 9*B*b^5)*d^5)*e*f^4 + (6*C*b^5*c^2*d^3 - (2*C*a
*b^4 + 9*B*b^5)*c*d^4 + 3*(2*C*a^2*b^3 - 3*B*a*b^4 - 21*A*b^5)*d^5)*f^5)*x)
*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (16*C*b^5*d^5*e^5 - 8*(2*C*b^5
*c*d^4 + (2*C*a*b^4 + 3*B*b^5)*d^5)*e^4*f - (5*C*b^5*c^2*d^3 - (20*C*a*b^4
+ 27*B*b^5)*c*d^4 + (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^5)*d^5)*e^3*f^2 - (5
*C*b^5*c^3*d^2 - 6*(C*a*b^4 + 2*B*b^5)*c^2*d^3 - 3*(2*C*a^2*b^3 - 14*B*a*b^
4 - 21*A*b^5)*c*d^4 + (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*d^5)*e^2*f^
3 - (16*C*b^5*c^4*d - (20*C*a*b^4 + 27*B*b^5)*c^3*d^2 - 3*(2*C*a^2*b^3 - 14
*B*a*b^4 - 21*A*b^5)*c^2*d^3 - 2*(10*C*a^3*b^2 - 21*B*a^2*b^3 + 126*A*a*b^4
)*c*d^4 + (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*e*f^4 + (16*C*b^5
*c^5 - 8*(2*C*a*b^4 + 3*B*b^5)*c^4*d - (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^5
)*c^3*d^2 - (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*c^2*d^3 - (16*C*a^4*b
 - 27*B*a^3*b^2 + 63*A*a^2*b^3)*c*d^4 + 2*(8*C*a^5 - 12*B*a^4*b + 21*A*a^3*
b^2)*d^5)*f^5)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d
+ a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2
*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d
^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*
d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) +
 3*(16*C*b^5*d^5*e^4*f - 8*(C*b^5*c*d^4 + (C*a*b^4 + 3*B*b^5)*d^5)*e^3*f^2
- 3*(2*C*b^5*c^2*d^3 - (2*C*a*b^4 + 5*B*b^5)*c*d^4 + (2*C*a^2*b^3 - 5*B*a*b
^4 - 14*A*b^5)*d^5)*e^2*f^3 - (8*C*b^5*c^3*d^2 - 3*(2*C*a*b^4 + 5*B*b^5)*c^
2*d^3 - 6*(C*a^2*b^3 - 3*B*a*b^4 - 7*A*b^5)*c*d^4 + (8*C*a^3*b^2 - 15*B*a^2
*b^3 + 42*A*a*b^4)*d^5)*e*f^4 + (16*C*b^5*c^4*d - 8*(C*a*b^4 + 3*B*b^5)*c^3
*d^2 - 3*(2*C*a^2*b^3 - 5*B*a*b^4 - 14*A*b^5)*c^2*d^3 - (8*C*a^3*b^2 - 15*B
*a^2*b^3 + 42*A*a*b^4)*c*d^4 + 2*(8*C*a^4*b - 12*B*a^3*b^2 + 21*A*a^2*b^3)*
d^5)*f^5)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2
)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*
e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*
```

```
b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)
/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*
e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^
3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*
d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(
b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^5*d^5*f
^5)
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Giac [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)\,dx$$

```
[In] int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
[Out] int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2), x)
```

3.62 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 774

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = -\frac{2(5bdf(3aC(de+cf)+b(cCe-7Adf))-(2bde-bcf+4adf)(6aCdf-b(7Bdf-4C(de+cf))))\sqrt{a}}{105b^3d^2f^2} - \frac{2(6aCdf-b(7Bdf-4C(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} - \frac{2\sqrt{-bc+ad}(3bdf(5bcf(3aC(de+cf)+b(cCe-7Adf))-(bce+ade+3acf)(6aCdf-b(7Bdf-4C}{} - \frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf-\right.}{105b^4d^{5/2}f^3\sqrt{c+d}}$$

```
[Out] 2/7*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)*(b*x+a)^(1/2)/b/d/f-2/35*(6*a*C*d*f-b*(7*
B*d*f-4*C*(c*f+d*e)))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d/f^2-2
/105*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d*f-b*c*f+2*b*d*e)*
(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^
(1/2)/b^3/d^2/f^2-2/105*(3*b*d*f*(5*b*c*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*
e))-(3*a*c*f+a*d*e+b*c*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e))))+2*(1/2*b*d
*e-(a*d+b*c)*f)*(5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e))-(4*a*d*f-b*c*
f+2*b*d*e)*(6*a*C*d*f-b*(7*B*d*f-4*C*(c*f+d*e)))))*EllipticE(d^(1/2)*(b*x+a
)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*
(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(5/2)/f^3/(d*x+c)^(1/2)/(b
```

```
*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(-c*f+d*e)*(24*a^2*C*d^2*f^2+a*
b*d*f*(-28*B*d*f-5*C*c*f+13*C*d*e)-b^2*(7*d*f*(-5*A*d*f-B*c*f+2*B*d*e)-C*(-
4*c^2*f^2-c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1
/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c
))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(5/2)/f^3/(d*x+c)^(1/2)/(f*x+e)
^(1/2)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 769, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx =$$

$$-\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-(b^2(7d\right.}{105b^4d^{5/2}f^3\sqrt{c+}}$$

$$-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5bcf(3aC(cf+de)+b(cCe-7Adf))+(3acf+ade+bce)(-6aCdf\right.}{}$$

$$-\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(5(3aC(cf+de)+b(cCe-7Adf))+\frac{(4adf-bcf+2bde)(-6aCdf+7bBdf-4bC(cf+de))}{bdf}\right)}{105b^2df}$$

$$+\frac{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}(-6aCdf+7bBdf-4bC(cf+de))}{35b^2df^2}$$

$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*f - 4*b*C*(d*e + c*f
)))/(b*d*f) + 5*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f)))*Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d*f) + (2*(7*b*B*d*f - 6*a*C*d*f - 4*b*
C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*d*f^2)
+ (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - (2*Sqrt[-
(b*c) + a*d]*(3*b*d*f*((b*c*e + a*d*e + 3*a*c*f)*(7*b*B*d*f - 6*a*C*d*f - 4
*b*C*(d*e + c*f)) + 5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))) + 2*
((b*d*e)/2 - (b*c + a*d)*f)*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C
*d*f - 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f
))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(10
5*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[
-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e
```

```
 - 5*c*C*f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*
e^2 - c*d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f
*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*
d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*d^(5/2)*f^3*Sqrt[c + d*x]*S
qrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} + \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{1}{2}b(3aC(de+cf)+b(cCe-7Adf))+\frac{1}{2}b(7bBdf-6aCdf-4bC(de+cf))x\right)}{\sqrt{a+bx}}\,dx}{7b^2df}$$

$$= \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} + \frac{4\int \frac{\sqrt{e+fx}\left(-\frac{1}{4}b((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-\frac{1}{4}b((2bde-bcf+4adf)(7bB\right.}{\sqrt{a+bx}\sqrt{c+dx}}}{35b^3df^2}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)+b(cCe-7Ad}{105b^3d^2f^2}$$
$$+\frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2}$$
$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$
$$+\frac{8\int\frac{-\frac{1}{8}b(3bde((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-(bce+ade+acf)((2bde-bcf-}{}}{}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)+b(cCe-7Ad}{105b^3d^2f^2}$$
$$+\frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2}$$
$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$
$$-\frac{((be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf-5A\right.}{105b^3d^2f^3}$$
$$-\frac{\left(3bdf((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7\right.}{}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)+b(cCe-7Ad}{105b^3d^2f^2}$$
$$+\frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2}$$
$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$
$$-\frac{\left((be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf-5A\right.\right.}{105b^3d^2f^3\sqrt{c+dx}}$$
$$-\frac{\left(\left(3bdf((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-\right.\right.}{}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)+b(cCe-7Adf)}{105b^3d^2f^2} + \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} - \frac{2\sqrt{-bc+ad}\big(3bdf((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+}{} - \frac{\Big((be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf-5A\right.}{105b^3d^2f^3\sqrt{c+d}}$$

$$= -\frac{2((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+cf)+b(cCe-7Adf)}{105b^3d^2f^2} + \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{35b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} - \frac{2\sqrt{-bc+ad}\big(3bdf((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+}{} - \frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-\right.}{105b^4d^{5/2}f^3\sqrt{c+}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.73 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = -\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(48a^3Cd^3f^3-8a^2bd^2f^2(7Bdf+2C(de+cf))+ab^2df(7df(3Bde+3Bcf+10Adf)+C\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f +
2*C*(d*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*
d^2*e^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*
c^2*d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*
e*f + c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x
)*(c + d*x)*(e + f*x)*(-24*a^2*C*d^2*f^2 + a*b*d*f*(28*B*d*f + C*(5*d*e + 5
*c*f + 18*d*f*x)) + b^2*(-7*d*f*(B*c*f + 5*A*d*f + B*d*(e + 3*f*x)) + C*(4*
c^2*f^2 - c*d*f*(2*e + 3*f*x) + d^2*(4*e^2 - 3*e*f*x - 15*f^2*x^2)))) + I*(
b*c - a*d)*f*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f + 2*C*(d*e + c*f)
) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*d^2*e^2 + 8*c*d
*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8
*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*e*f + c^2*f^2))
))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*
(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e -
 a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(24*a^2*C*d^2*f^2
+ a*b*d*f*(-5*C*d*e + 13*c*C*f - 28*B*d*f) + b^2*(7*d*f*(B*d*e - 2*B*c*f +
5*A*d*f) - C*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(1
05*b^5*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]
)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.56

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	9543

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/7*C/b*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/
d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)+2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*
e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f
+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a
*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*c*e-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*
f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a
*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3
*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*
c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/
```

```
d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e
*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e
/f+a/b))^(1/2))+2*(A*c*f+A*d*e+B*c*e-4/7*C/b*a*c*e-2/5*(B*d*f+C*c*f+C*d*e-2
/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3
*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*d*
f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b
*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a
/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f
*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE((
(x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e
/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 - 4*C*b^4*d^4*e^2*f^2 + (2*C*b^4*c*d^3 - (5*
C*a*b^3 - 7*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 + (5*C*a*b^3 - 7*B*b^4)*c*
d^3 - (24*C*a^2*b^2 - 28*B*a*b^3 + 35*A*b^4)*d^4)*f^4 + 3*(C*b^4*d^4*e*f^3
+ (C*b^4*c*d^3 - (6*C*a*b^3 - 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(f*x + e) - (8*C*b^4*d^4*e^4 - (9*C*b^4*c*d^3 - (5*C*a*b^3 - 14*B*
b^4)*d^4)*e^3*f - (4*C*b^4*c^2*d^2 + 7*(C*a*b^3 - 3*B*b^4)*c*d^3 - (10*C*a^
2*b^2 - 14*B*a*b^3 + 35*A*b^4)*d^4)*e^2*f^2 - (9*C*b^4*c^3*d + 7*(C*a*b^3 -
 3*B*b^4)*c^2*d^2 + 14*(3*C*a^2*b^2 - 4*B*a*b^3 + 10*A*b^4)*c*d^3 - (40*C*a
^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 + (8*C*b^4*c^4 + (5*C*a*b^3 -
14*B*b^4)*c^3*d + (10*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*c^2*d^2 + (40*C*a^
3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*c*d^3 - 2*(24*C*a^4 - 28*B*a^3*b + 35*A*a^
2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*
d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*
(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c
*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^
3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))
 - 3*(8*C*b^4*d^4*e^3*f - (5*C*b^4*c*d^3 - (9*C*a*b^3 - 14*B*b^4)*d^4)*e^2*
f^2 - (5*C*b^4*c^2*d^2 + 2*(4*C*a*b^3 - 7*B*b^4)*c*d^3 - (16*C*a^2*b^2 - 21
*B*a*b^3 + 35*A*b^4)*d^4)*e*f^3 + (8*C*b^4*c^3*d + (9*C*a*b^3 - 14*B*b^4)*c
^2*d^2 + (16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*c*d^3 - 2*(24*C*a^3*b - 28*
B*a^2*b^2 + 35*A*a*b^3)*d^4)*f^4)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*
```

```
e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2
*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*
d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^
2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
- 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*
f)/(b*d*f))))/(b^5*d^4*f^4)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/sqrt(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a+b\,x}}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2),x)
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2), x)
```

3.63 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 706

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)}{15b^3df(be-af)}$$
$$+\frac{2(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)}$$
$$-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$+\frac{2\sqrt{-bc+ad}(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-cdef+}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2\sqrt{-bc+ad}(de-cf)(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf)))\sqrt{}}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)+2/5*(6*a^2*C*d*f+b^2*(5*A*d*f+C*c*e)-a*b*(5*B*d*f+C*c*f+C*d*e
))*(f*x+e)^(3/2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+2/
15*(24*a^2*C*d*f^2-a*b*f*(20*B*d*f+C*c*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*
e*(-c*f+2*d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/f/(-a*f+b*
e)+2/15*(48*a^2*C*d^2*f^2-8*a*b*d*f*(5*B*d*f+C*c*f+C*d*e)+b^2*(5*d*f*(6*A*d
*f+B*c*f+B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b*x+a)^(
1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*
(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f
*x+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(24*a^2*C*d*f^2-a*b*f*(20*B*d*f+C*c
*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*e*(-c*f+2*d*e)))*EllipticF(d^(1/2)*(b*
x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/
```

```
2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/d^(3/2)/f^
2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(48a^2Cd^2f^2-8abdf(5Bdf+cCf+C\cdots}{\cdots}$$

$$-\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(24a^2Cdf^2-abf(20Bdf+cCf+7Cde)+b^2(5df(3Af+Be)-Ce(\cdots}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

$$+\frac{2\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\left(6a^2Cdf-ab(5Bdf+cCf+Cde)+b^2(5Adf+cCe)\right)}{5b^2f(bc-ad)(be-af)}$$

$$+\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(24a^2Cdf^2-abf(20Bdf+cCf+7Cde)+b^2(5df(3Af+Be)-Ce(2de-cf)))}{15b^3df(be-af)}$$

$$-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x]
```

```
[Out] (2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e +
 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(1
5*b^3*d*f*(b*e - a*f)) + (2*(6*a^2*C*d*f + b^2*(c*C*e + 5*A*d*f) - a*b*(C*d
*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b^2*
(b*c - a*d)*f*(b*e - a*f)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) +
a*d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(
B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/
Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*
Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e
 - c*f)*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(
B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c
) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^4*d^(3/2)*f^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
```

```
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
```

```
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(cCe+3Bde+3Bcf-Adf)+b^2(Bce+2A(de+cf))}{2b}+\frac{1}{2}\left(-\frac{6a^2Cdf}{b}-b(cCe+5Adf)+a(Cde+cCf+5Bdf)\right)x\right)}{\sqrt{a+bx}}\,dx}{(bc-ad)(be-af)}$$

$$= \frac{2(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)} - \frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{4\int \frac{\sqrt{e+fx}\left(-\frac{(bc-ad)\left(6a^2Cf(de+3cf)-b^2\left(cCe^2-5Adef-5cf(Be+2Af)\right)-ab(5Bf(de+3cf)+Ce(de+7cf))\right)}{4b}-\frac{(bc-ad)\left(24a^2Cdf^2-abf(7Cde\cdots\right.}{\cdots}\right.}{\sqrt{a+bx}\sqrt{c+dx}}}{5b(bc-ad)f(be-af)}$$

$$= \frac{2(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf)))\sqrt{a+bx}\sqrt{c+dx}\cdots}{15b^3df(be-af)} + \frac{2(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)} - \frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{8\int \frac{-\frac{(bc-ad)(be-af)\left(24a^2Cdf(de+cf)-ab\left(20Bdf(de+cf)+C\left(d^2e^2+14cdef+c^2f^2\right)\right)-b^2\left(c^2Cef-15Ad^2ef+cd\left(Ce^2-5f(2Be+3Af)\right)\right)\right)}{8b}-\frac{(bc\cdots}{\cdots}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{15b^2d(bc-ad)f(be-a\cdots}$$

$$= \frac{2(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{a+bx}\sqrt{c+}}{15b^3df(be-af)}$$

$$+ \frac{2(6a^2Cdf + b^2(cCe + 5Adf) - ab(Cde + cCf + 5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$- \frac{((de-cf)(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf))))\int}{15b^3df^2}$$

$$+ \frac{(48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e^2 - cdef +}{15b^3df^2}$$

$$= \frac{2(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{a+bx}\sqrt{c+}}{15b^3df(be-af)}$$

$$+ \frac{2(6a^2Cdf + b^2(cCe + 5Adf) - ab(Cde + cCf + 5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$- \frac{\left((de-cf)(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{}\right.}{15b^3df^2\sqrt{c+dx}}$$

$$+ \frac{\left((48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e^2 - cdef +\right.}{15b^3df^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= \frac{2(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{a+bx}\sqrt{c+}}{15b^3df(be-af)}$$

$$+ \frac{2(6a^2Cdf + b^2(cCe + 5Adf) - ab(Cde + cCf + 5Bdf))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5b^2(bc-ad)f(be-af)}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$+ \frac{2\sqrt{-bc+ad}(48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$- \frac{\left((de-cf)(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{}\right.}{15b^3df^2\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de - cf)))\sqrt{a + bx}\sqrt{c + dx}}{15b^3df(be - af)}$$

$$+ \frac{2(6a^2Cdf + b^2(cCe + 5Adf) - ab(Cde + cCf + 5Bdf))\sqrt{a + bx}\sqrt{c + dx}(e + fx)^{3/2}}{5b^2(bc - ad)f(be - af)}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c + dx)^{3/2}(e + fx)^{3/2}}{b(bc - ad)(be - af)\sqrt{a + bx}}$$

$$+ \frac{2\sqrt{-bc + ad}(48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e^2}{15b^4d^{3/2}f^2\sqrt{c + dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$- \frac{2\sqrt{-bc + ad}(de - cf)(24a^2Cdf^2 - abf(7Cde + cCf + 20Bdf) + b^2(5df(Be + 3Af) - Ce(2de}{15b^4d^{3/2}f^2\sqrt{c + dx}\sqrt{e + fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.90 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c + dx}\sqrt{e + fx}(A + Bx + Cx^2)}{(a + bx)^{3/2}}\, dx =$$

$$-\frac{2\left(-b^2\sqrt{-a + \frac{bc}{d}}(48a^2Cd^2f^2 - 8abdf(Cde + cCf + 5Bdf) + b^2(5df(Bde + Bcf + 6Adf) - 2C(d^2e^2 - cde\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x
]
```

```
[Out] (-2*(-(b^2*Sqrt[-a + (b*c)/d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f
+ 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f
+ c^2*f^2)))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e
 + f*x)*(15*(A*b^2 + a*(-(b*B) + a*C))*d*f - (-9*a*C*d*f + b*(C*d*e + c*C*f
 + 5*B*d*f))*(a + b*x) - 3*b*C*d*f*x*(a + b*x)) - I*(b*c - a*d)*f*(48*a^2*C
*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f
+ 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c
 + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSi
nh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*
b*f*(d*e - c*f)*(24*a^2*C*d^2*f - a*b*d*(C*d*e + 7*c*C*f + 20*B*d*f) + b^2*
(-2*c^2*C*f + 15*A*d^2*f + c*d*(C*e + 5*B*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
 d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh
[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*
b^5*Sqrt[-a + (b*c)/d]*d^2*f^2*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1163, normalized size of antiderivative = 1.65

method	result	size
elliptic	Expression too large to display	1163
default	Expression too large to display	5787

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^4/((x+a/b)*(b*d*
f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/5*C/b^2*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*
x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2/3*(1/b^2*(B*b*d*f-C*a*
d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+
a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(-(A*a
*b^2*d*f-A*b^3*c*f-A*b^3*d*e-B*a^2*b*d*f+B*a*b^2*c*f+B*a*b^2*d*e-B*b^3*c*e+
C*a^3*d*f-C*a^2*b*c*f-C*a^2*b*d*e+C*a*b^2*c*e)/b^4+(A*b^2-B*a*b+C*a^2)/b^4*
(a*d*f-b*c*f-b*d*e)+(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^4-2/5*C/b^2*a*c*e-2
/3*(1/b^2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b*c*f+2*b*
d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^
(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*
f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((
x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(1/b^3*(A*b^2*d*f-
B*a*b*d*f+B*b^2*c*f+B*b^2*d*e+C*a^2*d*f-C*a*b*c*f-C*a*b*d*e+C*b^2*c*e)+(A*b
^2-B*a*b+C*a^2)/b^3*d*f-2/5*C/b^2*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(1/b^
2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b*c*f+2*b*d*e))/b/
d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f
+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(
e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-
c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 1463, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^4*d^3*f^3*x^2 + C*a*b^3*d^3*e*f^2 + (C*a*b^3*c*d^2 - (24*C*a
^2*b^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3 + (C*b^4*d^3*e*f^2 + (C*b^4*c*d^2
- (6*C*a*b^3 - 5*B*b^4)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x +
 e) + (2*C*a*b^3*d^3*e^3 - (3*C*a*b^3*c*d^2 - (7*C*a^2*b^2 - 5*B*a*b^3)*d^3
)*e^2*f - (3*C*a*b^3*c^2*d + 4*(7*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - (32*C*a^3*
b - 25*B*a^2*b^2 + 15*A*a*b^3)*d^3)*e*f^2 + (2*C*a*b^3*c^3 + (7*C*a^2*b^2 -
 5*B*a*b^3)*c^2*d + (32*C*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*c*d^2 - 2*(24*
C*a^4 - 20*B*a^3*b + 15*A*a^2*b^2)*d^3)*f^3 + (2*C*b^4*d^3*e^3 - (3*C*b^4*c
*d^2 - (7*C*a*b^3 - 5*B*b^4)*d^3)*e^2*f - (3*C*b^4*c^2*d + 4*(7*C*a*b^3 - 5
*B*b^4)*c*d^2 - (32*C*a^2*b^2 - 25*B*a*b^3 + 15*A*b^4)*d^3)*e*f^2 + (2*C*b^
4*c^3 + (7*C*a*b^3 - 5*B*b^4)*c^2*d + (32*C*a^2*b^2 - 25*B*a*b^3 + 15*A*b^4
)*c*d^2 - 2*(24*C*a^3*b - 20*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3)*x)*sqrt(b*d*
f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^
2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*
d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3),
 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*a*b^3*d^3*e^2*f
- (2*C*a*b^3*c*d^2 - (8*C*a^2*b^2 - 5*B*a*b^3)*d^3)*e*f^2 + (2*C*a*b^3*c^2*
d + (8*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - 2*(24*C*a^3*b - 20*B*a^2*b^2 + 15*A*a
*b^3)*d^3)*f^3 + (2*C*b^4*d^3*e^2*f - (2*C*b^4*c*d^2 - (8*C*a*b^3 - 5*B*b^4
)*d^3)*e*f^2 + (2*C*b^4*c^2*d + (8*C*a*b^3 - 5*B*b^4)*c*d^2 - 2*(24*C*a^2*b
^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3)*x)*sqrt(b*d*f)*weierstrassZeta(4/3*(b
^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(
b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b
^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d -
3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2
*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^
2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3
*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c
+ a*d)*f)/(b*d*f))))/(b^6*d^3*f^3*x + a*b^5*d^3*f^3)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(3/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(3/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2),x)
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2), x)
```

3.64 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 687

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \frac{2(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))\sqrt{}}{3b^3(bc-ad)(be-af)}$$

$$-\frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}}-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+\frac{2(16a^3Cd^2f^2-8a^2bdf(Bdf+2C(de+cf))-b^3(c^2Cef+Ad^2ef+cd(Ce^2+6Bef+Af^2))+ab^2(df(7Ba}{3b^4\sqrt{d}\sqrt{-bc+ad}f(be-af}$$

$$+\frac{2(de-cf)(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\text{arc}\right.}{3b^4\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)-2*(B*b-2*C*a)*(f*x+e)^(3/2)*(d*x+c)^(1/2)/b^2/(-a*f+b*e)/(b
*x+a)^(1/2)+2/3*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*e)-a*b*(4*B*d*f+7*C*c*f
+C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f+b*e
)+2/3*(16*a^3*C*d^2*f^2-8*a^2*b*d*f*(B*d*f+2*C*(c*f+d*e))-b^3*(c^2*C*e*f+A*
d^2*e*f+c*d*(A*f^2+6*B*e*f+C*e^2))+a*b^2*(d*f*(2*A*d*f+7*B*c*f+7*B*d*e)+C*(
c^2*f^2+16*c*d*e*f+d^2*e^2)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/
2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^
(1/2)/b^4/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a
*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*e)-a*b*(4
*B*d*f+7*C*c*f+C*d*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*
f+b*e))^(1/2)/b^4/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1628, 155, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(8a^2Cdf-ab(4Bdf+7cCf+Cde}{3b^4\sqrt{d}f\sqrt{c+dx}}$$
$$+\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe))}{3b^3(bc-ad)(be-af)}$$
$$+\frac{2\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(16a^3Cd^2f^2-8a^2bdf(Bdf+2C(cf+de))+ab^2(df(2Adf+7Bcf+7Bde)+C(c^2f^2}{3b^4\sqrt{d}f\sqrt{c+dx}\sqrt{ad-bc}}$$
$$-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}(e+fx)^{3/2}(bB-2aC)}{b^2\sqrt{a+bx}(be-af)}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*
B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^3*(b*c - a*d)*(b*e
- a*f)) - (2*(b*B - 2*a*C)*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b^2*(b*e - a*f)*
Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2)
)/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/2)) + (2*(16*a^3*C*d^2*f^2 - 8*
a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e
^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*
e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]
*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[
c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(8*a^2*C*d*f + b
^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
3*b^4*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3\left(b^2Bce+a^2C(de+cf)-ab(cCe+Bde+Bcf-Adf)\right)}{2b}-\frac{3}{2}\left(\frac{2a^2Cdf}{b}+b(cCe+Adf)-a(Cde+cCf+Bdf)\right)x\right)}{(a+bx)^{3/2}}\,dx}{3(bc-ad)(be-af)}$$

$$= -\frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{4\int \frac{\sqrt{e+fx}\left(-\frac{3(be-af)\left(2a^2Cd(de+3cf)+b^2c(cCe+Bde+2Bcf+Adf)-ab\left(Bd^2e+5c^2Cf+3cd(Ce+Bf)\right)\right)}{4b}-\frac{3d(be-af)\left(8a^2Cdf+b^2(cCe+3B\right.}{}\right.}{\sqrt{a+bx}\sqrt{c+dx}}}{3b(bc-ad)(be-af)^2}$$

$$= \frac{2\left(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^3(bc-ad)(be-af)} - \frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{8\int \frac{\frac{3d(be-af)\left(8a^3Cdf(de+cf)-b^3ce(2cCe+3Bde+3Bcf+2Adf)+ab^2\left(d^2e(3Be+Af)+3c^2f(3Ce+Bf)+cd\left(9Ce^2+8Bef+Af^2\right)\right)-a^2b\left(4Bdf(d\right.\right.}{8b}}{}}{}$$

$$= \frac{2(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^3(bc-ad)(be-af)}$$
$$- \frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$- \frac{((de-cf)(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf)))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3b^3(bc-ad)f}$$
$$- \frac{(16a^3Cd^2f^2 - 8a^2bdf(Bdf + 2C(de+cf)) - b^3(c^2Cef + Ad^2ef + cd(Ce^2 + 6Bef + Af^2)) + a \ldots}{3b^3(bc-ad)f(be-af \ldots}$$

$$= \frac{2(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^3(bc-ad)(be-af)}$$
$$- \frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$- \frac{\left((de-cf)(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{\ldots}{\sqrt{a+bx}\sqrt{\ldots}}}{3b^3(bc-ad)f\sqrt{c+dx}}$$
$$- \frac{\left((16a^3Cd^2f^2 - 8a^2bdf(Bdf + 2C(de+cf)) - b^3(c^2Cef + Ad^2ef + cd(Ce^2 + 6Bef + Af^2)) + \ldots\right.}{3b^3(bc-ad)f(be \ldots}$$

$$= \frac{2(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^3(bc-ad)(be-af)}$$
$$- \frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$+ \frac{2(16a^3Cd^2f^2 - 8a^2bdf(Bdf + 2C(de+cf)) - b^3(c^2Cef + Ad^2ef + cd(Ce^2 + 6Bef + Af^2)) + \ldots}{3b^4\sqrt{d}\sqrt{-bc+ad \ldots}}$$
$$- \frac{\left((de-cf)(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\right)\int \ldots}{3b^3(bc-ad)f\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^3(bc-ad)(be-af)}$$

$$- \frac{2(bB-2aC)\sqrt{c+dx}(e+fx)^{3/2}}{b^2(be-af)\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+ \frac{2(16a^3Cd^2f^2 - 8a^2bdf(Bdf + 2C(de+cf)) - b^3(c^2Cef + Ad^2ef + cd(Ce^2 + 6Bef + Af^2)) +}{3b^4\sqrt{d}\sqrt{-bc+a}}$$

$$+ \frac{2(de-cf)(8a^2Cdf + b^2(cCe + 3Bcf + Adf) - ab(Cde + 7cCf + 4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}F}{3b^4\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.99 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}df(c+dx)(e+fx)\left((Ab^2 + a(-bB+aC))(bc-ad)(be-af) + (-8a^3Cdf + b^3(3Bce + \right.\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a
*C))*(b*c - a*d)*(b*e - a*f) + (-8*a^3*C*d*f + b^3*(3*B*c*e + A*d*e + A*c*f
) - 2*a*b^2*(3*c*C*e + 2*B*d*e + 2*B*c*f + A*d*f) + a^2*b*(5*B*d*f + 7*C*(d
*e + c*f)))*(a + b*x) - C*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (a + b*x)*
(b^2*Sqrt[-a + (b*c)/d]*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d*f + 2*C*(d*e +
 c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f + A*f^2)) + a*b^
2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e*f + c^2*f^2)))
*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d
*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f +
 A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e
*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(8*a^
2*C*d*f + b^2*(c*C*e + 3*B*d*e + A*d*f) - a*b*(7*C*d*e + c*C*f + 4*B*d*f))*
(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a
+ b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*
d*f)/(b*c*f - a*d*f)])))/(3*b^5*Sqrt[-a + (b*c)/d]*d*(b*c - a*d)*f*(b*e - a
*f)*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. 2(627) = 1254.

Time = 5.45 (sec) , antiderivative size = 1378, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1378
default	Expression too large to display	15769

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2/3*(A*b^2-B*a*b+C*a^2)/b^5*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*
c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d
*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7*C*a^
2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*
c*e))^(1/2)+2/3*C/b^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*
e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d*f-2*B*a*b*d*f+B*b^2*c*f+B*b^2*d*e+3*C*
a^2*d*f-2*C*a*b*c*f-2*C*a*b*d*e+C*b^2*c*e)/b^4-1/3*(A*b^2-B*a*b+C*a^2)/b^4*
d*f-1/3/b^4*(a*d*f-b*c*f-b*d*e)*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-5*B*a^2*
b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7*C*a^2*b*c*f-7*C
*a^2*b*d*e+6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)-1/3*(b*c*f+b*d*
e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e
-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7*C*a^2*
b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)-2/3*C/b^3*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e
))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/
(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+
b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+
a/b))^(1/2))+2*(1/b^3*(B*b*d*f-2*C*a*d*f+C*b*c*f+C*b*d*e)-1/3/b^3*d*f*(2*A*
a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B
*b^3*c*e+8*C*a^3*d*f-7*C*a^2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/(a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)-2/3*C/b^3*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e
/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f
*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-
e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))
-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 2588, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*(((6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d^2 - (7*C*a^3*b^3 - 3*B*a^2*b
^4)*d^3)*e*f^2 - ((7*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 - (8*C*a^4*b^2 - 4*B*a^
3*b^3 + A*a^2*b^4)*d^3)*f^3 + ((C*b^6*c*d^2 - C*a*b^5*d^3)*e*f^2 - (C*a*b^5
*c*d^2 - C*a^2*b^4*d^3)*f^3)*x^2 + (((8*C*a*b^5 - 3*B*b^6)*c*d^2 - (9*C*a^2
*b^4 - 4*B*a*b^5 + A*b^6)*d^3)*e*f^2 - ((9*C*a^2*b^4 - 4*B*a*b^5 + A*b^6)*c
*d^2 - (10*C*a^3*b^3 - 5*B*a^2*b^4 + 2*A*a*b^5)*d^3)*f^3)*x)*sqrt(b*x + a)*
sqrt(d*x + c)*sqrt(f*x + e) - ((C*a^2*b^4*c*d^2 - C*a^3*b^3*d^3)*e^3 - (4*C
*a^2*b^4*c^2*d - (11*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 + (6*C*a^4*b^2 - 2*B*a^
3*b^3 - A*a^2*b^4)*d^3)*e^2*f + (C*a^2*b^4*c^3 + (11*C*a^3*b^3 - 3*B*a^2*b^
4)*c^2*d - 2*(19*C*a^4*b^2 - 8*B*a^3*b^3 + 2*A*a^2*b^4)*c*d^2 + (24*C*a^5*b
 - 11*B*a^4*b^2 + 2*A*a^3*b^3)*d^3)*e*f^2 - (C*a^3*b^3*c^3 + (6*C*a^4*b^2 -
 2*B*a^3*b^3 - A*a^2*b^4)*c^2*d - (24*C*a^5*b - 11*B*a^4*b^2 + 2*A*a^3*b^3)
*c*d^2 + 2*(8*C*a^6 - 4*B*a^5*b + A*a^4*b^2)*d^3)*f^3 + ((C*b^6*c*d^2 - C*a
*b^5*d^3)*e^3 - (4*C*b^6*c^2*d - (11*C*a*b^5 - 3*B*b^6)*c*d^2 + (6*C*a^2*b^
4 - 2*B*a*b^5 - A*b^6)*d^3)*e^2*f + (C*b^6*c^3 + (11*C*a*b^5 - 3*B*b^6)*c^2
*d - 2*(19*C*a^2*b^4 - 8*B*a*b^5 + 2*A*b^6)*c*d^2 + (24*C*a^3*b^3 - 11*B*a^
2*b^4 + 2*A*a*b^5)*d^3)*e*f^2 - (C*a*b^5*c^3 + (6*C*a^2*b^4 - 2*B*a*b^5 - A
*b^6)*c^2*d - (24*C*a^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*c*d^2 + 2*(8*C*a^4*
b^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d^3)*f^3)*x^2 + 2*((C*a*b^5*c*d^2 - C*a^2*b^
4*d^3)*e^3 - (4*C*a*b^5*c^2*d - (11*C*a^2*b^4 - 3*B*a*b^5)*c*d^2 + (6*C*a^3
*b^3 - 2*B*a^2*b^4 - A*a*b^5)*d^3)*e^2*f + (C*a*b^5*c^3 + (11*C*a^2*b^4 - 3
*B*a*b^5)*c^2*d - 2*(19*C*a^3*b^3 - 8*B*a^2*b^4 + 2*A*a*b^5)*c*d^2 + (24*C*
a^4*b^2 - 11*B*a^3*b^3 + 2*A*a^2*b^4)*d^3)*e*f^2 - (C*a^2*b^4*c^3 + (6*C*a^
3*b^3 - 2*B*a^2*b^4 - A*a*b^5)*c^2*d - (24*C*a^4*b^2 - 11*B*a^3*b^3 + 2*A*a
^2*b^4)*c*d^2 + 2*(8*C*a^5*b - 4*B*a^4*b^2 + A*a^3*b^3)*d^3)*f^3)*x)*sqrt(b
*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2
*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3
*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^
3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*((C*a^2*b^4*c*d^2
- C*a^3*b^3*d^3)*e^2*f + (C*a^2*b^4*c^2*d - 2*(8*C*a^3*b^3 - 3*B*a^2*b^4)*c
*d^2 + (16*C*a^4*b^2 - 7*B*a^3*b^3 + A*a^2*b^4)*d^3)*e*f^2 - (C*a^3*b^3*c^2
*d - (16*C*a^4*b^2 - 7*B*a^3*b^3 + A*a^2*b^4)*c*d^2 + 2*(8*C*a^5*b - 4*B*a^
4*b^2 + A*a^3*b^3)*d^3)*f^3 + ((C*b^6*c*d^2 - C*a*b^5*d^3)*e^2*f + (C*b^6*c
```

```
^2*d - 2*(8*C*a*b^5 - 3*B*b^6)*c*d^2 + (16*C*a^2*b^4 - 7*B*a*b^5 + A*b^6)*d
^3)*e*f^2 - (C*a*b^5*c^2*d - (16*C*a^2*b^4 - 7*B*a*b^5 + A*b^6)*c*d^2 + 2*(
8*C*a^3*b^3 - 4*B*a^2*b^4 + A*a*b^5)*d^3)*f^3)*x^2 + 2*((C*a*b^5*c*d^2 - C*
a^2*b^4*d^3)*e^2*f + (C*a*b^5*c^2*d - 2*(8*C*a^2*b^4 - 3*B*a*b^5)*c*d^2 + (
16*C*a^3*b^3 - 7*B*a^2*b^4 + A*a*b^5)*d^3)*e*f^2 - (C*a^2*b^4*c^2*d - (16*C
*a^3*b^3 - 7*B*a^2*b^4 + A*a*b^5)*c*d^2 + 2*(8*C*a^4*b^2 - 4*B*a^3*b^3 + A*
a^2*b^4)*d^3)*f^3)*x)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c
*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27
*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*
c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a
^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d
 + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(
2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*
d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3
*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)))
)/((a^2*b^7*c*d^2 - a^3*b^6*d^3)*e*f^2 - (a^3*b^6*c*d^2 - a^4*b^5*d^3)*f^3
+ ((b^9*c*d^2 - a*b^8*d^3)*e*f^2 - (a*b^8*c*d^2 - a^2*b^7*d^3)*f^3)*x^2 + 2
*((a*b^8*c*d^2 - a^2*b^7*d^3)*e*f^2 - (a^2*b^7*c*d^2 - a^3*b^6*d^3)*f^3)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(5/2),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(5/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(5/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2),x)
```

```
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2), x)
```

3.65 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 964

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \frac{2(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce - 2Ad\cdots}{15b\cdots}$$

$$+\frac{2(6a^3Cdf + ab^2(10cCe+3Bde+3Bcf-4Adf) - b^3(5Bce-2A(de+cf)) - a^2b(Bdf+8C(de+cf)))\sqrt{\cdots}}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(48a^4Cd^2f^2 - 8a^3bdf(Bdf+11C(de+cf)) - b^4(2Ad^2e^2 - cde(5Be+2Af) - c^2(30Ce^2+5Bef-2A\cdots}{\cdots}$$

$$+\frac{2(de-cf)(24a^3Cd^2f - a^2bd(23Cde+41cCf+4Bdf) - b^3(15c^2Ce-2Ad^2e+cd(5Be+Af)) + ab^2(15c^2\cdots}{15b^4\sqrt{d}(-bc+ad)^{3/2}(be-\cdots}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(6*a^3*C*d*f+a*b^2*(-4*A*d*f+3*B*c*f+3*B*d*e+10*C*c*e)
-b^3*(5*B*c*e-2*A*(c*f+d*e))-a^2*b*(B*d*f+8*C*(c*f+d*e)))*(f*x+e)^(3/2)*(d*
x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(24*a^3*C*d^2*f-a
^2*b*d*(4*B*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5*B*e
))+a*b^2*(15*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*(d*x+c)^(1/2)*
(f*x+e)^(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^(1/2)+2/15*(48*a^4*C*d^2*
f^2-8*a^3*b*d*f*(B*d*f+11*C*(c*f+d*e))-b^4*(2*A*d^2*e^2-c*d*e*(2*A*f+5*B*e)
-c^2*(-2*A*f^2+5*B*e*f+30*C*e^2))-a*b^3*(d^2*e*(-2*A*f+3*B*e)+c^2*f*(3*B*f+
70*C*e)+2*c*d*(-A*f^2+11*B*e*f+35*C*e^2))+a^2*b^2*(2*C*(19*c^2*f^2+81*c*d*e
*f+19*d^2*e^2)-d*f*(2*A*d*f-13*B*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-
```

```
a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2
)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(24*a^3*C*d^2*f-a^2*b*d*(4*B
*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5*B*e))+a*b^2*(1
5*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*EllipticF(d^(1/2)*(b*x+a)
^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+
b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)/d^(
1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 964, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 155, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf)))\,\sqrt{\cdots}}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$+\frac{2\sqrt{d}(48Cd^2f^2a^4-8bdf(Bdf+11C(de+cf))a^3+b^2(2C(19d^2e^2+81cdfe+19c^2f^2)-df(2Adf-13B(d\cdots}{\cdots}$$

$$+\frac{2(24Cd^2fa^3-bd(23Cde+41cCf+4Bdf)a^2+b^2(15Cfc^2+(40Cde+6Bdf)c+d^2(3Be-Af))\,a-b^3(\cdots}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$

$$+\frac{2(de-cf)\,(24Cd^2fa^3-bd(23Cde+41cCf+4Bdf)a^2+b^2(15Cfc^2+(40Cde+6Bdf)c+d^2(3Be-Af\cdots}{15b^4\sqrt{d}(ad-bc)^{3/2}(be-\cdots}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x]
```

```
[Out] (2*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*
C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f
) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^3*(b*c - a*
d)^2*(b*e - a*f)*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*B*d
*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*f +
8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(15*b^2*(b*c - a*d)*(b*e -
 a*f)^2*(a + b*x)^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e +
f*x)^(3/2))/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(48*
a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*
f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d*f*(2*A*d*f
- 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipti
cE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(
b*e - a*f))])/(15*b^4*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt
```

```
[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*b*d*(23
*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A
*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqr
t[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcS
in[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a
*f))])/(15*b^4*Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[
e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
```

```
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(5cCe+3Bde+3Bcf-5Adf)+b^2(5Bce-2A(de+cf))}{2b}+\frac{1}{2}\left(aBdf-\frac{6a^2Cdf}{b}+5aC(de+cf)-b(5cCe+Adf)\right)x\right)}{(a+bx)^{5/2}}}{5(bc-ad)(be-af)}$$

$$= \frac{2(6a^3Cdf+ab^2(10cCe+3Bde+3Bcf-4Adf)-b^3(5Bce-2A(de+cf))-a^2b(Bdf+8C(de}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$- \frac{4\int \frac{\sqrt{e+fx}\left(\frac{6a^3Cdf(de+3cf)-b^3e\left(15c^2Ce-2Ad^2e+cd(5Be+Af)\right)+ab^2\left(30c^2Cef+d^2e(3Be-4Af)+cd\left(25Ce^2+6Bef+3Af^2\right)\right)-a^2b\left(Bdf(de\right.}{4b}\right.}{}}{15}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2C}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$
$$+ \frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de + cf)) - a^2b(Bdf + 8C(de}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$
$$- \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$
$$- \frac{8\int -\frac{24a^4Cd^2f^2(de+cf)+b^4ce\left(15c^2Cef-Ad^2ef+cd\left(15Ce^2+10Bef-Af^2\right)\right)-ab^3\left(Ad^3e^2f+30c^3Cef^2+2cd^2e\left(15Ce^2+7Bef-3Af^2\right)+c^2df\left(80\right.\right.}{}}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2C}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$
$$+ \frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de + cf)) - a^2b(Bdf + 8C(de}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$
$$- \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$
$$+ \frac{((de-cf)\,(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af))}{15b^3(bc-ad)^2(be-a}$$
$$+ \frac{(d(48a^4Cd^2f^2 - 8a^3bdf(Bdf + 11C(de + cf)) - b^4(2Ad^2e^2 - cde(5Be + 2Af) - c^2(30Ce^2 + 5B}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2C}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$
$$+ \frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de + cf)) - a^2b(Bdf + 8C(de}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$
$$- \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$
$$+ \frac{\Big((de-cf)\,(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af))}{15b^3(bc-ad)^2(be}$$
$$+ \frac{\Big(d(48a^4Cd^2f^2 - 8a^3bdf(Bdf + 11C(de + cf)) - b^4(2Ad^2e^2 - cde(5Be + 2Af) - c^2(30Ce^2 + 5A}{}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$

$$+ \frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de+cf)) - a^2b(Bdf + 8C(}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2\sqrt{d}(48a^4Cd^2f^2 - 8a^3bdf(Bdf + 11C(de+cf)) - b^4(2Ad^2e^2 - cde(5Be + 2Af) - c^2(30Ce^2 +}{}$$

$$+ \frac{\Big((de-cf)(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af}{15b^3(bc - a}$$

$$= \frac{2(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(15c^2}{15b^3(bc-ad)^2(be-af)\sqrt{a+bx}}$$

$$+ \frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de+cf)) - a^2b(Bdf + 8C(}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2\sqrt{d}(48a^4Cd^2f^2 - 8a^3bdf(Bdf + 11C(de+cf)) - b^4(2Ad^2e^2 - cde(5Be + 2Af) - c^2(30Ce^2 +}{}$$

$$+ \frac{2(de-cf)(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)}{15b^4\sqrt{d}(-bc+ad)^{3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.80 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\Bigg(-\frac{2(Ab^2-abB+a^2C)}{5b^3(a+bx)^3}$$
$$-\frac{2(5b^3Bce-10ab^2cCe+Ab^3de-6ab^2Bde+11a^2bCde+Ab^3cf-6ab^2Bcf+11a^2bcCf-2aAb^2df+7a^2b}{15b^3(bc-ad)(be-af)(a+bx)^2}$$
$$-\frac{2(15b^4c^2Ce^2+5b^4Bcde^2-40ab^3cCde^2-2Ab^4d^2e^2-3ab^3Bd^2e^2+23a^2b^2Cd^2e^2+5b^4Bc^2ef-40ab^3c^2Ce}{}$$
$$+\frac{2(a+bx)^{3/2}\left(\sqrt{-a+\frac{bc}{d}}(48a^4Cd^2f^2-8a^3bdf(Bdf+11C(de+cf))+b^4(-2Ad^2e^2+cde(5Be+2Af)+c\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x
]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5*
b^3*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e - 6*a*b^2*B
*d*e + 11*a^2*b*C*d*e + A*b^3*c*f - 6*a*b^2*B*c*f + 11*a^2*b*c*C*f - 2*a*A*
b^2*d*f + 7*a^2*b*B*d*f - 12*a^3*C*d*f))/(15*b^3*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*c*C*d*e^2 -
2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 + 5*b^4*B*c^2*e*
f - 40*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 22*a*b^3*B*c*d*e*f + 102*a^2*b^2
*c*C*d*e*f + 2*a*A*b^3*d^2*e*f + 13*a^2*b^2*B*d^2*e*f - 58*a^3*b*C*d^2*e*f
- 2*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 23*a^2*b^2*c^2*C*f^2 + 2*a*A*b^3*c*
d*f^2 + 13*a^2*b^2*B*c*d*f^2 - 58*a^3*b*c*C*d*f^2 - 2*a^2*A*b^2*d^2*f^2 - 8
*a^3*b*B*d^2*f^2 + 33*a^4*C*d^2*f^2))/(15*b^3*(b*c - a*d)^2*(b*e - a*f)^2*(
a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(48*a^4*C*d^2*f^2 - 8*a
^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) + b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*
A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e*(3*B*e - 2*A*f) +
 c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2)) + a^2*b^2*(2
*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f + 13*B*(d*e + c*f
))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a + b*x) - (a*f)/(
a + b*x)) + (I*(-(b*c) + a*d)*f*(-48*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f + 1
1*C*(d*e + c*f)) - b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*A*f) + c^2*(30*C*e^
2 + 5*B*e*f - 2*A*f^2)) + a*b^3*(d^2*e*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*
B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2)) - a^2*b^2*(2*C*(19*d^2*e^2 + 81
*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f + 13*B*(d*e + c*f))))*Sqrt[1 - a/(a
+ b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*E
```

```
llipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)])/Sqrt[a + b*x] - (I*b*(-(b*c) + a*d)*(d*e - c*f)*(-24*a^3*C*d*f
^2 + a^2*b*f*(41*C*d*e + 23*c*C*f + 4*B*d*f) + b^3*(15*c*C*e^2 + A*d*e*f +
c*f*(5*B*e - 2*A*f)) - a*b^2*(5*C*e*(3*d*e + 8*c*f) + f*(6*B*d*e + 3*B*c*f
- A*d*f)))*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x)
 + (b*e)/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x
]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x]))/(15*b^5*Sqrt[-a + (b*
c)/d]*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt[c + ((a + b*x)*(d - (a*d)/(a + b*x))
)/b]*Sqrt[e + ((a + b*x)*(f - (a*f)/(a + b*x)))/b])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2291 vs. $2(902) = 1804$.

Time = 4.64 (sec) , antiderivative size = 2292, normalized size of antiderivative = 2.38

method	result	size
elliptic	Expression too large to display	2292
default	Expression too large to display	34614

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2/5*(A*b^2-B*a*b+C*a^2)/b^6*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(2*A*a*b^2*d*f-A*b^3*c*f-A*b
^3*d*e-7*B*a^2*b*d*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d*f-1
1*C*a^2*b*c*f-11*C*a^2*b*d*e+10*C*a*b^2*c*e)/b^5/(a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*
c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)^2/b^4*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A*a*b^3*d
^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d^2*f^2-13
*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*b^3*c*d*e*
f+3*B*a*b^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2*f^2+58*C*a
^3*b*c*d*f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*b^2*c*d*e*f-
23*C*a^2*b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*C*b^4*c^2*e^2
)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d*f-3*C*a*d*f+C
*b*c*f+C*b*d*e)/b^4+1/15*d*f*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-7*B*a^2*b*d
*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d*f-11*C*a^2*b*c*f-11*C
*a^2*b*d*e+10*C*a*b^2*c*e)/b^4/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)-1/15/b^4*(
a*d*f-b*c*f-b*d*e)*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A*a*b^3*d^2*e*f
+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d^2*f^2-13*B*a^2
*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*b^3*c*d*e*f+3*B*
a*b^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2*f^2+58*C*a^3*b*c
*d*f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*b^2*c*d*e*f-23*C*a
```

```
^2*b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*C*b^4*c^2*e^2)/(a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2-1/15*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)^2/b^4*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A*a*b^3*d^2*e*f+2*
A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d^2*f^2-13*B*a^2*b^
2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*b^3*c*d*e*f+3*B*a*b
^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2*f^2+58*C*a^3*b*c*d*
f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*b^2*c*d*e*f-23*C*a^2*
b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*C*b^4*c^2*e^2))*(e/f-c
/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d
))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a
*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/
2))+2*(C*d*f/b^3-1/15/b^3*d*f*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A*a*
b^3*d^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d^2*f
^2-13*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*b^3*c
*d*e*f+3*B*a*b^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2*f^2+5
8*C*a^3*b*c*d*f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*b^2*c*d
*e*f-23*C*a^2*b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*C*b^4*c^
2*e^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2)*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(
1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f
*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*E
llipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*Ellip
ticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 4721, normalized size of antiderivative = 4.90

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="fricas")
```

```
[Out] -2/45*(3*((15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 + 2*B*a*b^7 + 3*A*b^8)*c^2*d - 5
*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*e^2*f - (5*(5*C*a^3*b^5 + A*a*b^7)*c^2*d -
10*(7*C*a^4*b^4 - B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (41*C*a^5*b^3 - 6*B*a^4*b^
4 + A*a^3*b^5)*d^3)*e*f^2 + (15*C*a^4*b^4*c^2*d - (41*C*a^5*b^3 - 6*B*a^4*b
^4 + A*a^3*b^5)*c*d^2 + (24*C*a^6*b^2 - 4*B*a^5*b^3 - A*a^4*b^4)*d^3)*f^3 +
 ((15*C*b^8*c^2*d - 5*(8*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7
 - 2*A*b^8)*d^3)*e^2*f - (5*(8*C*a*b^7 - B*b^8)*c^2*d - 2*(51*C*a^2*b^6 - 1
1*B*a*b^7 + A*b^8)*c*d^2 + (58*C*a^3*b^5 - 13*B*a^2*b^6 - 2*A*a*b^7)*d^3)*e
*f^2 + ((23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*c^2*d - (58*C*a^3*b^5 - 13*B*a
^2*b^6 - 2*A*a*b^7)*c*d^2 + (33*C*a^4*b^4 - 8*B*a^3*b^5 - 2*A*a^2*b^6)*d^3)
```

*f^3)*x^2 + ((5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*b^7 - A*b^8
)*c*d^2 + 5*(7*C*a^3*b^5 - A*a*b^7)*d^3)*e^2*f - ((59*C*a^2*b^6 + B*a*b^7 -
A*b^8)*c^2*d - 20*(8*C*a^3*b^5 - B*a^2*b^6)*c*d^2 + (93*C*a^4*b^4 - 13*B*a
^3*b^5 - 7*A*a^2*b^6)*d^3)*e*f^2 + (5*(7*C*a^3*b^5 - A*a*b^7)*c^2*d - (93*C
*a^4*b^4 - 13*B*a^3*b^5 - 7*A*a^2*b^6)*c*d^2 + 3*(18*C*a^5*b^3 - 3*B*a^4*b^
4 - 2*A*a^3*b^5)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (
(15*C*a^3*b^5*c^2*d - 5*(4*C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (7*C*a^5*b^3 + 3*
B*a^4*b^4 + 2*A*a^3*b^5)*d^3)*e^3 + (15*C*a^3*b^5*c^3 - 10*(13*C*a^4*b^4 -
2*B*a^3*b^5)*c^2*d + (182*C*a^5*b^3 - 22*B*a^4*b^4 - 3*A*a^3*b^5)*c*d^2 - (
73*C*a^6*b^2 - 8*B*a^5*b^3 + 3*A*a^4*b^4)*d^3)*e^2*f - (5*(4*C*a^4*b^4 + B*
a^3*b^5)*c^3 - (182*C*a^5*b^3 - 22*B*a^4*b^4 - 3*A*a^3*b^5)*c^2*d + 2*(134*
C*a^6*b^2 - 19*B*a^5*b^3 - 6*A*a^4*b^4)*c*d^2 - (112*C*a^7*b - 17*B*a^6*b^2
- 3*A*a^5*b^3)*d^3)*e*f^2 + ((7*C*a^5*b^3 + 3*B*a^4*b^4 + 2*A*a^3*b^5)*c^3
- (73*C*a^6*b^2 - 8*B*a^5*b^3 + 3*A*a^4*b^4)*c^2*d + (112*C*a^7*b - 17*B*a
^6*b^2 - 3*A*a^5*b^3)*c*d^2 - 2*(24*C*a^8 - 4*B*a^7*b - A*a^6*b^2)*d^3)*f^3
+ ((15*C*b^8*c^2*d - 5*(4*C*a*b^7 + B*b^8)*c*d^2 + (7*C*a^2*b^6 + 3*B*a*b^
7 + 2*A*b^8)*d^3)*e^3 + (15*C*b^8*c^3 - 10*(13*C*a*b^7 - 2*B*b^8)*c^2*d + (
182*C*a^2*b^6 - 22*B*a*b^7 - 3*A*b^8)*c*d^2 - (73*C*a^3*b^5 - 8*B*a^2*b^6 +
3*A*a*b^7)*d^3)*e^2*f - (5*(4*C*a*b^7 + B*b^8)*c^3 - (182*C*a^2*b^6 - 22*B
*a*b^7 - 3*A*b^8)*c^2*d + 2*(134*C*a^3*b^5 - 19*B*a^2*b^6 - 6*A*a*b^7)*c*d^
2 - (112*C*a^4*b^4 - 17*B*a^3*b^5 - 3*A*a^2*b^6)*d^3)*e*f^2 + ((7*C*a^2*b^6
+ 3*B*a*b^7 + 2*A*b^8)*c^3 - (73*C*a^3*b^5 - 8*B*a^2*b^6 + 3*A*a*b^7)*c^2*
d + (112*C*a^4*b^4 - 17*B*a^3*b^5 - 3*A*a^2*b^6)*c*d^2 - 2*(24*C*a^5*b^3 -
4*B*a^4*b^4 - A*a^3*b^5)*d^3)*f^3)*x^3 + 3*((15*C*a*b^7*c^2*d - 5*(4*C*a^2*
b^6 + B*a*b^7)*c*d^2 + (7*C*a^3*b^5 + 3*B*a^2*b^6 + 2*A*a*b^7)*d^3)*e^3 + (
15*C*a*b^7*c^3 - 10*(13*C*a^2*b^6 - 2*B*a*b^7)*c^2*d + (182*C*a^3*b^5 - 22*
B*a^2*b^6 - 3*A*a*b^7)*c*d^2 - (73*C*a^4*b^4 - 8*B*a^3*b^5 + 3*A*a^2*b^6)*d
^3)*e^2*f - (5*(4*C*a^2*b^6 + B*a*b^7)*c^3 - (182*C*a^3*b^5 - 22*B*a^2*b^6
- 3*A*a*b^7)*c^2*d + 2*(134*C*a^4*b^4 - 19*B*a^3*b^5 - 6*A*a^2*b^6)*c*d^2 -
(112*C*a^5*b^3 - 17*B*a^4*b^4 - 3*A*a^3*b^5)*d^3)*e*f^2 + ((7*C*a^3*b^5 +
3*B*a^2*b^6 + 2*A*a*b^7)*c^3 - (73*C*a^4*b^4 - 8*B*a^3*b^5 + 3*A*a^2*b^6)*c
^2*d + (112*C*a^5*b^3 - 17*B*a^4*b^4 - 3*A*a^3*b^5)*c*d^2 - 2*(24*C*a^6*b^2
- 4*B*a^5*b^3 - A*a^4*b^4)*d^3)*f^3)*x^2 + 3*((15*C*a^2*b^6*c^2*d - 5*(4*C
*a^3*b^5 + B*a^2*b^6)*c*d^2 + (7*C*a^4*b^4 + 3*B*a^3*b^5 + 2*A*a^2*b^6)*d^3
)*e^3 + (15*C*a^2*b^6*c^3 - 10*(13*C*a^3*b^5 - 2*B*a^2*b^6)*c^2*d + (182*C*
a^4*b^4 - 22*B*a^3*b^5 - 3*A*a^2*b^6)*c*d^2 - (73*C*a^5*b^3 - 8*B*a^4*b^4 +
3*A*a^3*b^5)*d^3)*e^2*f - (5*(4*C*a^3*b^5 + B*a^2*b^6)*c^3 - (182*C*a^4*b^
4 - 22*B*a^3*b^5 - 3*A*a^2*b^6)*c^2*d + 2*(134*C*a^5*b^3 - 19*B*a^4*b^4 - 6
*A*a^3*b^5)*c*d^2 - (112*C*a^6*b^2 - 17*B*a^5*b^3 - 3*A*a^4*b^4)*d^3)*e*f^2
+ ((7*C*a^4*b^4 + 3*B*a^3*b^5 + 2*A*a^2*b^6)*c^3 - (73*C*a^5*b^3 - 8*B*a^4
*b^4 + 3*A*a^3*b^5)*c^2*d + (112*C*a^6*b^2 - 17*B*a^5*b^3 - 3*A*a^4*b^4)*c*
d^2 - 2*(24*C*a^7*b - 4*B*a^6*b^2 - A*a^5*b^3)*d^3)*f^3)*x)*sqrt(b*d*f)*wei
erstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*
b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 +
a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3

```
*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(
3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*((30*C*a^3*b^5*c^2*d - 5*(1
4*C*a^4*b^4 - B*a^3*b^5)*c*d^2 + (38*C*a^5*b^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)
*d^3)*e^2*f - (5*(14*C*a^4*b^4 - B*a^3*b^5)*c^2*d - 2*(81*C*a^5*b^3 - 11*B*
a^4*b^4 + A*a^3*b^5)*c*d^2 + (88*C*a^6*b^2 - 13*B*a^5*b^3 - 2*A*a^4*b^4)*d^
3)*e*f^2 + ((38*C*a^5*b^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)*c^2*d - (88*C*a^6*b^
2 - 13*B*a^5*b^3 - 2*A*a^4*b^4)*c*d^2 + 2*(24*C*a^7*b - 4*B*a^6*b^2 - A*a^5
*b^3)*d^3)*f^3 + ((30*C*b^8*c^2*d - 5*(14*C*a*b^7 - B*b^8)*c*d^2 + (38*C*a^
2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*e^2*f - (5*(14*C*a*b^7 - B*b^8)*c^2*d - 2
*(81*C*a^2*b^6 - 11*B*a*b^7 + A*b^8)*c*d^2 + (88*C*a^3*b^5 - 13*B*a^2*b^6 -
 2*A*a*b^7)*d^3)*e*f^2 + ((38*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*c^2*d - (88*
C*a^3*b^5 - 13*B*a^2*b^6 - 2*A*a*b^7)*c*d^2 + 2*(24*C*a^4*b^4 - 4*B*a^3*b^5
 - A*a^2*b^6)*d^3)*f^3)*x^3 + 3*((30*C*a*b^7*c^2*d - 5*(14*C*a^2*b^6 - B*a*
b^7)*c*d^2 + (38*C*a^3*b^5 - 3*B*a^2*b^6 - 2*A*a*b^7)*d^3)*e^2*f - (5*(14*C
*a^2*b^6 - B*a*b^7)*c^2*d - 2*(81*C*a^3*b^5 - 11*B*a^2*b^6 + A*a*b^7)*c*d^2
 + (88*C*a^4*b^4 - 13*B*a^3*b^5 - 2*A*a^2*b^6)*d^3)*e*f^2 + ((38*C*a^3*b^5
 - 3*B*a^2*b^6 - 2*A*a*b^7)*c^2*d - (88*C*a^4*b^4 - 13*B*a^3*b^5 - 2*A*a^2*b
^6)*c*d^2 + 2*(24*C*a^5*b^3 - 4*B*a^4*b^4 - A*a^3*b^5)*d^3)*f^3)*x^2 + 3*((
30*C*a^2*b^6*c^2*d - 5*(14*C*a^3*b^5 - B*a^2*b^6)*c*d^2 + (38*C*a^4*b^4 - 3
*B*a^3*b^5 - 2*A*a^2*b^6)*d^3)*e^2*f - (5*(14*C*a^3*b^5 - B*a^2*b^6)*c^2*d
 - 2*(81*C*a^4*b^4 - 11*B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (88*C*a^5*b^3 - 13*B*
a^4*b^4 - 2*A*a^3*b^5)*d^3)*e*f^2 + ((38*C*a^4*b^4 - 3*B*a^3*b^5 - 2*A*a^2*
b^6)*c^2*d - (88*C*a^5*b^3 - 13*B*a^4*b^4 - 2*A*a^3*b^5)*c*d^2 + 2*(24*C*a^
6*b^2 - 4*B*a^5*b^3 - A*a^4*b^4)*d^3)*f^3)*x)*sqrt(b*d*f)*weierstrassZeta(4
/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f
^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f -
 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3
*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2
)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3
*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e +
(b*c + a*d)*f)/(b*d*f))))/((a^3*b^9*c^2*d - 2*a^4*b^8*c*d^2 + a^5*b^7*d^3)*
e^2*f - 2*(a^4*b^8*c^2*d - 2*a^5*b^7*c*d^2 + a^6*b^6*d^3)*e*f^2 + (a^5*b^7*
c^2*d - 2*a^6*b^6*c*d^2 + a^7*b^5*d^3)*f^3 + ((b^12*c^2*d - 2*a*b^11*c*d^2
+ a^2*b^10*d^3)*e^2*f - 2*(a*b^11*c^2*d - 2*a^2*b^10*c*d^2 + a^3*b^9*d^3)*e
*f^2 + (a^2*b^10*c^2*d - 2*a^3*b^9*c*d^2 + a^4*b^8*d^3)*f^3)*x^3 + 3*((a*b^
11*c^2*d - 2*a^2*b^10*c*d^2 + a^3*b^9*d^3)*e^2*f - 2*(a^2*b^10*c^2*d - 2*a^
3*b^9*c*d^2 + a^4*b^8*d^3)*e*f^2 + (a^3*b^9*c^2*d - 2*a^4*b^8*c*d^2 + a^5*b
^7*d^3)*f^3)*x^2 + 3*((a^2*b^10*c^2*d - 2*a^3*b^9*c*d^2 + a^4*b^8*d^3)*e^2*
f - 2*(a^3*b^9*c^2*d - 2*a^4*b^8*c*d^2 + a^5*b^7*d^3)*e*f^2 + (a^4*b^8*c^2*
d - 2*a^5*b^7*c*d^2 + a^6*b^6*d^3)*f^3)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(7/2),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(7/2), x
)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(7/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{7/2}}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2),x)
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2), x)
```

3.66 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1716

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx =$$

$$-\frac{2(24a^4Cd^2f^2 - a^3bdf(61Cde + 43cCf - 4Bdf) - 3ab^3(d^2e(Be-3Af) + 2c^2f(7Ce-Bf) + cd(28Ce^2 - 5\cdots}{}$$

$$+\frac{2(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de+cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be+5Af) + c^2de(35Ce^2 + 14Bef\cdots}{}$$

$$+\frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de+cf)) + a^2b(Bdf - 10C(de+cf)))\sqrt{\cdots}}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}}$$

$$-\frac{2(Ab^2 - a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$+\frac{2\sqrt{d}(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de+cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be+5Af) + c^2de(35Ce^2 + 14B\cdots}{}$$

$$+\frac{2\sqrt{d}(de-cf)(24a^4Cd^2f^2 - a^3bdf(43Cde + 61cCf - 4Bdf) + b^4(8Ad^2e^2 - cde(14Be+Af) + c^2(35Ce^2 + \cdots}{}$$

```
[Out] -2/7*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(7/2)+2/35*(6*a^3*C*d*f+a*b^2*(-8*A*d*f+3*B*c*f+3*B*d*e+14*C*c*e)
-b^3*(7*B*c*e-4*A*(c*f+d*e))+a^2*b*(B*d*f-10*C*(c*f+d*e)))*(f*x+e)^(3/2)*(d
*x+c)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(5/2)-2/105*(24*a^4*C*d^2*f
^2-a^3*b*d*f*(-4*B*d*f+43*C*c*f+61*C*d*e)-3*a*b^3*(d^2*e*(-3*A*f+B*e)+2*c^2
*f*(-B*f+7*C*e)+c*d*(5*A*f^2-5*B*e*f+28*C*e^2))-b^4*(4*A*d^2*e^2-c*d*e*(-A*
f+7*B*e)-c^2*(8*A*f^2-14*B*e*f+35*C*e^2))-3*a^2*b^2*(d*f*(-A*d*f+2*B*c*f+3*
B*d*e)-C*(5*c^2*f^2+37*c*d*e*f+15*d^2*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
```

```
3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/105*(48*a^5*C*d^3*f^3+8*a^4*b*d
^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+14*B*e)+c^2
*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e^2))-a*b^4*
(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*e^2-19*f*(-A
*f+B*e))-c*d^2*e*(42*C*e^2-f*(20*A*f+19*B*e)))+a^3*b^2*d*f*(C*(103*c^2*f^2+
344*c*d*e*f+103*d^2*e^2)+d*f*(6*A*d*f-19*B*(c*f+d*e)))-3*a^2*b^3*(C*(5*c^3*
f^3+94*c^2*d*e*f^2+94*c*d^2*e^2*f+5*d^3*e^3)+d*f*(3*A*d*f*(c*f+d*e)-B*(3*c^
2*f^2+16*c*d*e*f+3*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^3
/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/105*(48*a^5*C*d^3*f^3+8*a^4*b*d^2*f^2*(B*d*f-
16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*f+14*B*e)+c^2*d*e*(-5*A*f^2
+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*C*e^2))-a*b^4*(d^3*e^2*(-19*
A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(238*C*e^2-19*f*(-A*f+B*e))-c*d^2
*e*(42*C*e^2-f*(20*A*f+19*B*e)))+a^3*b^2*d*f*(C*(103*c^2*f^2+344*c*d*e*f+10
3*d^2*e^2)+d*f*(6*A*d*f-19*B*(c*f+d*e)))-3*a^2*b^3*(C*(5*c^3*f^3+94*c^2*d*e
*f^2+94*c*d^2*e^2*f+5*d^3*e^3)+d*f*(3*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+16*c*d*e
*f+3*d^2*e^2))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c
)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)
/b^4/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2
)+2/105*(-c*f+d*e)*(24*a^4*C*d^2*f^2-a^3*b*d*f*(-4*B*d*f+61*C*c*f+43*C*d*e)
+b^4*(8*A*d^2*e^2-c*d*e*(A*f+14*B*e)+c^2*(-4*A*f^2+7*B*e*f+35*C*e^2))+3*a*b
^3*(d^2*e*(-5*A*f+2*B*e)-c^2*f*(B*f+28*C*e)-c*d*(-3*A*f^2-5*B*e*f+14*C*e^2)
)-3*a^2*b^2*(d*f*(-A*d*f+3*B*c*f+2*B*d*e)-C*(15*c^2*f^2+37*c*d*e*f+5*d^2*e^
2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+
b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1
/2)/b^4/(a*d-b*c)^(5/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 5.04 (sec) , antiderivative size = 1716, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{1628, 155, 157, 164, 115, 114, 122, 121\}$

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$+\frac{2(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf)))\,\sqrt{}}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(48Cd^3f^3a^5+8bd^2f^2(Bdf-16C(de+cf))a^4+b^2df(C(103d^2e^2+344cdfe+103c^2f^2)+df(6Adf-1}{}$$

$$+\frac{2(48Cd^3f^3a^5+8bd^2f^2(Bdf-16C(de+cf))a^4+b^2df(C(103d^2e^2+344cdfe+103c^2f^2)+df(6Adf-19B}{}$$

$$-\frac{2(24Cd^2f^2a^4-bdf(61Cde+43cCf-4Bdf)a^3-3b^2(df(3Bde+2Bcf-Adf)-C(15d^2e^2+37cdfe+5c}{}$$

$$+\frac{2\sqrt{d}(de-cf)\,(24Cd^2f^2a^4-bdf(43Cde+61cCf-4Bdf)a^3-3b^2(df(2Bde+3Bcf-Adf)-C(5d^2e^2+3}{}$$

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4*B*d*f) - 3*a*b^3
*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C*e^2 - 5*B*e*f + 5
*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*(35*C*e^2 - 14*B*e*
f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d*f) - C*(15*d^2*e^2
+ 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*
d)^2*(b*e - a*f)^2*(a + b*x)^(3/2)) + (2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^
2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f
) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f +
8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f) - c^2
*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e + 20*A*
f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A*
d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c^
2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*e
*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^3*(b*c - a*d)^3*(b*e
 - a*f)^3*Sqrt[a + b*x]) + (2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*B*d*e + 3*
B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*f - 10*C*(d
*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(35*b^2*(b*c - a*d)*(b*e - a*f)^
2*(a + b*x)^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(
3/2))/(7*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(7/2)) + (2*Sqrt[d]*(48*a^5*C*
d^3*f^3 + 8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c
*d^2*e^2*(14*B*e + 5*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f
*(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3
*f^2*(7*C*e - B*f) - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C
*e^2 - f*(19*B*e + 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f +
103*c^2*f^2) + d*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3
```

```
+ 94*c*d^2*e^2*f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) -
 B*(3*d^2*e^2 + 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]],
 ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*f
)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)
*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(43*C*d*e + 61*c*C*f - 4*B*d*f) + b^4*(8*A*d
^2*e^2 - c*d*e*(14*B*e + A*f) + c^2*(35*C*e^2 + 7*B*e*f - 4*A*f^2)) + 3*a*b
^3*(d^2*e*(2*B*e - 5*A*f) - c^2*f*(28*C*e + B*f) - c*d*(14*C*e^2 - 5*B*e*f
- 3*A*f^2)) - 3*a^2*b^2*(d*f*(2*B*d*e + 3*B*c*f - A*d*f) - C*(5*d^2*e^2 + 3
7*c*d*e*f + 15*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^4*(-(b*c) + a*d)^(5/2)*(b*e - a*
f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
```

```
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$-\frac{2\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(-\frac{3a^2C(de+cf)-ab(7cCe+3Bde+3Bcf-7Adf)+b^2(7Bce-4A(de+cf))}{2b}+\frac{1}{2}\left(-7bcCe+7aCde+7acCf+Abdf-aBdf-\frac{6a^2Cdf}{b}\right)\cdots\right.}{(a+bx)^{7/2}}}{7(bc-ad)(be-af)}$$

$$= \frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de+cf)) + a^2b(Bdf - 10C(de \cdots}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$- \frac{4\int \frac{\sqrt{e+fx}\left(\frac{6a^3Cdf(de+3cf)+b^3\left(4Ad^2e^2-cde(7Be-Af)-c^2\left(35Ce^2-14Bef+8Af^2\right)\right)+ab^2\left(d^2e(3Be-8Af)+6c^2f(7Ce-Bf)+cd\left(49Ce^2-8\cdots\right.\right.}{4b}\right.}{\cdots}}{\cdots}$$

$$=$$

$$-\frac{2(24a^4Cd^2f^2 - a^3bdf(61Cde + 43cCf - 4Bdf) - 3ab^3(d^2e(Be - 3Af) + 2c^2f(7Ce - Bf) + c\cdots}{\cdots}$$

$$+ \frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de+cf)) + a^2b(Bdf - 10C(\cdots}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$- \frac{8\int -\frac{24a^4Cd^2f^2(de+cf)+3ab^3\left(d^3e^2(2Be-5Af)-2c^3f^2(7Ce-Bf)-2cd^2e\left(7Ce^2-2Bef-3Af^2\right)-c^2df\left(56Ce^2-4Bef+5Af^2\right)\right)+b^4\left(8Ad^3e^3-\cdots\right.}{\cdots}}{\cdots}$$

$$=$$

$$-\frac{2(24a^4Cd^2f^2 - a^3bdf(61Cde + 43cCf - 4Bdf) - 3ab^3(d^2e(Be - 3Af) + 2c^2f(7Ce - Bf) + c\cdots}{\cdots}$$

$$+ \frac{2(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de+cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be + 5Af) + c^2de(35C\cdots}{\cdots}$$

$$+ \frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de+cf)) + a^2b(Bdf - 10C(\cdots}{35b^2(bc-ad)(be-af)^2(a+bx)^{5/2}}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$+ \frac{16\int -\frac{df\left(24a^5Cd^2f^2(de+cf)-3a^2b^3\left(d^3e^2(Be-3Af)+c^3f^2(48Ce+Bf)+3c^2df\left(34Ce^2-4Bef-Af^2\right)+12cd^2e\left(4Ce^2-Bef+Af^2\right)\right)-b^5ce\left(4A\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= -\frac{2(24a^4Cd^2f^2 - a^3bdf(61Cde + 43cCf - 4Bdf) - 3ab^3(d^2e(Be - 3Af) + 2c^2f(7Ce - Bf) + cd}{}$$
$$+ \frac{2(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de + cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be + 5Af) + c^2de(35Ce}{}$$
$$+ \frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de + cf)) + a^2b(Bdf - 10C(d}{35b^2(bc - ad)(be - af)^2(a + bx)^{5/2}}$$
$$- \frac{2(Ab^2 - a(bB - aC))(c + dx)^{3/2}(e + fx)^{3/2}}{7b(bc - ad)(be - af)(a + bx)^{7/2}}$$
$$- \frac{(d(de - cf)(24a^4Cd^2f^2 - a^3bdf(43Cde + 61cCf - 4Bdf) + b^4(8Ad^2e^2 - cde(14Be + Af) + c^2}{}$$
$$- \frac{(d(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de + cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be + 5Af) + c^2de(35C}{}$$

$$= -\frac{2(24a^4Cd^2f^2 - a^3bdf(61Cde + 43cCf - 4Bdf) - 3ab^3(d^2e(Be - 3Af) + 2c^2f(7Ce - Bf) + cd}{}$$
$$+ \frac{2(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de + cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be + 5Af) + c^2de(35Ce}{}$$
$$+ \frac{2(6a^3Cdf + ab^2(14cCe + 3Bde + 3Bcf - 8Adf) - b^3(7Bce - 4A(de + cf)) + a^2b(Bdf - 10C(d}{35b^2(bc - ad)(be - af)^2(a + bx)^{5/2}}$$
$$- \frac{2(Ab^2 - a(bB - aC))(c + dx)^{3/2}(e + fx)^{3/2}}{7b(bc - ad)(be - af)(a + bx)^{7/2}}$$
$$- \frac{\left(d(de - cf)(24a^4Cd^2f^2 - a^3bdf(43Cde + 61cCf - 4Bdf) + b^4(8Ad^2e^2 - cde(14Be + Af) + c^2\right.}{}$$
$$- \frac{\left(d(48a^5Cd^3f^3 + 8a^4bd^2f^2(Bdf - 16C(de + cf)) - b^5(8Ad^3e^3 - cd^2e^2(14Be + 5Af) + c^2de(35C\right.}{}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.94 (sec) , antiderivative size = 2437, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x
]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(7*
b^3*(a + b*x)^4) - (2*(7*b^3*B*c*e - 14*a*b^2*c*C*e + A*b^3*d*e - 8*a*b^2*B
*d*e + 15*a^2*b*C*d*e + A*b^3*c*f - 8*a*b^2*B*c*f + 15*a^2*b*c*C*f - 2*a*A*
b^2*d*f + 9*a^2*b*B*d*f - 16*a^3*C*d*f))/(35*b^3*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^3) - (2*(35*b^4*c^2*C*e^2 + 7*b^4*B*c*d*e^2 - 84*a*b^3*c*C*d*e^2 -
4*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 45*a^2*b^2*C*d^2*e^2 + 7*b^4*B*c^2*e*
f - 84*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 30*a*b^3*B*c*d*e*f + 198*a^2*b^2
*c*C*d*e*f + 6*a*A*b^3*d^2*e*f + 15*a^2*b^2*B*d^2*e*f - 106*a^3*b*C*d^2*e*f
 - 4*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 45*a^2*b^2*c^2*C*f^2 + 6*a*A*b^3*c
*d*f^2 + 15*a^2*b^2*B*c*d*f^2 - 106*a^3*b*c*C*d*f^2 - 6*a^2*A*b^2*d^2*f^2 -
 8*a^3*b*B*d^2*f^2 + 57*a^4*C*d^2*f^2))/(105*b^3*(b*c - a*d)^2*(b*e - a*f)^
2*(a + b*x)^2) - (2*(35*b^5*c^2*C*d*e^3 - 14*b^5*B*c*d^2*e^3 - 42*a*b^4*c*C
*d^2*e^3 + 8*A*b^5*d^3*e^3 + 6*a*b^4*B*d^3*e^3 + 15*a^2*b^3*C*d^3*e^3 + 35*
b^5*c^3*C*e^2*f + 14*b^5*B*c^2*d*e^2*f - 238*a*b^4*c^2*C*d*e^2*f - 5*A*b^5*
c*d^2*e^2*f + 19*a*b^4*B*c*d^2*e^2*f + 282*a^2*b^3*c*C*d^2*e^2*f - 19*a*A*b
^4*d^3*e^2*f - 9*a^2*b^3*B*d^3*e^2*f - 103*a^3*b^2*C*d^3*e^2*f - 14*b^5*B*c
^3*e*f^2 - 42*a*b^4*c^3*C*e*f^2 - 5*A*b^5*c^2*d*e*f^2 + 19*a*b^4*B*c^2*d*e*
f^2 + 282*a^2*b^3*c^2*C*d*e*f^2 + 20*a*A*b^4*c*d^2*e*f^2 - 48*a^2*b^3*B*c*d
^2*e*f^2 - 344*a^3*b^2*c*C*d^2*e*f^2 + 9*a^2*A*b^3*d^3*e*f^2 + 19*a^3*b^2*B
*d^3*e*f^2 + 128*a^4*b*C*d^3*e*f^2 + 8*A*b^5*c^3*f^3 + 6*a*b^4*B*c^3*f^3 +
15*a^2*b^3*c^3*C*f^3 - 19*a*A*b^4*c^2*d*f^3 - 9*a^2*b^3*B*c^2*d*f^3 - 103*a
^3*b^2*c^2*C*d*f^3 + 9*a^2*A*b^3*c*d^2*f^3 + 19*a^3*b^2*B*c*d^2*f^3 + 128*a
^4*b*c*C*d^2*f^3 - 6*a^3*A*b^2*d^3*f^3 - 8*a^4*b*B*d^3*f^3 - 48*a^5*C*d^3*f
^3))/(105*b^3*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x))) - (2*(a + b*x)^(3/2)*
(-(Sqrt[-a + (b*c)/d]*(-48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^2*(-(B*d*f) + 16*C
*(d*e + c*f)) + b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f) + c^2*d*e*(35
*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f + 8*A*f^2)) + a*b
^4*(d^3*e^2*(6*B*e - 19*A*f) + 6*c^3*f^2*(-7*C*e + B*f) + c^2*d*f*(-238*C*e
^2 + 19*f*(B*e - A*f)) + c*d^2*e*(-42*C*e^2 + f*(19*B*e + 20*A*f))) - a^3*b
^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A*d*f - 19*B*(
d*e + c*f))) + 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c^2*d*e*f^2 +
5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*e*f + 3*c^2*f
^2))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a + b*x) - (a*f)
/(a + b*x))) + (I*(-(b*c) + a*d)*f*(-48*a^5*C*d^3*f^3 + 8*a^4*b*d^2*f^2*(-(
```

```
B*d*f) + 16*C*(d*e + c*f)) + b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5*A*f)
+ c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e*f + 8*
A*f^2)) + a*b^4*(d^3*e^2*(6*B*e - 19*A*f) + 6*c^3*f^2*(-7*C*e + B*f) + c^2*
d*f*(-238*C*e^2 + 19*f*(B*e - A*f)) + c*d^2*e*(-42*C*e^2 + f*(19*B*e + 20*A
*f))) - a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d*f*(6*A
*d*f - 19*B*(d*e + c*f))) + 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*f + 94*c
^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 16*c*d*
e*f + 3*c^2*f^2))))*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/
(a + b*x) + (b*e)/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqr
t[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x] - (I*b*(-(b*c)
 + a*d)*f*(d*e - c*f)*(-24*a^4*C*d^2*f^2 + a^3*b*d*f*(61*C*d*e + 43*c*C*f -
 4*B*d*f) + b^4*(4*A*d^2*e^2 + c*d*e*(-7*B*e + A*f) + c^2*(-35*C*e^2 + 14*B
*e*f - 8*A*f^2)) + 3*a*b^3*(d^2*e*(B*e - 3*A*f) - 2*c^2*f*(-7*C*e + B*f) +
c*d*(28*C*e^2 - 5*B*e*f + 5*A*f^2)) - 3*a^2*b^2*(d*f*(-3*B*d*e - 2*B*c*f +
A*d*f) + C*(15*d^2*e^2 + 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[1 - a/(a + b*x) + (
b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*EllipticF[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)
])/Sqrt[a + b*x]))/(105*b^5*Sqrt[-a + (b*c)/d]*(b*c - a*d)^3*(b*e - a*f)^3*
Sqrt[c + ((a + b*x)*(d - (a*d)/(a + b*x)))/b]*Sqrt[e + ((a + b*x)*(f - (a*f
)/(a + b*x)))/b])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3899 vs. 2(1646) = 3292.

Time = 5.38 (sec) , antiderivative size = 3900, normalized size of antiderivative = 2.27

method	result	size
elliptic	Expression too large to display	3900
default	Expression too large to display	65231

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2/7*(A*b^2-B*a*b+C*a^2)/b^7*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^4+2/35*(2*A*a*b^2*d*f-A*b^3*c*f-A*b
^3*d*e-9*B*a^2*b*d*f+8*B*a*b^2*c*f+8*B*a*b^2*d*e-7*B*b^3*c*e+16*C*a^3*d*f-1
5*C*a^2*b*c*f-15*C*a^2*b*d*e+14*C*a*b^2*c*e)/b^6/(a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*
c*e)^(1/2)/(x+a/b)^3+2/105*(6*A*a^2*b^2*d^2*f^2-6*A*a*b^3*c*d*f^2-6*A*a*b^3
*d^2*e*f+4*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+4*A*b^4*d^2*e^2+8*B*a^3*b*d^2*f^2-
15*B*a^2*b^2*c*d*f^2-15*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+30*B*a*b^3*c*d*
e*f+3*B*a*b^3*d^2*e^2-7*B*b^4*c^2*e*f-7*B*b^4*c*d*e^2-57*C*a^4*d^2*f^2+106*
C*a^3*b*c*d*f^2+106*C*a^3*b*d^2*e*f-45*C*a^2*b^2*c^2*f^2-198*C*a^2*b^2*c*d*
```

e*f-45*C*a^2*b^2*d^2*e^2+84*C*a*b^3*c^2*e*f+84*C*a*b^3*c*d*e^2-35*C*b^4*c^2
*e^2)/b^5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^
2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/105*(b*d*f*x^2
+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/b^4*(6*A*a^3*b^
2*d^3*f^3-9*A*a^2*b^3*c*d^2*f^3-9*A*a^2*b^3*d^3*e*f^2+19*A*a*b^4*c^2*d*f^3-
20*A*a*b^4*c*d^2*e*f^2+19*A*a*b^4*d^3*e^2*f-8*A*b^5*c^3*f^3+5*A*b^5*c^2*d*e
*f^2+5*A*b^5*c*d^2*e^2*f-8*A*b^5*d^3*e^3+8*B*a^4*b*d^3*f^3-19*B*a^3*b^2*c*d
^2*f^3-19*B*a^3*b^2*d^3*e*f^2+9*B*a^2*b^3*c^2*d*f^3+48*B*a^2*b^3*c*d^2*e*f^
2+9*B*a^2*b^3*d^3*e^2*f-6*B*a*b^4*c^3*f^3-19*B*a*b^4*c^2*d*e*f^2-19*B*a*b^4
*c*d^2*e^2*f-6*B*a*b^4*d^3*e^3+14*B*b^5*c^3*e*f^2-14*B*b^5*c^2*d*e^2*f+14*B
*b^5*c*d^2*e^3+48*C*a^5*d^3*f^3-128*C*a^4*b*c*d^2*f^3-128*C*a^4*b*d^3*e*f^2
+103*C*a^3*b^2*c^2*d*f^3+344*C*a^3*b^2*c*d^2*e*f^2+103*C*a^3*b^2*d^3*e^2*f-
15*C*a^2*b^3*c^3*f^3-282*C*a^2*b^3*c^2*d*e*f^2-282*C*a^2*b^3*c*d^2*e^2*f-15
*C*a^2*b^3*d^3*e^3+42*C*a*b^4*c^3*e*f^2+238*C*a*b^4*c^2*d*e^2*f+42*C*a*b^4*
c*d^2*e^3-35*C*b^5*c^3*e^2*f-35*C*b^5*c^2*d*e^3)/((x+a/b)*(b*d*f*x^2+b*c*f*
x+b*d*e*x+b*c*e))^(1/2)+2*(d*f*C/b^4+1/105*d*f*(6*A*a^2*b^2*d^2*f^2-6*A*a*b
^3*c*d*f^2-6*A*a*b^3*d^2*e*f+4*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+4*A*b^4*d^2*e^
2+8*B*a^3*b*d^2*f^2-15*B*a^2*b^2*c*d*f^2-15*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2
*f^2+30*B*a*b^3*c*d*e*f+3*B*a*b^3*d^2*e^2-7*B*b^4*c^2*e*f-7*B*b^4*c*d*e^2-5
7*C*a^4*d^2*f^2+106*C*a^3*b*c*d*f^2+106*C*a^3*b*d^2*e*f-45*C*a^2*b^2*c^2*f^
2-198*C*a^2*b^2*c*d*e*f-45*C*a^2*b^2*d^2*e^2+84*C*a*b^3*c^2*e*f+84*C*a*b^3*
c*d*e^2-35*C*b^4*c^2*e^2)/b^4/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2-1/105/b^4
*(a*d*f-b*c*f-b*d*e)*(6*A*a^3*b^2*d^3*f^3-9*A*a^2*b^3*c*d^2*f^3-9*A*a^2*b^3
*d^3*e*f^2+19*A*a*b^4*c^2*d*f^3-20*A*a*b^4*c*d^2*e*f^2+19*A*a*b^4*d^3*e^2*f
-8*A*b^5*c^3*f^3+5*A*b^5*c^2*d*e*f^2+5*A*b^5*c*d^2*e^2*f-8*A*b^5*d^3*e^3+8*
B*a^4*b*d^3*f^3-19*B*a^3*b^2*c*d^2*f^3-19*B*a^3*b^2*d^3*e*f^2+9*B*a^2*b^3*c
^2*d*f^3+48*B*a^2*b^3*c*d^2*e*f^2+9*B*a^2*b^3*d^3*e^2*f-6*B*a*b^4*c^3*f^3-1
9*B*a*b^4*c^2*d*e*f^2-19*B*a*b^4*c*d^2*e^2*f-6*B*a*b^4*d^3*e^3+14*B*b^5*c^3
*e*f^2-14*B*b^5*c^2*d*e^2*f+14*B*b^5*c*d^2*e^3+48*C*a^5*d^3*f^3-128*C*a^4*b
*c*d^2*f^3-128*C*a^4*b*d^3*e*f^2+103*C*a^3*b^2*c^2*d*f^3+344*C*a^3*b^2*c*d^
2*e*f^2+103*C*a^3*b^2*d^3*e^2*f-15*C*a^2*b^3*c^3*f^3-282*C*a^2*b^3*c^2*d*e*
f^2-282*C*a^2*b^3*c*d^2*e^2*f-15*C*a^2*b^3*d^3*e^3+42*C*a*b^4*c^3*e*f^2+238
*C*a*b^4*c^2*d*e^2*f+42*C*a*b^4*c*d^2*e^3-35*C*b^5*c^3*e^2*f-35*C*b^5*c^2*d
*e^3)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3-1/105*(b*c*f+b*d*e)/(a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)^3/b^4*(6*A*a^3*b^2*d^3*f^3-9*A*a^2*b^3*c*d^2*f^3-9*A*a
^2*b^3*d^3*e*f^2+19*A*a*b^4*c^2*d*f^3-20*A*a*b^4*c*d^2*e*f^2+19*A*a*b^4*d^3
*e^2*f-8*A*b^5*c^3*f^3+5*A*b^5*c^2*d*e*f^2+5*A*b^5*c*d^2*e^2*f-8*A*b^5*d^3*
e^3+8*B*a^4*b*d^3*f^3-19*B*a^3*b^2*c*d^2*f^3-19*B*a^3*b^2*d^3*e*f^2+9*B*a^2
*b^3*c^2*d*f^3+48*B*a^2*b^3*c*d^2*e*f^2+9*B*a^2*b^3*d^3*e^2*f-6*B*a*b^4*c^3
*f^3-19*B*a*b^4*c^2*d*e*f^2-19*B*a*b^4*c*d^2*e^2*f-6*B*a*b^4*d^3*e^3+14*B*b
^5*c^3*e*f^2-14*B*b^5*c^2*d*e^2*f+14*B*b^5*c*d^2*e^3+48*C*a^5*d^3*f^3-128*C
*a^4*b*c*d^2*f^3-128*C*a^4*b*d^3*e*f^2+103*C*a^3*b^2*c^2*d*f^3+344*C*a^3*b^
2*c*d^2*e*f^2+103*C*a^3*b^2*d^3*e^2*f-15*C*a^2*b^3*c^3*f^3-282*C*a^2*b^3*c^
2*d*e*f^2-282*C*a^2*b^3*c*d^2*e^2*f-15*C*a^2*b^3*d^3*e^3+42*C*a*b^4*c^3*e*f
^2+238*C*a*b^4*c^2*d*e^2*f+42*C*a*b^4*c*d^2*e^3-35*C*b^5*c^3*e^2*f-35*C*b^5

```
*c^2*d*e^3))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)
*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*
x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c
/d)/(-e/f+a/b))^(1/2))-2/105/b^3*d*f*(6*A*a^3*b^2*d^3*f^3-9*A*a^2*b^3*c*d^2
*f^3-9*A*a^2*b^3*d^3*e*f^2+19*A*a*b^4*c^2*d*f^3-20*A*a*b^4*c*d^2*e*f^2+19*A
*a*b^4*d^3*e^2*f-8*A*b^5*c^3*f^3+5*A*b^5*c^2*d*e*f^2+5*A*b^5*c*d^2*e^2*f-8*
A*b^5*d^3*e^3+8*B*a^4*b*d^3*f^3-19*B*a^3*b^2*c*d^2*f^3-19*B*a^3*b^2*d^3*e*f
^2+9*B*a^2*b^3*c^2*d*f^3+48*B*a^2*b^3*c*d^2*e*f^2+9*B*a^2*b^3*d^3*e^2*f-6*B
*a*b^4*c^3*f^3-19*B*a*b^4*c^2*d*e*f^2-19*B*a*b^4*c*d^2*e^2*f-6*B*a*b^4*d^3*
e^3+14*B*b^5*c^3*e*f^2-14*B*b^5*c^2*d*e^2*f+14*B*b^5*c*d^2*e^3+48*C*a^5*d^3
*f^3-128*C*a^4*b*c*d^2*f^3-128*C*a^4*b*d^3*e*f^2+103*C*a^3*b^2*c^2*d*f^3+34
4*C*a^3*b^2*c*d^2*e*f^2+103*C*a^3*b^2*d^3*e^2*f-15*C*a^2*b^3*c^3*f^3-282*C*
a^2*b^3*c^2*d*e*f^2-282*C*a^2*b^3*c*d^2*e^2*f-15*C*a^2*b^3*d^3*e^3+42*C*a*b
^4*c^3*e*f^2+238*C*a*b^4*c^2*d*e^2*f+42*C*a*b^4*c*d^2*e^3-35*C*b^5*c^3*e^2*
f-35*C*b^5*c^2*d*e^3)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3*(e/f-c/d)*((x+e/f
)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b
*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)
*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1
/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))
))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.01 (sec) , antiderivative size = 9150, normalized size of antiderivative = 5.33

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^(9/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{9/2}}\,dx$$

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2),x)
```

```
[Out] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2), x)
```

3.67 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1235

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf))}{}$$

$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf)))\sqrt{a}}{315bd^3f^3}$$

$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$$+\frac{2\sqrt{-bc+ad}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7Adf)-C(}{}$$

$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16Bde+3Bcf\right.}{}$$

```
[Out] -2/63*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e))*(b*x+a)^(3/2)*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/d^2/f^2+2/9*C*(b*x+a)^(5/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d
/f-2/315*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*c*e)-(-3*a*d*f+4*b*c*
f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e)))*(d*x+c)^(3/2)*(b*x+a)^
(1/2)*(f*x+e)^(1/2)/b/d^3/f^3-2/945*(5*b*d*f*(7*a*d*f*(-9*A*b*d*f+C*a*c*f+3
*C*a*d*e+5*C*b*c*e)-(a*c*f+3*a*d*e+3*b*c*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+
8*C*d*e)))+2*(1/2*a*d*f-b*(c*f+2*d*e))*(7*b*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d
*e+5*C*b*c*e)-(-3*a*d*f+4*b*c*f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C
```

```
*d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d^3/f^4+2/315*(8*a^4
*C*d^4*f^4+a^3*b*d^3*f^3*(-18*B*d*f-7*C*c*f+11*C*d*e)-3*a^2*b^2*d^2*f^2*(3*
d*f*(-7*A*d*f-3*B*c*f+4*B*d*e)-C*(-3*c^2*f^2-5*c*d*e*f+9*d^2*e^2))-a*b^3*d*
f*(2*C*(-16*c^3*f^3-18*c^2*d*e*f^2-33*c*d^2*e^2*f+92*d^3*e^3)+3*d*f*(7*A*d*
f*(-7*c*f+13*d*e)-B*(-19*c^2*f^2-29*c*d*e*f+72*d^2*e^2)))+b^4*(C*(-16*c^4*f
^4-16*c^3*d*e*f^3-21*c^2*d^2*e^2*f^2-40*c*d^3*e^3*f+128*d^4*e^4)+3*d*f*(7*A
*d*f*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-B*(-8*c^3*f^3-9*c^2*d*e*f^2-16*c*d^2*
e^2*f+48*d^3*e^3))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
f*x+e)^(1/2)/b^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3
15*(-a*f+b*e)*(-c*f+d*e)*(4*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-3*B*d*f-C*c*f+3
*C*d*e)-3*a*b^2*d*f*(3*d*f*(-21*A*d*f+3*B*c*f+16*B*d*e)-5*C*(c^2*f^2+2*c*d*
e*f+8*d^2*e^2))-b^3*(C*(8*c^3*f^3+15*c^2*d*e*f^2+24*c*d^2*e^2*f+128*d^3*e^3
)+3*d*f*(7*A*d*f*(c*f+8*d*e)-4*B*(c^2*f^2+2*c*d*e*f+12*d^2*e^2))))*Elliptic
F(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*
(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b
^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 2.96 (sec) , antiderivative size = 1235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{2C(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^{5/2}}{9bdf}$$
$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(c+dx)^{3/2}\sqrt{e+fx}(a+bx)^{3/2}}{63bd^2f^2}$$
$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9Bdf)))(c}{315bd^3f^3}$$
$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf}{}$$
$$+\frac{2\sqrt{ad-bc}((C(128d^4e^4-40cd^3fe^3-21c^2d^2f^2e^2-16c^3df^3e-16c^4f^4)+3df(7Adf(8d^2e^2-3cdfe-2c^2f}{}$$
$$+\frac{2\sqrt{ad-bc}(be-af)(de-cf)\left(-(((C(128d^3e^3+24cd^2fe^2+15c^2df^2e+8c^3f^3)+3df(7Adf(8de+cf)-4.\right.}{}$$

```
[In] Int[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]

[Out] (-2*(5*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*
c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*(
(a*d*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*
```

```
A*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(945*b^2*d^3*f^4)
- (2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (6*b*d*e + 4*
b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f)))*Sqrt[a + b*
x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(315*b*d^3*f^3) - (2*(4*a*C*d*f + b*(8*C*
d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(6
3*b*d^2*f^2) + (2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f
) + (2*Sqrt[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*
C*f - 18*B*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) -
C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(2*C*(92*d^3*e^3 - 33*c*
d^2*e^2*f - 18*c^2*d*e*f^2 - 16*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f)
- B*(72*d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d
^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d
*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9
*c^2*d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))]])/(315*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/
(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d^3*
f^3 + 3*a^2*b*d^2*f^2*(3*C*d*e - c*C*f - 3*B*d*f) - 3*a*b^2*d*f*(3*d*f*(16*
B*d*e + 3*B*c*f - 21*A*d*f) - 5*C*(8*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - b^3*
(C*(128*d^3*e^3 + 24*c*d^2*e^2*f + 15*c^2*d*e*f^2 + 8*c^3*f^3) + 3*d*f*(7*A
*d*f*(8*d*e + c*f) - 4*B*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3
15*b^3*d^(7/2)*f^5*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
```

```
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{2\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(-\frac{1}{2}b(5bcCe+3aCde+acCf-9Abdf)-\frac{1}{2}b(4aCdf+b(8Cde+6cCf-9Bdf))x)}{\sqrt{e+fx}}\,dx}{9b^2df}$$

$$= -\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2} + \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{4\int \frac{\sqrt{a+bx}\sqrt{c+dx}(-\frac{1}{4}b(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-\frac{1}{4}b(7bdf(5bcCe}{\sqrt{e+fx}}}{63b^2d^2f^2}$$

$$= -\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9}{315bd^3f^3} - \frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2} + \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{8\int \frac{\sqrt{c+dx}\left(-\frac{1}{8}b(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf}{}}{}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf}{} - \frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9}{315bd^3f^3} - \frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2} + \frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} + \frac{16\int \frac{-\frac{1}{16}b\Big(3bcf(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf}{}}{}$$

$$=$$

$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cC$$

$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-}{315bd^3f^3}$$

$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$$+\frac{((be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16Bde+3Bcf\right.}{}$$

$$+\frac{(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7Adf)-C(9}{}$$

$$=$$

$$-\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cC$$

$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-}{315bd^3f^3}$$

$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$$+\frac{\Big((be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16Bde+3Bc\right.}{}$$

$$+\frac{\Big((8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7Adf)-C}{}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf}{}$$
$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9}{315bd^3f^3}$$
$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$
$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$
$$+\frac{2\sqrt{-bc+ad}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7}{}$$
$$+\frac{\Big((be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16Bde+3Bcf\right.}{}$$

$$= -\frac{2\big(5bdf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf}{}$$
$$-\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9}{315bd^3f^3}$$
$$-\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{63bd^2f^2}$$
$$+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$
$$+\frac{2\sqrt{-bc+ad}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)-3a^2b^2d^2f^2(3df(4Bde-3Bcf-7}{}$$
$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^3Cd^3f^3+3a^2bd^2f^2(3Cde-cCf-3Bdf)-3ab^2df(3df(16B\right.}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.13 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$
$$-\frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}(8a^4Cd^4f^4+a^3bd^3f^3(11Cde-7cCf-18Bdf)+3a^2b^2d^2f^2(3df(-4Bde+3Bcf+7Adf\right.}{}$$

```
[In] Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x
]
```

```
[Out] (-2*(-(b^2*Sqrt[-a + (b*c)/d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e -
7*c*C*f - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*f + 7*A*d*
f) + C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184*d^3*e^3 +
66*c*d^2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*(13*d*e - 7*
c*f) + B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 -
40*c*d^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*
(7*A*d*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + B*(-48*d^3*e^3 + 16*c*d^2*e^
2*f + 9*c^2*d*e*f^2 + 8*c^3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b
*c)/d]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(4*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2
*(3*B*d*f + C*(-2*d*e + c*f + d*f*x)) - a*b^2*d*f*(9*d*f*(14*A*d*f + B*(-11
*d*e + 3*c*f + 8*d*f*x)) + C*(-15*c^2*f^2 + c*d*f*(-19*e + 11*f*x) + d^2*(8
4*e^2 - 61*e*f*x + 50*f^2*x^2))) + b^3*(C*(-8*c^3*f^3 + 3*c^2*d*f^2*(-3*e +
 2*f*x) + c*d^2*f*(-12*e^2 + 7*e*f*x - 5*f^2*x^2) + d^3*(64*e^3 - 48*e^2*f*
x + 40*e*f^2*x^2 - 35*f^3*x^3)) - 3*d*f*(7*A*d*f*(-4*d*e + c*f + 3*d*f*x) +
 B*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x + 5*f^2*x^2)
)))) - I*(b*c - a*d)*f*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*C*f
 - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*f + 7*A*d*f) + C*
(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184*d^3*e^3 + 66*c*d^
2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*(13*d*e - 7*c*f) +
B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d^
3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d*
f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + B*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9
*c^2*d*e*f^2 + 8*c^3*f^3))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x
))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d
]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(d*e
 - c*f)*(4*a^3*C*d^3*f^3 + 3*a^2*b*d^3*f^2*(2*C*e - 3*B*f) - 3*a*b^2*d*f*(3
*d*f*(-11*B*d*e - 5*B*c*f + 14*A*d*f) + 2*C*(14*d^2*e^2 + 7*c*d*e*f + 4*c^2
*f^2)) + b^3*(4*C*(16*d^3*e^3 + 9*c*d^2*e^2*f + 6*c^2*d*e*f^2 + 4*c^3*f^3)
+ 3*d*f*(14*A*d*f*(2*d*e + c*f) - B*(24*d^2*e^2 + 13*c*d*e*f + 8*c^2*f^2)))
)*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(
a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e -
a*d*f)/(b*c*f - a*d*f)]))/(315*b^4*Sqrt[-a + (b*c)/d]*d^4*f^5*Sqrt[a + b*x]
*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 2108, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	2108
default	Expression too large to display	15736

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/9*C*b/f*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+
4*b*d*e))/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*
x+b*c*e*x+a*c*e)^(1/2)+2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*a^2*d+2*C*a*b*c-2/9
*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C
*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*x*(b
*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)
+2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*B*b*c*a+C*a^2*c-2/3*C*b/f*a*c*e-2/7*(b^2*
B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(5/2*a*c*f
+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*a^2*d+2*C*a*b*c-2/9*
C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*
b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*
d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*
x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a^2*c-2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*
a^2*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*
a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3
*b*d*e))/b/d/f*a*c*e-2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*B*b*c*a+C*a^2*c-2/3*C
*b/f*a*c*e-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*
e))/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*
a^2*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*
a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3
*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*
c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/
d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e
*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e
/f+a/b))^(1/2))+2*(a^2*A*d+2*A*a*b*c+B*a^2*c-4/7*(b^2*B*d+2*C*a*b*d+C*b^2*c
-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*a*c*e-2/5*(A*b^2*d+2*B*a*b*d+B*
b^2*c+C*a^2*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*
B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3
*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(2*A*a*b*d+b^2*A
*c+B*a^2*d+2*B*b*c*a+C*a^2*c-2/3*C*b/f*a*c*e-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c
-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)
-2/5*(A*b^2*d+2*B*a*b*d+B*b^2*c+C*a^2*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*
```

```
a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+
4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))
/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-
e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b
*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f
)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e
/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1931, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/945*(3*(35*C*b^5*d^5*f^5*x^3 - 64*C*b^5*d^5*e^3*f^2 + 12*(C*b^5*c*d^4 + (
7*C*a*b^4 + 6*B*b^5)*d^5)*e^2*f^3 + (9*C*b^5*c^2*d^3 - (19*C*a*b^4 + 15*B*b
^5)*c*d^4 - 3*(2*C*a^2*b^3 + 33*B*a*b^4 + 28*A*b^5)*d^5)*e*f^4 + (8*C*b^5*c
^3*d^2 - 3*(5*C*a*b^4 + 4*B*b^5)*c^2*d^3 + 3*(C*a^2*b^3 + 9*B*a*b^4 + 7*A*b
^5)*c*d^4 - (4*C*a^3*b^2 - 9*B*a^2*b^3 - 126*A*a*b^4)*d^5)*f^5 - 5*(8*C*b^5
*d^5*e*f^4 - (C*b^5*c*d^4 + (10*C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 + (48*C*b^
5*d^5*e^2*f^3 - (7*C*b^5*c*d^4 + (61*C*a*b^4 + 54*B*b^5)*d^5)*e*f^4 - (6*C*
b^5*c^2*d^3 - (11*C*a*b^4 + 9*B*b^5)*c*d^4 - 3*(C*a^2*b^3 + 24*B*a*b^4 + 21
*A*b^5)*d^5)*f^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (128*C*b^5
*d^5*e^5 - 8*(13*C*b^5*c*d^4 + (31*C*a*b^4 + 18*B*b^5)*d^5)*e^4*f - (25*C*b
^5*c^2*d^3 - 2*(113*C*a*b^4 + 60*B*b^5)*c*d^4 - (95*C*a^2*b^3 + 288*B*a*b^4
 + 168*A*b^5)*d^5)*e^3*f^2 - (10*C*b^5*c^3*d^2 - 15*(3*C*a*b^4 + 2*B*b^5)*c
^2*d^3 + 3*(37*C*a^2*b^3 + 91*B*a*b^4 + 49*A*b^5)*c*d^4 - (20*C*a^3*b^2 - 1
17*B*a^2*b^3 - 357*A*a*b^4)*d^5)*e^2*f^3 - (8*C*b^5*c^4*d - (22*C*a*b^4 + 1
5*B*b^5)*c^3*d^2 + 3*(5*C*a^2*b^3 + 21*B*a*b^4 + 14*A*b^5)*c^2*d^3 + 7*(2*C
*a^3*b^2 - 21*B*a^2*b^3 - 54*A*a*b^4)*c*d^4 - (7*C*a^4*b - 27*B*a^3*b^2 + 1
68*A*a^2*b^3)*d^5)*e*f^4 - (16*C*b^5*c^5 - 8*(5*C*a*b^4 + 3*B*b^5)*c^4*d +
(22*C*a^2*b^3 + 69*B*a*b^4 + 42*A*b^5)*c^3*d^2 + (7*C*a^3*b^2 - 51*B*a^2*b^
3 - 168*A*a*b^4)*c^2*d^3 + (11*C*a^4*b - 36*B*a^3*b^2 + 357*A*a^2*b^3)*c*d^
4 - (8*C*a^5 - 18*B*a^4*b + 63*A*a^3*b^2)*d^5)*f^5)*sqrt(b*d*f)*weierstrass
PInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d +
a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^
3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*
x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(128*C*b^5*d^5*e^4*f - 8*(5*C*b^5*c
```

```
*d^4 + (23*C*a*b^4 + 18*B*b^5)*d^5)*e^3*f^2 - 3*(7*C*b^5*c^2*d^3 - 2*(11*C*
a*b^4 + 8*B*b^5)*c*d^4 - (9*C*a^2*b^3 + 72*B*a*b^4 + 56*A*b^5)*d^5)*e^2*f^3
 - (16*C*b^5*c^3*d^2 - 9*(4*C*a*b^4 + 3*B*b^5)*c^2*d^3 + 3*(5*C*a^2*b^3 + 2
9*B*a*b^4 + 21*A*b^5)*c*d^4 - (11*C*a^3*b^2 - 36*B*a^2*b^3 - 273*A*a*b^4)*d
^5)*e*f^4 - (16*C*b^5*c^4*d - 8*(4*C*a*b^4 + 3*B*b^5)*c^3*d^2 + 3*(3*C*a^2*
b^3 + 19*B*a*b^4 + 14*A*b^5)*c^2*d^3 + (7*C*a^3*b^2 - 27*B*a^2*b^3 - 147*A*
a*b^4)*c*d^4 - (8*C*a^4*b - 18*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*f^5)*sqrt(b*d
*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 -
 a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2
 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*
b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), we
ierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a
*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 +
 a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^
3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*
(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^4*d^5*f^6)
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x
)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}}\,dx$$

```
[In] int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x)
```

3.68 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 766

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7Bdf)))\sqrt{\ldots}}{105b^2d^2f^3}$$

$$-\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}(3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cC\ldots}{\ldots}$$

$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-10A\ldots\right.}{105b^3d^{5/2}f^4\sqrt{c+dx}\ldots}$$

```
[Out] 2/7*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-2/35*(4*a*C*d*f+b*(-7
*B*d*f+4*C*c*f+6*C*d*e))*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^
2-2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+2
*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2
)*(f*x+e)^(1/2)/b^2/d^2/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f+3*C
*a*d*e+3*C*b*c*e)-(a*c*f+3*a*d*e+b*c*e)*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*
d*e)))+2*(1/2*b*c*f-d*(a*f+b*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C
*b*c*e)+(a*d*f-2*b*(c*f+2*d*e))*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e))))*
EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))
^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(5
```

```
/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/105*(-a*f+b*e)*(-c*f+d
*e)*(4*a^2*C*d^2*f^2+a*b*d*f*(-7*B*d*f-2*C*c*f+8*C*d*e)-b^2*(7*d*f*(-10*A*d
*f+B*c*f+8*B*d*e)-4*C*(c^2*f^2+2*c*d*e*f+12*d^2*e^2)))*EllipticF(d^(1/2)*(b
*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1
/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/d^(5/2)/f
^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4a^2Cd^2f^2+abdf(-7Bdf-2cCf+8Cde)-(b^2(7df(-1\ldots}{105b^3d^{5/2}f^4\sqrt{c+dx}\ldots}$$

$$-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCd\ldots}{\ldots}$$

$$-\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(acCf+3aCde-7Abdf+3bcCe)+(adf-2b(cf+2de))(4aCdf+b\ldots}{105b^2d^2f^3}$$

$$-\frac{2\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}(4aCdf+b(-7Bdf+4cCf+6Cde))}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(
2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f)))*Sqrt[a + b*x]*
Sqrt[c + d*x]*Sqrt[e + f*x])/(105*b^2*d^2*f^3) - (2*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(35*b*d
^2*f^2) + (2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - (
2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7
*A*b*d*f) - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7
*B*d*f))) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e
 + 4*c*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(
d*(b*e - a*f))])/(105*b^3*d^(5/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e
 - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2
+ a*b*d*f*(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*
```

```
A*d*f) - 4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(105*b^3*d^(5/2
)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
```

```
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{1}{2}b(3bcCe+3aCde+acCf-7Abdf)-\frac{1}{2}b(4aCdf+b(6Cde+4cCf-7Bdf))x\right)}{\sqrt{e+fx}}\,dx}{7b^2df}$$

$$= -\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{4\int \frac{\sqrt{c+dx}\left(-\frac{1}{4}b(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-\frac{1}{4}b(5bdf(3bcCe+3aC\right.}{\sqrt{a+bx}\sqrt{e+fx}}}{35b^2d^2f^2}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7\ldots}{105b^2d^2f^3} - \frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{8\int \frac{-\frac{1}{8}b(3bcf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-(bce+ade+acf)(5bdf(3bcC\ldots}{}}{}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7\ldots}{105b^2d^2f^3} - \frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{((be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-10Adf)\ldots\right.}{105b^2d^2f^4} - \frac{(3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-\ldots}{}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-7\ldots}{105b^2d^2f^3} - \frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} + \frac{\left((be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-10Adf)\ldots\right.\right.}{105b^2d^2f^4\sqrt{c+dx}} - \frac{\left((3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf\ldots\right.}{}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-}{105b^2d^2f^3}$$
$$-\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2}$$
$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$
$$-\frac{2\sqrt{-bc+ad}\left(3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6C\right.}{}$$
$$+\frac{\left((be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-10Adf\right.\right.}{105b^2d^2f^4\sqrt{c+}}$$

$$= -\frac{2(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b(6Cde+4cCf-}{105b^2d^2f^3}$$
$$-\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35bd^2f^2}$$
$$+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$
$$-\frac{2\sqrt{-bc+ad}\left(3bdf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6C\right.}{}$$
$$+\frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bc\right.}{105b^3d^{5/2}f^4\sqrt{c}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.61 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(8a^3Cd^3f^3+a^2bd^2f^2(9Cde-5cCf-14Bdf)+ab^2df(7df(-3Bde+2Bcf+5Adf)+C\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (2*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*
C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d
^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9
*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 -
3*c*d*e*f - 2*c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*
(a + b*x)*(c + d*x)*(e + f*x)*(-4*a^2*C*d^2*f^2 + a*b*d*f*(7*B*d*f + C*(-5*
d*e + 2*c*f + 3*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e + c*f + 3*d*f*x))
 + C*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x + 5*f^2*x^
2)))) + I*(b*c - a*d)*f*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c*C*f
 - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16*d^2*
e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^2 - 3*c
*d*e*f - 2*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sq
rt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt
[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f
)*(4*a^2*C*d^2*f^2 + a*b*d*f*(5*C*d*e + c*C*f - 7*B*d*f) - b^2*(7*d*f*(-4*B
*d*e - 2*B*c*f + 5*A*d*f) + C*(24*d^2*e^2 + 13*c*d*e*f + 8*c^2*f^2)))*(a +
b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x
))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/
(b*c*f - a*d*f)]))/(105*b^4*Sqrt[-a + (b*c)/d]*d^3*f^4*Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.57

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	10271

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/7/f*C*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/
d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)+2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f*C*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*
e)-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f
+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a
*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*
f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f*C*(5/2*a
*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3
*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*
c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/
```

```
d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e
*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e
/f+a/b))^(1/2))+2*(A*a*d+A*b*c+B*a*c-4/7/f*C*a*c*e-2/5*(B*b*d+C*a*d+C*b*c-2
/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3
*(A*b*d+B*a*d+B*b*c+C*a*c-2/7/f*C*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b*
d+C*a*d+C*b*c-2/7/f*C*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b
*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a
/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f
*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE((
(x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e
/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 - (5*C*b^4*c*d^3 + (5
*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 - (2*C*a*b^3 + 7*B*b^4)*
c*d^3 + (4*C*a^2*b^2 - 7*B*a*b^3 - 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4*d^4*e*f^
3 - (C*b^4*c*d^3 + (C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 - 8*(5*C*b^4*c*d^3 + (5*C*a*b^3 + 7*
B*b^4)*d^4)*e^3*f - (10*C*b^4*c^2*d^2 - 7*(6*C*a*b^3 + 7*B*b^4)*c*d^3 + (10
*C*a^2*b^2 - 49*B*a*b^3 - 70*A*b^4)*d^4)*e^2*f^2 - (5*C*b^4*c^3*d - 7*(C*a*
b^3 + 2*B*b^4)*c^2*d^2 - 7*(C*a^2*b^2 - 8*B*a*b^3 - 10*A*b^4)*c*d^3 + (5*C*
a^3*b - 14*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 - (8*C*b^4*c^4 - (9*C*a*b^3 +
 14*B*b^4)*c^3*d - (4*C*a^2*b^2 - 21*B*a*b^3 - 35*A*b^4)*c^2*d^2 - (9*C*a^3
*b - 21*B*a^2*b^2 + 140*A*a*b^3)*c*d^3 + (8*C*a^4 - 14*B*a^3*b + 35*A*a^2*b
^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d +
 a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*
b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^
2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d
^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) +
3*(48*C*b^4*d^4*e^3*f - 8*(2*C*b^4*c*d^3 + (2*C*a*b^3 + 7*B*b^4)*d^4)*e^2*f
^2 - (9*C*b^4*c^2*d^2 - (8*C*a*b^3 + 21*B*b^4)*c*d^3 + (9*C*a^2*b^2 - 21*B*
a*b^3 - 70*A*b^4)*d^4)*e*f^3 - (8*C*b^4*c^3*d - (5*C*a*b^3 + 14*B*b^4)*c^2*
d^2 - (5*C*a^2*b^2 - 14*B*a*b^3 - 35*A*b^4)*c*d^3 + (8*C*a^3*b - 14*B*a^2*b
^2 + 35*A*a*b^3)*d^4)*f^4)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (
```

```
b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2),
-4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a
*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^
2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4
/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b
^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d
*f))))/(b^4*d^4*f^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{\sqrt{a+bx}\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}}\,dx$$

```
[In] int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
[Out] int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x)
```

3.69 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 527

$$
\begin{aligned}
&\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx \\
&= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} \\
&\quad + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} \\
&\quad - \frac{2\sqrt{-bc+ad}(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2cCf-}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} \\
&\quad - \frac{2\sqrt{-bc+ad}(de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)\sqrt{}}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}
\end{aligned}
$$

```
[Out] 2/5*C*(d*x+c)^(3/2)*(b*x+a)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(4*a*C*d*f+b*(-5
*B*d*f+2*C*c*f+4*C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^
2-2/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+3*C*a*d*e+C*b*c*e)-(2*a*d*f-b*c*f+2*b*d
*e)*(4*a*C*d*f+b*(-5*B*d*f+2*C*c*f+4*C*d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d
*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/d^(3/2)/f^3/(d*x+c)^(1/2)/(b*(f*x
+e)/(-a*f+b*e))^(1/2)-2/15*(-c*f+d*e)*(4*a^2*C*d*f^2+a*b*f*(-5*B*d*f-C*c*f+
3*C*d*e)-b^2*(5*d*f*(-3*A*f+2*B*e)-C*e*(c*f+8*d*e)))*EllipticF(d^(1/2)*(b*x
+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2
)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/d^(3/2)/f^3
/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-(b^2(5df(2Be-3Af)}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

$$-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+3aCde-5Abdf+bcCe)-(2adf-bcf+2bde)(4aCdf+b(}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$-\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(4aCdf+b(-5Bdf+2cCf+4Cde))}{15b^2df^2}$$

$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a*C*d*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x])/(15*b^2*d*f^2) + (2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e +
 a*c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e
 + 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellip
ticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e
 - c*C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt
[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSi
n[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*
f))])/(15*b^3*d^(3/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
```

```
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}b(bcCe+3aCde+acCf-5Abdf)-\frac{1}{2}b(4aCdf+b(4Cde+2cCf-5Bdf))x\right)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx}{5b^2df}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{4\int \frac{-\frac{1}{4}b(3bcf(bcCe+3aCde+acCf-5Abdf)-(bce+ade+acf)(4aCdf+b(4Cde+2cCf-5Bdf)))-\frac{1}{4}b(3bdf(bcCe+3aCde+acCf-5\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{15b^3df^2}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{\left((de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)\right)\int \cdots}{15b^2df^3} - \frac{(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2cCf-5B\cdots}{15b^2df^3}$$

$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{\left((de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)\sqrt{\cdots}\right.}{15b^2df^3\sqrt{c+dx}} - \frac{\left((3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2cCf-5\cdots\right.}{15b^2df^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= -\frac{2(4aCdf + b(4Cde + 2cCf - 5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{2\sqrt{-bc+ad}(3bdf(bcCe+3aCde+acCf-5Abdf) - (2bde-bcf+2adf)(4aCdf+b(4Cde+2}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{\left((de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)\sqrt{\frac{b(c}{bc}}}{15b^2df^3\sqrt{c+dx}\sqrt{e+fx}}$$

$$= -\frac{2(4aCdf + b(4Cde + 2cCf - 5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2} + \frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} - \frac{2\sqrt{-bc+ad}(3bdf(bcCe+3aCde+acCf-5Abdf) - (2bde-bcf+2adf)(4aCdf+b(4Cde+2}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{-bc+ad}(de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.83 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

$$= \frac{2\sqrt{a+bx}\left(\frac{b^2(8a^2Cd^2f^2+abdf(7Cde-3cCf-10Bdf)+b^2(5df(-2Bde+Bcf+3Adf)+C(8d^2e^2-3cdef-2c^2f^2)))(c+dx)(e+fx)}{a+bx} + b^2df(c\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*Sqrt[a + b*x]*((b^2*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B
*d*f) + b^2*(5*d*f*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f
- 2*c^2*f^2)))*(c + d*x)*(e + f*x))/(a + b*x) + b^2*d*f*(c + d*x)*(e + f*x)
*(5*b*B*d*f - 4*a*C*d*f + b*C*(-4*d*e + c*f + 3*d*f*x)) + (I*(b*c - a*d)*f*
(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B*d*f) + b^2*(5*d*f*(-2*
```

```
B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2)))*Sqrt[a +
 b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*E
llipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)])/Sqrt[-a + (b*c)/d] + I*b*Sqrt[-a + (b*c)/d]*d*f*(d*e - c*f)*(5
*b*B*d*f - 4*a*C*d*f - 2*b*C*(2*d*e + c*f))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x)
)/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt
[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^4*d
^2*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5fb}+\frac{2\left(Bd+Cc-\frac{2C(2adf+2bcf+2bde)}{5fb}\right)\sqrt{bdf\,x^3+adf\,x^2-}}{3bdf}\right.}{}$
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/5*C/f/b*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d
*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2
*(A*c-2/5*C/f/b*a*c*e-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d
/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x
+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c
*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e
/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*d+B*c-2/5*C/f/b*(3/2*a*c
*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e))/b
/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/
f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d
*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/
(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f
-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$
$$= \frac{2\left(3\left(3\,Cb^3d^3f^3x-4\,Cb^3d^3ef^2+\left(Cb^3cd^2-\left(4\,Cab^2-5\,Bb^3\right)d^3\right)f^3\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}-\left(8\,Cb^3d^3e^3\right.\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 + (C*b^3*c*d^2 - (4*C*a*b^2
- 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3*d
^3*e^3 - (7*C*b^3*c*d^2 - (3*C*a*b^2 - 10*B*b^3)*d^3)*e^2*f - (2*C*b^3*c^2*
d + 2*(C*a*b^2 - 5*B*b^3)*c*d^2 - (3*C*a^2*b - 5*B*a*b^2 + 15*A*b^3)*d^3)*e
*f^2 - (2*C*b^3*c^3 + (2*C*a*b^2 - 5*B*b^3)*c^2*d + (7*C*a^2*b - 10*B*a*b^2
 + 30*A*b^3)*c*d^2 - (8*C*a^3 - 10*B*a^2*b + 15*A*a*b^2)*d^3)*f^3)*sqrt(b*d
*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c
^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c
*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
 (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3)
, 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*d^3*e^2*f -
 (3*C*b^3*c*d^2 - (7*C*a*b^2 - 10*B*b^3)*d^3)*e*f^2 - (2*C*b^3*c^2*d + (3*C
*a*b^2 - 5*B*b^3)*c*d^2 - (8*C*a^2*b - 10*B*a*b^2 + 15*A*b^3)*d^3)*f^3)*sqr
t(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*
c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*
c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2
+ (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3
), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^
2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*
d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3),
 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^4*d^3*f^4)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}}\,dx$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),x
)
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

3.70 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 540

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{}}{3b^2(bc-ad)f(be-af)}$$

$$-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$+\frac{2\sqrt{-bc+ad}(8a^2Cdf^2-abf(3Cde+cCf+6Bdf)+b^2(3df(Be+Af)-Ce(2de-cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}}{3b^3\sqrt{d}f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$+\frac{2\sqrt{-bc+ad}(de-cf)(2bCe-3bBf+4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)+2/3*(4*a^2*C*d*f+b^2*(3*A*d*f+C*c*e)-a*b*(3*B*d*f+C*c*f+C*d*e
))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)+2/
3*(8*a^2*C*d*f^2-a*b*f*(6*B*d*f+C*c*f+3*C*d*e)+b^2*(3*d*f*(A*f+B*e)-C*e*(-c
*f+2*d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d
/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1
/2)/b^3/f^2/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2
/3*(-c*f+d*e)*(-3*B*b*f+4*C*a*f+2*C*b*e)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a
*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)
/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f^2/d^(1/2)/(d*x+c)^(1/
2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3}{3b^3\sqrt{d}f^2\sqrt{c+dx}(be-}$$
$$+\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(4a^2Cdf-ab(3Bdf+cCf+Cde)+b^2(3Adf+cCe))}{3b^2f(bc-ad)(be-af)}$$
$$-\frac{2(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{b\sqrt{a+bx}(bc-ad)(be-af)}$$
$$+\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4aCf-3bBf+2bCe)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]),x]
```

```
[Out] (2*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*Sq
rt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b^2*(b*c - a*d)*f*(b*e - a*f))
- (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + (2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(
3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sq
rt[d]*f^2*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*S
qrt[-(b*c) + a*d]*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*
Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^
3*Sqrt[d]*f^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
```

```
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
```

```
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{b^2(Bc+2Ad)e+a^2C(3de+cf)-ab(cCe+3Bde+Bcf-Adf)}{2b}+\frac{1}{2}\left(-\frac{4a^2Cdf}{b}-b(cCe+3Adf)+a(Cde+cCf+3Bdf)\right)x\right)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx}{(bc-ad)(be-af)}$$

$$= \frac{2(4a^2Cdf + b^2(cCe+3Adf) - ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{4\int \frac{-\frac{(bc-ad)\left(4a^2Cf(de+cf)-b^2e(cCe-3Bcf-3Adf)-ab(3Bf(de+cf)+Ce(de+3cf))\right)}{4b}-\frac{(bc-ad)\left(8a^2Cdf^2-abf(3Cde+cCf+6Bdf)+b^2(3df(\ldots\right.}{4b}\ldots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\ldots}{3b(bc-ad)f(be-af)}$$

$$= \frac{2(4a^2Cdf + b^2(cCe+3Adf) - ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{((de-cf)(2bCe-3bBf+4aCf))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3b^2f^2} + \frac{(8a^2Cdf^2 - abf(3Cde+cCf+6Bdf) + b^2(3df(Be+Af) - Ce(2de-cf)))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{3b^2f^2(be-af)}$$

$$= \frac{2(4a^2Cdf + b^2(cCe+3Adf) - ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)} - \frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{\left((de-cf)(2bCe-3bBf+4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}\,dx}{3b^2f^2\sqrt{c+dx}} + \frac{\left((8a^2Cdf^2 - abf(3Cde+cCf+6Bdf) + b^2(3df(Be+Af) - Ce(2de-cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+\ldots}\right.}{3b^2f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= \frac{2(4a^2Cdf + b^2(cCe + 3Adf) - ab(Cde + cCf + 3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)}$$
$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$+ \frac{2\sqrt{-bc+ad}(8a^2Cdf^2 - abf(3Cde + cCf + 6Bdf) + b^2(3df(Be+Af) - Ce(2de-cf)))\sqrt{\frac{b(c+}{bc-}}}{3b^3\sqrt{d}f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+ \frac{\left((de-cf)(2bCe - 3bBf + 4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}\,dx}{3b^2f^2\sqrt{c+dx}\sqrt{e+fx}}$$
$$= \frac{2(4a^2Cdf + b^2(cCe + 3Adf) - ab(Cde + cCf + 3Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3b^2(bc-ad)f(be-af)}$$
$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$+ \frac{2\sqrt{-bc+ad}(8a^2Cdf^2 - abf(3Cde + cCf + 6Bdf) + b^2(3df(Be+Af) - Ce(2de-cf)))\sqrt{\frac{b(c+}{bc-}}}{3b^3\sqrt{d}f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+ \frac{2\sqrt{-bc+ad}(de-cf)(2bCe - 3bBf + 4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)\middle|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.02 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx =$$
$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(-8a^2Cdf^2 + abf(3Cde + cCf + 6Bdf) + b^2(-3df(Be+Af) + Ce(2de-cf)))(c+dx)(e\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(-8*a^2*C*d*f^2 + a*b*f*(3*C*d*e + c*C*f + 6*B*
d*f) + b^2*(-3*d*f*(B*e + A*f) + C*e*(2*d*e - c*f)))*(c + d*x)*(e + f*x) +
b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-(b*B) + a*C)
)*f - C*(b*e - a*f)*(a + b*x)) - I*(b*c - a*d)*f*(8*a^2*C*d*f^2 - a*b*f*(3*
C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) + C*e*(-2*d*e + c*f)))*(a
```

```
 + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a +
b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*
f)/(b*c*f - a*d*f)] - I*b*f*(d*e - c*f)*(4*a^2*C*d*f + b^2*(c*C*e + 3*A*d*f
) - a*b*(C*d*e + c*C*f + 3*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a
 + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (
b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(3*b^4*Sqrt[-a +
 (b*c)/d]*d*f^2*(b*e - a*f)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.59

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2\left(bdf\,x^{2}+bcfx+bdex+bce\right)\left(b^{2}A-abB+C\,a^{2}\right)}{b^{3}(af-be)\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^{2}+bcfx+bdex+bce\right)}}+\frac{2C\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+acfx+adex+bcex+ace}}{3b^{2}f}+\frac{2\left(A\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)/((x+a
/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/3*C/b^2/f*(b*d*f*x^3+a*d*f*x
^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d-B*a
*b*d+B*b^2*c+C*a^2*d-C*a*b*c)/b^3-(A*b^2-B*a*b+C*a^2)/b^3*(a*d*f-b*c*f-b*d*
e)/(a*f-b*e)-(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)-2/3*C/b^2/f*(1
/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)
/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^
2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/
d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(1/b^2*(B*b*d-C*a*d+C*b*c)-(A*b^
2-B*a*b+C*a^2)/b^2*d*f/(a*f-b*e)-2/3*C/b^2/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)
*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^
(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+
a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b)
)^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1336, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*(C*a*b^3*d^2*e*f^2 - (4*C*a^2*b^2 - 3*B*a*b^3 + 3*A*b^4)*d^2*f^3 + (
C*b^4*d^2*e*f^2 - C*a*b^3*d^2*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
+ e) + (2*C*a*b^3*d^2*e^3 - (2*C*a*b^3*c*d - (2*C*a^2*b^2 - 3*B*a*b^3)*d^2)
*e^2*f - (C*a*b^3*c^2 + 6*(C*a^2*b^2 - B*a*b^3)*c*d - (7*C*a^3*b - 6*B*a^2*
b^2 + 6*A*a*b^3)*d^2)*e*f^2 + (C*a^2*b^2*c^2 + (5*C*a^3*b - 3*B*a^2*b^2 - 3
*A*a*b^3)*c*d - (8*C*a^4 - 6*B*a^3*b + 3*A*a^2*b^2)*d^2)*f^3 + (2*C*b^4*d^2
*e^3 - (2*C*b^4*c*d - (2*C*a*b^3 - 3*B*b^4)*d^2)*e^2*f - (C*b^4*c^2 + 6*(C*
a*b^3 - B*b^4)*c*d - (7*C*a^2*b^2 - 6*B*a*b^3 + 6*A*b^4)*d^2)*e*f^2 + (C*a*
b^3*c^2 + (5*C*a^2*b^2 - 3*B*a*b^3 - 3*A*b^4)*c*d - (8*C*a^3*b - 6*B*a^2*b^
2 + 3*A*a*b^3)*d^2)*f^3)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^
2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
- 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*
f)/(b*d*f)) + 3*(2*C*a*b^3*d^2*e^2*f - (C*a*b^3*c*d - 3*(C*a^2*b^2 - B*a*b^
3)*d^2)*e*f^2 + (C*a^2*b^2*c*d - (8*C*a^3*b - 6*B*a^2*b^2 + 3*A*a*b^3)*d^2)
*f^3 + (2*C*b^4*d^2*e^2*f - (C*b^4*c*d - 3*(C*a*b^3 - B*b^4)*d^2)*e*f^2 + (
C*a*b^3*c*d - (8*C*a^2*b^2 - 6*B*a*b^3 + 3*A*b^4)*d^2)*f^3)*x)*sqrt(b*d*f)*
weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b
*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a
*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*
c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weiers
trassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c
*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b
^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^
3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b
*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(a*b^5*d^2*e*f^3 - a^2*b^4*d^2*f
^4 + (b^6*d^2*e*f^3 - a*b^5*d^2*f^4)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,(a+b\,x)^{3/2}}\,dx$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),x
)
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
x)
```

3.71 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 597

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\sqrt{d}(8a^3Cdf^2-a^2bf(13Cde+7cCf+2Bdf)+ab^2(3Ce(de+4cf)+f(4Bde+Bcf-Adf))-b^3(Adef+\cdots}{3b^3\sqrt{-bc+ad}f(be-af)^2\sqrt{c+dx}\sqrt{\cdots}} + \frac{2(de-cf)(4a^2Cdf+b^2(3cCe+Adf)-ab(Bdf+3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{\cdots}}{\sqrt{-b\cdots}}\right.\right.}{3b^3\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)-2/3*(4*a^2*C*f+b^2*(-2*A*f+3*B*e)-a*b*(B*f+6*C*e))*(d*x+c)^
(1/2)*(f*x+e)^(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^(1/2)+2/3*(8*a^3*C*d*f^2-a^2*b
*f*(2*B*d*f+7*C*c*f+13*C*d*e)+a*b^2*(3*C*e*(4*c*f+d*e)+f*(-A*d*f+B*c*f+4*B*
d*e))-b^3*(A*d*e*f+c*(-2*A*f^2+3*B*e*f+3*C*e^2)))*EllipticE(d^(1/2)*(b*x+a)
^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c
)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^3/f/(-a*f+b*e)^2/(a*d-b*c)^(1/2)/(d*x+c
)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(4*a^2*C*d*f+b^2*(A*d*f
+3*C*c*e)-a*b*(B*d*f+3*C*(c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
b*(f*x+e)/(-a*f+b*e))^(1/2)/b^3/f/(-a*f+b*e)/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c
)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1628, 155, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \frac{2(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf}{3b^3\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}}$$
$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}(4a^2Cf-ab(Bf+6Ce)+b^2(3Be-2Af))}{3b^2\sqrt{a+bx}(be-af)^2}$$
$$+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab^2(f(-Adf+Bcf+4Bde)+3Ce(4c}{3b^3f\sqrt{c+dx}\sqrt{ad-bc}(be-af)^2}$$
$$-\frac{2(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(4*a^2*C*f + b^2*(3*B*e - 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqr
t[e + f*x])/(3*b^2*(b*e - a*f)^2*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C)
)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(3/
2)) + (2*Sqrt[d]*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) -
b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f
) + f*(4*B*d*e + B*c*f - A*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)^2*Sqrt[c
+ d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*(d*e - c*f)*(4*a^2*C*d*f + b^2
*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c
 - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^3*Sqrt[d
]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
```

```
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(3Bce-2Acf)-ab(3cCe+3Bde+Bcf-3Adf)}{2b}+\frac{1}{2}\left(aBdf-\frac{4a^2Cdf}{b}+3aC(de+cf)-b(3cCe+Adf)\right)x\right)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx}{3(bc-ad)(be-af)}$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{4\int \frac{\frac{4a^3Cdf(de+cf)-b^3ce(3cCe+3Bde-Adf)+ab^2\left(6c^2Cef+d^2e(3Be-2Af)+cd\left(9Ce^2+2Bef+Af^2\right)\right)-a^2b\left(Bdf(de+cf)+C\left(6d^2e^2+11cdef+3\right.\right.}{4b}}{\sqrt{a+bx}\sqrt{c+dx}}}{3b(bc-ad)(be}$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{((de-cf)(4a^2Cdf+b^2(3cCe+Adf)-ab(Bdf+3C(de+cf))))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3b^2(bc-ad)f(be-af)} - \frac{(d(8a^3Cdf^2-a^2bf(13Cde+7cCf+2Bdf)-b^3(3cCe^2+Adef+cf(3Be-2Af))+ab^2(3Ce}{3b^2(bc-ad)f(be-af)^2}$$

$$= -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{\left((de-cf)(4a^2Cdf+b^2(3cCe+Adf)-ab(Bdf+3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{b}{bc}}}}{3b^2(bc-ad)f(be-af)\sqrt{c+dx}} - \frac{\left(d(8a^3Cdf^2-a^2bf(13Cde+7cCf+2Bdf)-b^3(3cCe^2+Adef+cf(3Be-2Af))+ab^2(3C\right.}{3b^2(bc-ad)f(be-af)^2\sqrt{c}}$$

$$= -\frac{2(4a^2Cf + b^2(3Be - 2Af) - ab(6Ce + Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\sqrt{d}(8a^3Cdf^2 - a^2bf(13Cde + 7cCf + 2Bdf) - b^3(3cCe^2 + Adef + cf(3Be - 2Af)) + ab^2(3C}{3b^3\sqrt{-bc+ad}f(be-af)^2} - \frac{\left((de-cf)(4a^2Cdf + b^2(3cCe + Adf) - ab(Bdf + 3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\right)\int \frac{}{\sqrt{a+bx}\sqrt{}}}{3b^2(bc-ad)f(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

$$= -\frac{2(4a^2Cf + b^2(3Be - 2Af) - ab(6Ce + Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}} - \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2\sqrt{d}(8a^3Cdf^2 - a^2bf(13Cde + 7cCf + 2Bdf) - b^3(3cCe^2 + Adef + cf(3Be - 2Af)) + ab^2(3C}{3b^3\sqrt{-bc+ad}f(be-af)^2} + \frac{2(de-cf)(4a^2Cdf + b^2(3cCe + Adf) - ab(Bdf + 3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}F\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3b^3\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.13 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = -\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}f(c+dx)(e+fx)\left((Ab^2 + a(-bB + aC))(bc-ad)(be-af) + (-5a^3Cdf + b^3(3Bce + Ad\right.\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x])
,x]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C
))*(b*c - a*d)*(b*e - a*f) + (-5*a^3*C*d*f + b^3*(3*B*c*e + A*d*e - 2*A*c*f
) - a*b^2*(6*c*C*e + 4*B*d*e + B*c*f - A*d*f) + a^2*b*(7*C*d*e + 4*c*C*f +
2*B*d*f))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d*f^2 - a
^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B
```

```
*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(
c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*d*e + 7
*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f)) + a*b^2
*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(a + b*x)^(3/2)*Sqrt[
(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*
ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]
 + I*b*(b*c - a*d)*f*(d*e - c*f)*(-4*a^2*C*f + b^2*(-3*B*e + 2*A*f) + a*b*(
6*C*e + B*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
+ f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^4*Sqrt[-a + (b*c)/d]*(b*c - a*d
)*f*(b*e - a*f)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(543) = 1086$.

Time = 4.20 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	15367

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/3*(A*b^2-B*a*b+C*a^2)/b^4/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*
x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*x+b
*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*a*b^2*d*f-2*A*b^3*c*
f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f
+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a*f-b*e)/((x+a/b)*(b*d*f*x^2+b
*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d-2*C*a*d+C*b*c)/b^3+1/3*(A*b^2-B*a*b+
C*a^2)/b^3*d*f/(a*f-b*e)-1/3/b^3*(a*d*f-b*c*f-b*d*e)*(A*a*b^2*d*f-2*A*b^3*c
*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^3*d*
f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/(a*f-b*e)-1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*a*
b^2*d*f-2*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b
^3*c*e-5*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a*f-b*e))*(e
/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f
+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e
*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))
^(1/2))+2*(C*d/b^2-1/3*d*f/b^2*(A*a*b^2*d*f-2*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b
*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^
2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(a*f-b*e))*(e/f-c/
d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d)
```

```
)^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*
c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/
f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/
b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 2429, normalized size of antiderivative = 4.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(2*C*a^3*b^3 - B*a^2*b^4
)*d^2)*e*f^2 - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (4*C*a^4*b^2 - B*a^3*b^3 - 2*
A*a^2*b^4)*d^2)*f^3 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (7*C*a^2*b^4 - 4*B*a*b^
5 + A*b^6)*d^2)*e*f^2 - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (5*C*a^3*b
^3 - 2*B*a^2*b^4 - A*a*b^5)*d^2)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f
*x + e) - (3*(C*a^2*b^4*c*d - C*a^3*b^3*d^2)*e^3 - (6*C*a^2*b^4*c^2 - 3*(5*
C*a^3*b^3 - 2*B*a^2*b^4)*c*d + (8*C*a^4*b^2 - 5*B*a^3*b^3 - A*a^2*b^4)*d^2)
*e^2*f + (3*(2*C*a^3*b^3 + B*a^2*b^4)*c^2 - (25*C*a^4*b^2 - 4*B*a^3*b^3 - 2
*A*a^2*b^4)*c*d + (17*C*a^5*b - 5*B*a^4*b^2 - 4*A*a^3*b^3)*d^2)*e*f^2 - ((2
*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (11*C*a^5*b - 2*B*a^4*b^2 + 2*A
*a^3*b^3)*c*d + (8*C*a^6 - 2*B*a^5*b - A*a^4*b^2)*d^2)*f^3 + (3*(C*b^6*c*d
- C*a*b^5*d^2)*e^3 - (6*C*b^6*c^2 - 3*(5*C*a*b^5 - 2*B*b^6)*c*d + (8*C*a^2*
b^4 - 5*B*a*b^5 - A*b^6)*d^2)*e^2*f + (3*(2*C*a*b^5 + B*b^6)*c^2 - (25*C*a^
2*b^4 - 4*B*a*b^5 - 2*A*b^6)*c*d + (17*C*a^3*b^3 - 5*B*a^2*b^4 - 4*A*a*b^5)
*d^2)*e*f^2 - ((2*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (11*C*a^3*b^3 - 2*B*
a^2*b^4 + 2*A*a*b^5)*c*d + (8*C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*d^2)*f^3
)*x^2 + 2*(3*(C*a*b^5*c*d - C*a^2*b^4*d^2)*e^3 - (6*C*a*b^5*c^2 - 3*(5*C*a^
2*b^4 - 2*B*a*b^5)*c*d + (8*C*a^3*b^3 - 5*B*a^2*b^4 - A*a*b^5)*d^2)*e^2*f +
 (3*(2*C*a^2*b^4 + B*a*b^5)*c^2 - (25*C*a^3*b^3 - 4*B*a^2*b^4 - 2*A*a*b^5)*
c*d + (17*C*a^4*b^2 - 5*B*a^3*b^3 - 4*A*a^2*b^4)*d^2)*e*f^2 - ((2*C*a^3*b^3
 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (11*C*a^4*b^2 - 2*B*a^3*b^3 + 2*A*a^2*b^4)*
c*d + (8*C*a^5*b - 2*B*a^4*b^2 - A*a^3*b^3)*d^2)*f^3)*x)*sqrt(b*d*f)*weiers
trassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c
*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b
^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^
3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b
*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(3*(C*a^2*b^4*c*d - C*a^3*b^3*
d^2)*e^2*f - (3*(4*C*a^3*b^3 - B*a^2*b^4)*c*d - (13*C*a^4*b^2 - 4*B*a^3*b^3
```

```
 + A*a^2*b^4)*d^2)*e*f^2 + ((7*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*c*d - (
8*C*a^5*b - 2*B*a^4*b^2 - A*a^3*b^3)*d^2)*f^3 + (3*(C*b^6*c*d - C*a*b^5*d^2
)*e^2*f - (3*(4*C*a*b^5 - B*b^6)*c*d - (13*C*a^2*b^4 - 4*B*a*b^5 + A*b^6)*d
^2)*e*f^2 + ((7*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (8*C*a^3*b^3 - 2*B*a^2
*b^4 - A*a*b^5)*d^2)*f^3)*x^2 + 2*(3*(C*a*b^5*c*d - C*a^2*b^4*d^2)*e^2*f -
(3*(4*C*a^2*b^4 - B*a*b^5)*c*d - (13*C*a^3*b^3 - 4*B*a^2*b^4 + A*a*b^5)*d^2
)*e*f^2 + ((7*C*a^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*c*d - (8*C*a^4*b^2 - 2*B*a
^3*b^3 - A*a^2*b^4)*d^2)*f^3)*x)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e
^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*
f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
 - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2
 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^
2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d -
 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f
)/(b*d*f))))/((a^2*b^7*c*d - a^3*b^6*d^2)*e^2*f^2 - 2*(a^3*b^6*c*d - a^4*b^
5*d^2)*e*f^3 + (a^4*b^5*c*d - a^5*b^4*d^2)*f^4 + ((b^9*c*d - a*b^8*d^2)*e^2
*f^2 - 2*(a*b^8*c*d - a^2*b^7*d^2)*e*f^3 + (a^2*b^7*c*d - a^3*b^6*d^2)*f^4)
*x^2 + 2*((a*b^8*c*d - a^2*b^7*d^2)*e^2*f^2 - 2*(a^2*b^7*c*d - a^3*b^6*d^2)
*e*f^3 + (a^3*b^6*c*d - a^4*b^5*d^2)*f^4)*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(5/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{5/2}}\,dx$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),x
)
```

```
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
x)
```

3.72 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1034

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6A}{15b^2(bc-ad)(be-af)^2(a+}$$
$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10Bef + 8}{}$$
$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$
$$+ \frac{2\sqrt{d}(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10Bef +}{}$$
$$+ \frac{2\sqrt{d}(de-cf)(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8Cde + 6}{15b^3(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{e}}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-6*
A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e))*(d*x+c)^(1/2)
*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)-2/15*(8*a^4*C*d^2*
f^2-a^3*b*d*f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5
*B*e)-c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d
*e)-C*(3*c^2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*
f*(-B*f+5*C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)
/b^2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(8*a^4*C*d^2*f^2-a^3*b*d*
f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5*B*e)-c^2*(8
*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d*e)-C*(3*c^2
*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*f*(-B*f+5*C*
```

```
e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*
c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^
(1/2)*(f*x+e)^(1/2)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+
e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B
*c*e)+a*b^2*(-6*A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e
))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*
e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2
)/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 1034, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1628, 155, 157, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{e+fx}(c+dx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$
$$-\frac{2(8Cd^2f^2a^4-bdf(23Cde+13cCf-2Bdf)a^3-b^2(df(7Bde+2Bcf-3Adf)-C(23d^2e^2+37cdfe+3c^2\ldots}{}$$
$$+\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\ldots}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$
$$+\frac{2\sqrt{d}(8Cd^2f^2a^4-bdf(23Cde+13cCf-2Bdf)a^3-b^2(df(7Bde+2Bcf-3Adf)-C(23d^2e^2+37cdfe+\ldots}{}$$
$$+\frac{2\sqrt{d}(de-cf)(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2A\ldots}{15b^3(ad-bc)^{3/2}(be-af)^2\sqrt{e+fx}\sqrt{c+\ldots}}$$

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]),x]
```

```
[Out] (2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B
*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^(3/2)) - (2*(8*a
^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)
) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 -
13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b^2*(b*c - a*d)^2*(b*e
 - a*f)^3*Sqrt[a + b*x]) - (2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[
e + f*x])/(5*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^(5/2)) + (2*Sqrt[d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2 -
 c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(d*
f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2))
- a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2 - 13
```

```
*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
- a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*
(e + f*x))/(b*e - a*f)]) + (2*Sqrt[d]*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c
*e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e
 + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(15*b^3*(-(b*c) + a*d)^(3/2)*(b*
e - a*f)^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(5Bce-2Ade-4Acf)-ab(5cCe+3Bde+Bcf-5Adf)}{2b}+\frac{1}{2}\left(-5bcCe+5aCde+5acCf+Abdf-aBdf-\frac{4a^2Cdf}{b}\right)x\right)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx}{5(bc-ad)(be-af)}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce-2Ade-4Acf) + ab^2(10cCe+3Bde+Bcf-6Adf) - a^2b(8Cde+6cCf}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$- \frac{4\int \frac{\frac{4a^3Cdf(de+cf)+b^3\left(2Ad^2e^2-cde(5Be-3Af)-c^2\left(15Ce^2-10Bef+8Af^2\right)\right)+ab^2\left(3d^2e(Be-2Af)+2c^2f(5Ce-Bf)+cd\left(25Ce^2-8Bef+9Af\right)\right)}{4b}}{}}{}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce-2Ade-4Acf) + ab^2(10cCe+3Bde+Bcf-6Adf) - a^2b(8Cde+6cCf}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde+13cCf-2Bdf) - b^4(2Ad^2e^2 - cde(5Be-3Af) - c^2(15Ce^2 - 1}{}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{8\int \frac{\frac{d\left(4a^4Cdf^2(de+cf)+b^4ce\left(15cCe^2-Adef-cf(5Be-4Af)\right)+a^3bf\left(Bdf(de+cf)-C\left(11d^2e^2+19cdef+6c^2f^2\right)\right)-ab^3\left(Ad^2e^2f+cde\left(30Ce^2-f\right)\right)\right)}{8b}}{}}{}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce-2Ade-4Acf) + ab^2(10cCe+3Bde+Bcf-6Adf) - a^2b(8Cde+6cCf}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde+13cCf-2Bdf) - b^4(2Ad^2e^2 - cde(5Be-3Af) - c^2(15Ce^2 - 1}{}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{(d(de-cf)(4a^3Cdf - b^3(5Bce-2Ade-4Acf) + ab^2(10cCe+3Bde+Bcf-6Adf) - a^2b(8 \cdots}{15b^2(bc-ad)^2(be-af)^2}$$

$$+ \frac{(d(8a^4Cd^2f^2 - a^3bdf(23Cde+13cCf-2Bdf) - b^4(2Ad^2e^2 - cde(5Be-3Af) - c^2(15Ce^2 - \cdots}{}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8Cde + 6cCf -}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10}{}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{\left(d(de-cf)(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8\right.}{15b^2(bc-ad)^2(be-af)^2\sqrt{c+dx}}$$

$$+ \frac{\left(d(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - \right.}{}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8Cde + 6cCf -}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10}{}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2\sqrt{d}(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 -}{}$$

$$+ \frac{\left(d(de-cf)(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8\right.}{15b^2(bc-ad)^2(be-af)^2\sqrt{c+}}$$

$$= \frac{2(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(8Cde + 6cCf -}{15b^2(bc-ad)(be-af)^2(a+bx)^{3/2}}$$

$$- \frac{2(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 - 10}{}$$

$$- \frac{2(Ab^2 - a(bB - aC))(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2\sqrt{d}(8a^4Cd^2f^2 - a^3bdf(23Cde + 13cCf - 2Bdf) - b^4(2Ad^2e^2 - cde(5Be - 3Af) - c^2(15Ce^2 -}{}$$

$$+ \frac{2\sqrt{d}(de-cf)(4a^3Cdf - b^3(5Bce - 2Ade - 4Acf) + ab^2(10cCe + 3Bde + Bcf - 6Adf) - a^2b(}{15b^3(-bc+ad)^{3/2}(be-af)^2\sqrt{c+d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.50 (sec) , antiderivative size = 1449, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\Bigg(-\frac{2(Ab^2-abB+a^2C)}{5b^2(be-af)(a+bx)^3}$$

$$-\frac{2(5b^3Bce-10ab^2cCe+Ab^3de-6ab^2Bde+11a^2bCde-4Ab^3cf-ab^2Bcf+6a^2bcCf+3aAb^2df+2a^2b}{15b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$-\frac{2(15b^4c^2Ce^2+5b^4Bcde^2-40ab^3cCde^2-2Ab^4d^2e^2-3ab^3Bd^2e^2+23a^2b^2Cd^2e^2-10b^4Bc^2ef-10ab^3c^2C}{}$$

$$+\frac{2(a+bx)^{3/2}\left(\sqrt{-a+\frac{bc}{d}}(8a^4Cd^2f^2+a^3bdf(-23Cde-13cCf+2Bdf)+b^4(-2Ad^2e^2+cde(5Be-3Af}{}$$

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x])
,x]
```

```
[Out] Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5*
b^2*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e
 - 6*a*b^2*B*d*e + 11*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f + 6*a^2*b*c*C
*f + 3*a*A*b^2*d*f + 2*a^2*b*B*d*f - 7*a^3*C*d*f))/(15*b^2*(b*c - a*d)*(b*e
 - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*
c*C*d*e^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 - 10
*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f - 3*A*b^4*c*d*e*f + 13*a*b^3*B*c*d*e*f
+ 37*a^2*b^2*c*C*d*e*f + 7*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*d^2*e*f - 23*a^3*b
*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*b^2*c^2*C*f^2 - 13
*a*A*b^3*c*d*f^2 - 2*a^2*b^2*B*c*d*f^2 - 13*a^3*b*c*C*d*f^2 + 3*a^2*A*b^2*d
^2*f^2 + 2*a^3*b*B*d^2*f^2 + 8*a^4*C*d^2*f^2))/(15*b^2*(b*c - a*d)^2*(b*e -
 a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(8*a^4*C*d^2*f
^2 + a^3*b*d*f*(-23*C*d*e - 13*c*C*f + 2*B*d*f) + b^4*(-2*A*d^2*e^2 + c*d*e
*(5*B*e - 3*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a^2*b^2*(d*f*(-7*
B*d*e - 2*B*c*f + 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)) + a*b
^3*(d^2*e*(-3*B*e + 7*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-40*C*e^2 + 13*f
*(B*e - A*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a + b*x
) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-8*a^4*C*d^2*f^2 + a^3*b*d*f*(2
3*C*d*e + 13*c*C*f - 2*B*d*f) + b^4*(2*A*d^2*e^2 + c*d*e*(-5*B*e + 3*A*f) +
 c^2*(-15*C*e^2 + 10*B*e*f - 8*A*f^2)) - a^2*b^2*(d*f*(-7*B*d*e - 2*B*c*f +
 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2)) + a*b^3*(d^2*e*(3*B*e
- 7*A*f) - 2*c^2*f*(-5*C*e + B*f) + c*d*(40*C*e^2 + 13*f*(-(B*e) + A*f))))*
Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt[1 - a/(a + b*x) + (b*e)/(f
```

```
*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
- a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x] - (I*b*(-(b*c) + a*d)*(d*e - c*f)*
(-4*a^3*C*d*f^2 + a^2*b*f*(11*C*d*e + 3*c*C*f - B*d*f) + b^3*(15*c*C*e^2 +
A*d*e*f + 2*c*f*(-5*B*e + 4*A*f)) + a*b^2*(-5*C*e*(3*d*e + 2*c*f) + f*(9*B*
d*e + 2*B*c*f - 9*A*d*f)))*Sqrt[1 - a/(a + b*x) + (b*c)/(d*(a + b*x))]*Sqrt
[1 - a/(a + b*x) + (b*e)/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)
/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[a + b*x]))/(15*b
^4*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f)^3*Sqrt[c + ((a + b*x)*(d -
(a*d)/(a + b*x)))/b]*Sqrt[e + ((a + b*x)*(f - (a*f)/(a + b*x)))/b])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2329 vs. $2(972) = 1944$.

Time = 5.68 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.25

method	result	size
elliptic	Expression too large to display	2330
default	Expression too large to display	36158

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/5*(A*b^2-B*a*b+C*a^2)/b^5/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*
x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(3*A*a*b^2*d*f-4*A*
b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7*C*a
^3*d*f+6*C*a^2*b*c*f+11*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^4/(a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a
*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e
)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2/b^3*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c
*d*f^2+7*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*
B*a^3*b*d^2*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+1
3*B*a*b^3*c*d*e*f-3*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d*e^2+8*C*a^
4*d^2*f^2-13*C*a^3*b*c*d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2*f^2+37*C*a^
2*b^2*c*d*e*f+23*C*a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b^3*c*d*e^2+15
*C*b^4*c^2*e^2)/(a*f-b*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)
+2*(C*d/b^3+1/15*d*f*(3*A*a*b^2*d*f-4*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a
*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7*C*a^3*d*f+6*C*a^2*b*c*f+11*C*a^2*b*d*e
-10*C*a*b^2*c*e)/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(a*f-b*e)-1/15/b^3*(
a*d*f-b*c*f-b*d*e)*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c*d*f^2+7*A*a*b^3*d^2*e*
f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*B*a^3*b*d^2*f^2-2*B*a^2
*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+13*B*a*b^3*c*d*e*f-3*B*a
*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d*e^2+8*C*a^4*d^2*f^2-13*C*a^3*b*c*
d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2*f^2+37*C*a^2*b^2*c*d*e*f+23*C*a^2*
```

```
b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)^2/(a*f-b*e)-1/15*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)^2/b^3*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c*d*f^2+7*A*a*b^3*d^
2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*B*a^3*b*d^2*f^2-2*B
*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+13*B*a*b^3*c*d*e*f-3
*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d*e^2+8*C*a^4*d^2*f^2-13*C*a^3*
b*c*d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2*f^2+37*C*a^2*b^2*c*d*e*f+23*C*
a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a*
f-b*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+
c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d
*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(
-e/f+a/b))^(1/2))-2/15*d*f/b^2*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c*d*f^2+7*A*
a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*B*a^3*b*d^2
*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+13*B*a*b^3*c
*d*e*f-3*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d*e^2+8*C*a^4*d^2*f^2-1
3*C*a^3*b*c*d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2*f^2+37*C*a^2*b^2*c*d*e
*f+23*C*a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b^3*c*d*e^2+15*C*b^4*c^2*
e^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2/(a*f-b*e)*(e/f-c/d)*((x+e/f)/(e/f-
c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^
3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f
+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/
b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 4867, normalized size of antiderivative = 4.71

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] -2/45*(3*((15*C*a^4*b^4*d^3 + (8*C*a^2*b^6 + 2*B*a*b^7 + 3*A*b^8)*c^2*d - 5
*(5*C*a^3*b^5 + A*a*b^7)*c*d^2)*e^2*f - (10*(B*a^2*b^6 + A*a*b^7)*c^2*d - 1
5*(C*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6)*c*d^2 + (11*C*a^5*b^3 + 9*B*a^4*b^4 +
 A*a^3*b^5)*d^3)*e*f^2 + (15*A*a^2*b^6*c^2*d - (6*C*a^5*b^3 - B*a^4*b^4 + 2
6*A*a^3*b^5)*c*d^2 + (4*C*a^6*b^2 + B*a^5*b^3 + 9*A*a^4*b^4)*d^3)*f^3 + ((1
5*C*b^8*c^2*d - 5*(8*C*a*b^7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7 - 2
*A*b^8)*d^3)*e^2*f - (10*(C*a*b^7 + B*b^8)*c^2*d - (37*C*a^2*b^6 + 13*B*a*b
^7 - 3*A*b^8)*c*d^2 + (23*C*a^3*b^5 + 7*B*a^2*b^6 - 7*A*a*b^7)*d^3)*e*f^2 +
 ((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b^8)*c^2*d - (13*C*a^3*b^5 + 2*B*a^2*b^6 +
 13*A*a*b^7)*c*d^2 + (8*C*a^4*b^4 + 2*B*a^3*b^5 + 3*A*a^2*b^6)*d^3)*f^3)*x^
```

2 + ((5*(4*C*a*b^7 + B*b^8)*c^2*d - (59*C*a^2*b^6 + B*a*b^7 - A*b^8)*c*d^2
+ 5*(7*C*a^3*b^5 - A*a*b^7)*d^3)*e^2*f - 2*((2*C*a^2*b^6 + 13*B*a*b^7 + 2*A
*b^8)*c^2*d - 20*(C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (14*C*a^4*b^4 + 11*B*a^3*b
^5 - 6*A*a^2*b^6)*d^3)*e*f^2 + (5*(B*a^2*b^6 + 4*A*a*b^7)*c^2*d - (13*C*a^4
*b^4 + 7*B*a^3*b^5 + 33*A*a^2*b^6)*c*d^2 + 3*(3*C*a^5*b^3 + 2*B*a^4*b^4 + 3
*A*a^3*b^5)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - ((30*C
*a^3*b^5*c^2*d - 5*(10*C*a^4*b^4 + B*a^3*b^5)*c*d^2 + (22*C*a^5*b^3 + 3*B*a
^4*b^4 + 2*A*a^3*b^5)*d^3)*e^3 - (15*C*a^3*b^5*c^3 + 5*(5*C*a^4*b^4 + 2*B*a
^3*b^5)*c^2*d - (67*C*a^5*b^3 + 33*B*a^4*b^4 + 2*A*a^3*b^5)*c*d^2 + (33*C*a
^6*b^2 + 17*B*a^5*b^3 + 8*A*a^4*b^4)*d^3)*e^2*f + (10*(C*a^4*b^4 + B*a^3*b^
5)*c^3 + (27*C*a^5*b^3 - 17*B*a^4*b^4 + 7*A*a^3*b^5)*c^2*d - (58*C*a^6*b^2
+ 7*B*a^5*b^3 + 18*A*a^4*b^4)*c*d^2 + (27*C*a^7*b + 8*B*a^6*b^2 + 17*A*a^5*
b^3)*d^3)*e*f^2 - ((3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*a^3*b^5)*c^3 + (8*C*a^6
*b^2 - 3*B*a^5*b^3 - 17*A*a^4*b^4)*c^2*d - (17*C*a^7*b + 3*B*a^6*b^2 - 8*A*
a^5*b^3)*c*d^2 + (8*C*a^8 + 2*B*a^7*b + 3*A*a^6*b^2)*d^3)*f^3 + ((30*C*b^8*
c^2*d - 5*(10*C*a*b^7 + B*b^8)*c*d^2 + (22*C*a^2*b^6 + 3*B*a*b^7 + 2*A*b^8)
*d^3)*e^3 - (15*C*b^8*c^3 + 5*(5*C*a*b^7 + 2*B*b^8)*c^2*d - (67*C*a^2*b^6 +
 33*B*a*b^7 + 2*A*b^8)*c*d^2 + (33*C*a^3*b^5 + 17*B*a^2*b^6 + 8*A*a*b^7)*d^
3)*e^2*f + (10*(C*a*b^7 + B*b^8)*c^3 + (27*C*a^2*b^6 - 17*B*a*b^7 + 7*A*b^8
)*c^2*d - (58*C*a^3*b^5 + 7*B*a^2*b^6 + 18*A*a*b^7)*c*d^2 + (27*C*a^4*b^4 +
 8*B*a^3*b^5 + 17*A*a^2*b^6)*d^3)*e*f^2 - ((3*C*a^2*b^6 + 2*B*a*b^7 + 8*A*b
^8)*c^3 + (8*C*a^3*b^5 - 3*B*a^2*b^6 - 17*A*a*b^7)*c^2*d - (17*C*a^4*b^4 +
3*B*a^3*b^5 - 8*A*a^2*b^6)*c*d^2 + (8*C*a^5*b^3 + 2*B*a^4*b^4 + 3*A*a^3*b^5
)*d^3)*f^3)*x^3 + 3*((30*C*a*b^7*c^2*d - 5*(10*C*a^2*b^6 + B*a*b^7)*c*d^2 +
 (22*C*a^3*b^5 + 3*B*a^2*b^6 + 2*A*a*b^7)*d^3)*e^3 - (15*C*a*b^7*c^3 + 5*(5
*C*a^2*b^6 + 2*B*a*b^7)*c^2*d - (67*C*a^3*b^5 + 33*B*a^2*b^6 + 2*A*a*b^7)*c
*d^2 + (33*C*a^4*b^4 + 17*B*a^3*b^5 + 8*A*a^2*b^6)*d^3)*e^2*f + (10*(C*a^2*
b^6 + B*a*b^7)*c^3 + (27*C*a^3*b^5 - 17*B*a^2*b^6 + 7*A*a*b^7)*c^2*d - (58*
C*a^4*b^4 + 7*B*a^3*b^5 + 18*A*a^2*b^6)*c*d^2 + (27*C*a^5*b^3 + 8*B*a^4*b^4
 + 17*A*a^3*b^5)*d^3)*e*f^2 - ((3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*c^3
+ (8*C*a^4*b^4 - 3*B*a^3*b^5 - 17*A*a^2*b^6)*c^2*d - (17*C*a^5*b^3 + 3*B*a^
4*b^4 - 8*A*a^3*b^5)*c*d^2 + (8*C*a^6*b^2 + 2*B*a^5*b^3 + 3*A*a^4*b^4)*d^3)
*f^3)*x^2 + 3*((30*C*a^2*b^6*c^2*d - 5*(10*C*a^3*b^5 + B*a^2*b^6)*c*d^2 + (
22*C*a^4*b^4 + 3*B*a^3*b^5 + 2*A*a^2*b^6)*d^3)*e^3 - (15*C*a^2*b^6*c^3 + 5*
(5*C*a^3*b^5 + 2*B*a^2*b^6)*c^2*d - (67*C*a^4*b^4 + 33*B*a^3*b^5 + 2*A*a^2*
b^6)*c*d^2 + (33*C*a^5*b^3 + 17*B*a^4*b^4 + 8*A*a^3*b^5)*d^3)*e^2*f + (10*(
C*a^3*b^5 + B*a^2*b^6)*c^3 + (27*C*a^4*b^4 - 17*B*a^3*b^5 + 7*A*a^2*b^6)*c^
2*d - (58*C*a^5*b^3 + 7*B*a^4*b^4 + 18*A*a^3*b^5)*c*d^2 + (27*C*a^6*b^2 + 8
*B*a^5*b^3 + 17*A*a^4*b^4)*d^3)*e*f^2 - ((3*C*a^4*b^4 + 2*B*a^3*b^5 + 8*A*a
^2*b^6)*c^3 + (8*C*a^5*b^3 - 3*B*a^4*b^4 - 17*A*a^3*b^5)*c^2*d - (17*C*a^6*
b^2 + 3*B*a^5*b^3 - 8*A*a^4*b^4)*c*d^2 + (8*C*a^7*b + 2*B*a^6*b^2 + 3*A*a^5
*b^3)*d^3)*f^3)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*
c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/2
7*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2
*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*

```
a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f
)) + 3*((15*C*a^3*b^5*c^2*d - 5*(8*C*a^4*b^4 - B*a^3*b^5)*c*d^2 + (23*C*a^5
*b^3 - 3*B*a^4*b^4 - 2*A*a^3*b^5)*d^3)*e^2*f - (10*(C*a^4*b^4 + B*a^3*b^5)*
c^2*d - (37*C*a^5*b^3 + 13*B*a^4*b^4 - 3*A*a^3*b^5)*c*d^2 + (23*C*a^6*b^2 +
 7*B*a^5*b^3 - 7*A*a^4*b^4)*d^3)*e*f^2 + ((3*C*a^5*b^3 + 2*B*a^4*b^4 + 8*A*
a^3*b^5)*c^2*d - (13*C*a^6*b^2 + 2*B*a^5*b^3 + 13*A*a^4*b^4)*c*d^2 + (8*C*a
^7*b + 2*B*a^6*b^2 + 3*A*a^5*b^3)*d^3)*f^3 + ((15*C*b^8*c^2*d - 5*(8*C*a*b^
7 - B*b^8)*c*d^2 + (23*C*a^2*b^6 - 3*B*a*b^7 - 2*A*b^8)*d^3)*e^2*f - (10*(C
*a*b^7 + B*b^8)*c^2*d - (37*C*a^2*b^6 + 13*B*a*b^7 - 3*A*b^8)*c*d^2 + (23*C
*a^3*b^5 + 7*B*a^2*b^6 - 7*A*a*b^7)*d^3)*e*f^2 + ((3*C*a^2*b^6 + 2*B*a*b^7
 + 8*A*b^8)*c^2*d - (13*C*a^3*b^5 + 2*B*a^2*b^6 + 13*A*a*b^7)*c*d^2 + (8*C*a
^4*b^4 + 2*B*a^3*b^5 + 3*A*a^2*b^6)*d^3)*f^3)*x^3 + 3*((15*C*a*b^7*c^2*d -
5*(8*C*a^2*b^6 - B*a*b^7)*c*d^2 + (23*C*a^3*b^5 - 3*B*a^2*b^6 - 2*A*a*b^7)*
d^3)*e^2*f - (10*(C*a^2*b^6 + B*a*b^7)*c^2*d - (37*C*a^3*b^5 + 13*B*a^2*b^6
 - 3*A*a*b^7)*c*d^2 + (23*C*a^4*b^4 + 7*B*a^3*b^5 - 7*A*a^2*b^6)*d^3)*e*f^2
 + ((3*C*a^3*b^5 + 2*B*a^2*b^6 + 8*A*a*b^7)*c^2*d - (13*C*a^4*b^4 + 2*B*a^3
*b^5 + 13*A*a^2*b^6)*c*d^2 + (8*C*a^5*b^3 + 2*B*a^4*b^4 + 3*A*a^3*b^5)*d^3)
*f^3)*x^2 + 3*((15*C*a^2*b^6*c^2*d - 5*(8*C*a^3*b^5 - B*a^2*b^6)*c*d^2 + (2
3*C*a^4*b^4 - 3*B*a^3*b^5 - 2*A*a^2*b^6)*d^3)*e^2*f - (10*(C*a^3*b^5 + B*a^
2*b^6)*c^2*d - (37*C*a^4*b^4 + 13*B*a^3*b^5 - 3*A*a^2*b^6)*c*d^2 + (23*C*a^
5*b^3 + 7*B*a^4*b^4 - 7*A*a^3*b^5)*d^3)*e*f^2 + ((3*C*a^4*b^4 + 2*B*a^3*b^5
 + 8*A*a^2*b^6)*c^2*d - (13*C*a^5*b^3 + 2*B*a^4*b^4 + 13*A*a^3*b^5)*c*d^2 +
 (8*C*a^6*b^2 + 2*B*a^5*b^3 + 3*A*a^4*b^4)*d^3)*f^3)*x)*sqrt(b*d*f)*weierst
rassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d +
a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^
3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPI
nverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^
2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)
*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x
+ b*d*e + (b*c + a*d)*f)/(b*d*f))))/((a^3*b^9*c^2*d - 2*a^4*b^8*c*d^2 + a^5
*b^7*d^3)*e^3*f - 3*(a^4*b^8*c^2*d - 2*a^5*b^7*c*d^2 + a^6*b^6*d^3)*e^2*f^2
 + 3*(a^5*b^7*c^2*d - 2*a^6*b^6*c*d^2 + a^7*b^5*d^3)*e*f^3 - (a^6*b^6*c^2*d
 - 2*a^7*b^5*c*d^2 + a^8*b^4*d^3)*f^4 + ((b^12*c^2*d - 2*a*b^11*c*d^2 + a^2
*b^10*d^3)*e^3*f - 3*(a*b^11*c^2*d - 2*a^2*b^10*c*d^2 + a^3*b^9*d^3)*e^2*f^
2 + 3*(a^2*b^10*c^2*d - 2*a^3*b^9*c*d^2 + a^4*b^8*d^3)*e*f^3 - (a^3*b^9*c^2
*d - 2*a^4*b^8*c*d^2 + a^5*b^7*d^3)*f^4)*x^3 + 3*((a*b^11*c^2*d - 2*a^2*b^1
0*c*d^2 + a^3*b^9*d^3)*e^3*f - 3*(a^2*b^10*c^2*d - 2*a^3*b^9*c*d^2 + a^4*b^
8*d^3)*e^2*f^2 + 3*(a^3*b^9*c^2*d - 2*a^4*b^8*c*d^2 + a^5*b^7*d^3)*e*f^3 -
(a^4*b^8*c^2*d - 2*a^5*b^7*c*d^2 + a^6*b^6*d^3)*f^4)*x^2 + 3*((a^2*b^10*c^2
*d - 2*a^3*b^9*c*d^2 + a^4*b^8*d^3)*e^3*f - 3*(a^3*b^9*c^2*d - 2*a^4*b^8*c*
d^2 + a^5*b^7*d^3)*e^2*f^2 + 3*(a^4*b^8*c^2*d - 2*a^5*b^7*c*d^2 + a^6*b^6*d
^3)*e*f^3 - (a^5*b^7*c^2*d - 2*a^6*b^6*c*d^2 + a^7*b^5*d^3)*f^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**(7/2)/(f*x+e)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{7/2}}\,dx$$

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),x
)
[Out] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
x)
```

3.73 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 838

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)+(3adf-4b(de+cf))(2aCdf-b(7Bdf-6C(de+cf))))\sqrt{a+}}{105bd^3f^3}$$

$$-\frac{2(2aCdf-b(7Bdf-6C(de+cf)))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)-(3bce+ade+acf)(2aCdf-b(7Bdf-6C(d\right.}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf(7df(3Bde+2Bcf-5Adf)-C(16d^2e^2+8cdef+1\right.}{}$$

```
[Out] -2/35*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/b/d^2/f^2+2/7*C*(b*x+a)^(5/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-
2/105*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(3*a*d*f-4*b*(c*f+d*e
))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b/d^3/f^3-2/105*(3*b*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*
b*c*e)-(a*c*f+a*d*e+3*b*c*e)*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e))))+2*(1/2*
a*d*f-b*(c*f+d*e))*(5*b*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)+(3*a*d*f
-4*b*(c*f+d*e))*(2*a*C*d*f-b*(7*B*d*f-6*C*(c*f+d*e)))))*EllipticE(d^(1/2)*(
b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(
1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(7/2)/f^4/(d*x+c)^(1/
```

```
2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(-a*f+b*e)*(3*a^2*C*d^2*f^2*(-c*f+d*e
)-3*a*b*d*f*(7*d*f*(-5*A*d*f+2*B*c*f+3*B*d*e)-C*(11*c^2*f^2+8*c*d*e*f+16*d^
2*e^2))-b^2*(C*(24*c^3*f^3+17*c^2*d*e*f^2+16*c*d^2*e^2*f+48*d^3*e^3)+7*d*f*
(5*A*d*f*(c*f+2*d*e)-B*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2))))*EllipticF(d^(1/2)
*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)
^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(7/2
)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 831, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{5/2}}{7bdf}$$
$$+\frac{2(7bBdf-2aCdf-6bC(de+cf))\sqrt{c+dx}\sqrt{e+fx}(a+bx)^{3/2}}{35bd^2f^2}$$
$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))\sqrt{c+d}}{105bd^3f^3}$$
$$-\frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+\right.}{}$$
$$-\frac{2\sqrt{ad-bc}(be-af)\left(-((C(48d^3e^3+16cd^2fe^2+17c^2df^2e+24c^3f^3)+7df(5Adf(2de+cf)-B(8d^2e^2+3\right.}{}$$

```
[In] Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) - (3*a*d*f - 4*b*(
d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f)))*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])/(105*b*d^3*f^3) + (2*(7*b*B*d*f - 2*a*C*d*f - 6*b*
C*(d*e + c*f))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(35*b*d^2*f^2)
+ (2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - (2*Sqrt[-(b
*c) + a*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) +
(3*b*c*e + a*d*e + a*c*f)*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))) + 2*
((a*d*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b
*d*f) - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c
*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqr
t[-(b*c) + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f
*(3*B*d*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) -
 b^2*(C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f
```

```
*(5*A*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(S
qrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]
)/(105*b^2*d^(7/2)*f^4*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
```

```
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} + \frac{2\int \frac{(a+bx)^{3/2}\left(-\frac{1}{2}b(5bcCe+aCde+acCf-7Abdf)+\frac{1}{2}b(7bBdf-2aCdf-6bC(de+cf))x\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{7b^2df}$$

$$= \frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2} + \frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} + \frac{4\int \frac{\sqrt{a+bx}\left(-\frac{1}{4}b(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-\frac{1}{4}b(5bdf(5bcCe+aCde+ac}{\sqrt{c+dx}\sqrt{e+fx}}}{35b^2d^2f^2}$$

$$=$$

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+}{105bd^3f^3}$$

$$+\frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$+\frac{8\int\frac{-\frac{1}{8}b(3adf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-(bce+ade+acf)(5bdf(5bcC}{}}{}$$

$$=$$

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+}{105bd^3f^3}$$

$$+\frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$-\frac{(3bdf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de}{}$$

$$-\frac{((be-af)\,(3a^2Cd^2f^2(de-cf)-3abdf(7df(3Bde+2Bcf-5Adf)-C(16d^2e^2+8cdef+11}{}$$

$$=$$

$$-\frac{2(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+}{105bd^3f^3}$$

$$+\frac{2(7bBdf-2aCdf-6bC(de+cf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35bd^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$-\frac{\Big((be-af)\,(3a^2Cd^2f^2(de-cf)-3abdf(7df(3Bde+2Bcf-5Adf)-C(16d^2e^2+8cdef+11}{}$$

$$-\frac{\Big((3bdf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(d}{}$$

$$= -\frac{2(5bdf(5bcCe + aCde + acCf - 7Abdf) - (3adf - 4b(de + cf))(7bBdf - 2aCdf - 6bC(de + c}{105bd^3f^3}$$
$$+ \frac{2(7bBdf - 2aCdf - 6bC(de + cf))(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}}{35bd^2f^2}$$
$$+ \frac{2C(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}}{7bdf}$$
$$- \frac{2\sqrt{-bc + ad}\big(3bdf(5adf(5bcCe + aCde + acCf - 7Abdf) + (3bce + ade + acf)(7bBdf - 2aCdf}{}$$
$$- \frac{\Big((be - af)\,(3a^2Cd^2f^2(de - cf) - 3abdf(7df(3Bde + 2Bcf - 5Adf) - C(16d^2e^2 + 8cdef + 11c}{}$$

$$= -\frac{2(5bdf(5bcCe + aCde + acCf - 7Abdf) - (3adf - 4b(de + cf))(7bBdf - 2aCdf - 6bC(de + c}{105bd^3f^3}$$
$$+ \frac{2(7bBdf - 2aCdf - 6bC(de + cf))(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}}{35bd^2f^2}$$
$$+ \frac{2C(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}}{7bdf}$$
$$- \frac{2\sqrt{-bc + ad}\big(3bdf(5adf(5bcCe + aCde + acCf - 7Abdf) + (3bce + ade + acf)(7bBdf - 2aCdf}{}$$
$$- \frac{2\sqrt{-bc + ad}(be - af)\,(3a^2Cd^2f^2(de - cf) - 3abdf(7df(3Bde + 2Bcf - 5Adf) - C(16d^2e^2 + 8}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.12 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.19

$$\int \frac{(a + bx)^{3/2}\,(A + Bx + Cx^2)}{\sqrt{c + dx}\sqrt{e + fx}}\,dx = \frac{2\Big(-b^2\sqrt{-a + \frac{bc}{d}}(6a^3Cd^3f^3 + 3a^2bd^2f^2(-7Bdf + 4C(de + cf)) - ab^2d}{}$$

```
[In] Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x])
,x]
```

```
[Out] (2*(-(b^2*Sqrt[-a + (b*c)/d]*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*f +
 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2) + 7
```

```
*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*f +
 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2 +
7*c*d*e*f + 8*c^2*f^2))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f
*(a + b*x)*(c + d*x)*(e + f*x)*(3*a^2*C*d^2*f^2 + 3*a*b*d*f*(14*B*d*f + C*(
-11*d*e - 11*c*f + 8*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e - 4*c*f + 3*
d*f*x)) + C*(24*c^2*f^2 + c*d*f*(23*e - 18*f*x) + 3*d^2*(8*e^2 - 6*e*f*x +
5*f^2*x^2)))) - I*(b*c - a*d)*f*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*
f + 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2)
+ 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*
f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^2
 + 7*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x
))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d
]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(3*a
^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(-2*B*d*e - 3*B*c*f + 5*A*d*f)
+ C*(11*d^2*e^2 + 8*c*d*e*f + 16*c^2*f^2)) + b^2*(C*(24*d^3*e^3 + 17*c*d^2*
e^2*f + 16*c^2*d*e*f^2 + 48*c^3*f^3) + 7*d*f*(5*A*d*f*(d*e + 2*c*f) - B*(4*
d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(
a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a +
(b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(105*b^3*Sqrt[-
a + (b*c)/d]*d^4*f^4*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	1233
default	Expression too large to display	9580

```
[In] int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/7*b*C/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*
c*e*x+a*c*e)^(1/2)+2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e)
)/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b*C/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*
b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a
*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(a^2*A-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*
a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b*C/d/f*(5
/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c
*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/
2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((
x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a
```

```
*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)
/(-e/f+a/b))^(1/2))+2*(2*a*b*A+a^2*B-4/7*b*C/d/f*a*c*e-2/5*(B*b^2+2*C*a*b-2
/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)
-2/3*(b^2*A+2*a*b*B+C*a^2-2/7*b*C/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(
B*b^2+2*C*a*b-2/7*b*C/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f
+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*(
(x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*Ellipti
cE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF((
(x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1388, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 + (23*C*b^4*c*d^3 - (
33*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 + (24*C*b^4*c^2*d^2 - (33*C*a*b^3 + 28*B*
b^4)*c*d^3 + (3*C*a^2*b^2 + 42*B*a*b^3 + 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4*d^
4*e*f^3 + (6*C*b^4*c*d^3 - (8*C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 + 8*(2*C*b^4*c*d^3 - (12*C
*a*b^3 + 7*B*b^4)*d^4)*e^3*f + (11*C*b^4*c^2*d^2 - 7*(4*C*a*b^3 + 3*B*b^4)*
c*d^3 + (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4)*d^4)*e^2*f^2 + (16*C*b^4*c^
3*d - 7*(4*C*a*b^3 + 3*B*b^4)*c^2*d^2 + 7*(C*a^2*b^2 + 7*B*a*b^3 + 5*A*b^4)
*c*d^3 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*d^4)*e*f^3 + (48*C*b^4*c^
4 - 8*(12*C*a*b^3 + 7*B*b^4)*c^3*d + (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4
)*c^2*d^2 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*c*d^3 + (6*C*a^4 - 21*
B*a^3*b + 175*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2
*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^
2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3
*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c
+ a*d)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*e^3*f + 8*(5*C*b^4*c*d^3 - (9*C*a*b^3
+ 7*B*b^4)*d^4)*e^2*f^2 + (40*C*b^4*c^2*d^2 - (62*C*a*b^3 + 49*B*b^4)*c*d^3
 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*d^4)*e*f^3 + (48*C*b^4*c^3*d - 8*
(9*C*a*b^3 + 7*B*b^4)*c^2*d^2 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*c*d^
3 + (6*C*a^3*b - 21*B*a^2*b^2 - 140*A*a*b^3)*d^4)*f^4)*sqrt(b*d*f)*weierstr
assZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a
```

```
^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3
)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPIn
verse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2
*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*
e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x +
 b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^5*f^5)
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((a + b*x)**(3/2)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x
)
```

```
[Out] int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

3.74 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 528

$$\begin{aligned}
&\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx \\
&= -\frac{2(2aCdf-b(5Bdf-4C(de+cf)))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} \\
&\quad + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} \\
&\quad - \frac{2\sqrt{-bc+ad}(3bdf(3bcCe+aCde+acCf-5Abdf)+(adf-2b(de+cf))(2aCdf-b(5Bdf-4C(de}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} \\
&\quad - \frac{2\sqrt{-bc+ad}(be-af)\left(aCdf(de-cf)-b(5df(2Bde+Bcf-3Adf)-C(8d^2e^2+3cdef+4c^2f^2))\right)\sqrt{}}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}
\end{aligned}$$

```
[Out] 2/5*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/15*(2*a*C*d*f-b*(5*
B*d*f-4*C*(c*f+d*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^2-2
/15*(3*b*d*f*(-5*A*b*d*f+C*a*c*f+C*a*d*e+3*C*b*c*e)+(a*d*f-2*b*(c*f+d*e))*(
2*a*C*d*f-b*(5*B*d*f-4*C*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-
b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-
a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/d^(5/2)/f^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*
f+b*e))^(1/2)-2/15*(-a*f+b*e)*(a*C*d*f*(-c*f+d*e)-b*(5*d*f*(-3*A*d*f+B*c*f+
2*B*d*e)-C*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2
)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*
x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(5/2)/f^3/(d*x+c)
^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {1629, 159, 164, 115, 114, 122, 121}

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC(4c^2f^2+3cde}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}} - \frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+aCde-5Abdf+3bcCe)-(adf-2b(cf+de))(-2aCdf+5}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} + \frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(-2aCdf+5bBdf-4bC(cf+de))}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}$$

```
[In] Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(5*b*B*d*f - 2*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*
Sqrt[e + f*x])/(15*b*d^2*f^2) + (2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e +
 f*x])/(5*b*d*f) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a*
c*C*f - 5*A*b*d*f) - (a*d*f - 2*b*(d*e + c*f))*(5*b*B*d*f - 2*a*C*d*f - 4*b
*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[Ar
cSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e -
 a*f))])/(15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)])
 - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) + b*C*(8*d^2*e^2
+ 3*c*d*e*f + 4*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(2*d*e + c*f)))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
15*b^2*d^(5/2)*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
```

```
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} + \frac{2\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}b(3bcCe+aCde+acCf-5Abdf)+\frac{1}{2}b(5bBdf-2aCdf-4bC(de+cf))x\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{5b^2df}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} + \frac{4\int \frac{-\frac{1}{4}b(3adf(3bcCe+aCde+acCf-5Abdf)+(bce+ade+acf)(5bBdf-2aCdf-4bC(de+cf)))-\frac{1}{4}b(3bdf(3bcCe+aCde+acCf-5Abc}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{15b^2d^2f^2}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4bC(de+cf))}{15bd^2f^3} - \frac{((be-af)(aCdf(de-cf)+bC(8d^2e^2+3cdef+4c^2f^2)+5bdf(3Adf-B(2de+cf))))\int \frac{}{\sqrt{a+bx}}}{15bd^2f^3}$$

$$= \frac{2(5bBdf-2aCdf-4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{\left((be-af)(aCdf(de-cf)+bC(8d^2e^2+3cdef+4c^2f^2)+5bdf(3Adf-B(2de+cf)))\sqrt{\frac{b(c+dx}{bc-ad}}\right.}{15bd^2f^3\sqrt{c+dx}} - \frac{\left((3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4bC(de+cf\right.}{15bd^2f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= \frac{2(5bBdf - 2aCdf - 4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{2\sqrt{-bc+ad}(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{\left((be-af)\left(aCdf(de-cf)+bC(8d^2e^2+3cdef+4c^2f^2)+5bdf(3Adf-B(2de+cf))\right)\sqrt{\frac{b(c+}{bc-}}\right.}{15bd^2f^3\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2(5bBdf - 2aCdf - 4bC(de+cf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15bd^2f^2} + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} - \frac{2\sqrt{-bc+ad}(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{-bc+ad}(be-af)\left(aCdf(de-cf)+bC(8d^2e^2+3cdef+4c^2f^2)+5bdf(3Adf-B(2de+c\right.}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.66 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(2a^2Cd^2f^2+abdf(-5Bdf+3C(de+cf))-b^2(C(8d^2e^2+7cdef+8c^2f^2)+5df(3Adf\right.}{}$$

```
[In] Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e
 + c*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*
B*(d*e + c*f))))*(c + d*x)*(e + f*x) - b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)
*(c + d*x)*(e + f*x)*(5*b*B*d*f + a*C*d*f + b*C*(-4*d*e - 4*c*f + 3*d*f*x))
 + I*(b*c - a*d)*f*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e + c*f))
- b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*B*(d*e +
```

```
c*f))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x)
)/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*
d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a*C*d*f*(-(d*e) + c*f) +
 b*C*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(d*e + 2*c*
f)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(
f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5df}+\frac{2\left(Bb+Ca-\frac{2C(2adf+2bcf+2bde)}{5df}\right)\sqrt{bdf\,x^3+adf\,x^2+b}}{3bdf}\right.}{}$
default	Expression too large to display

```
[In] int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/5*C/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d
*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2
*(A*a-2/5*C/d/f*a*c*e-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d
/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x
+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c
*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e
/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2*(A*b+B*a-2/5*C/d/f*(3/2*a*c
*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b
/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/
f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d
*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/
(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f
-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{2\Big(3\,(3\,Cb^3d^3f^3x - 4\,Cb^3d^3ef^2 - (4\,Cb^3cd^2 - (Cab^2 + 5\,Bb^3)d^3)f^3)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e} - (8\,Cb^3d^3}{}$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 - (4*C*b^3*c*d^2 - (C*a*b^2
+ 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3*d
^3*e^3 + (3*C*b^3*c*d^2 - (7*C*a*b^2 + 10*B*b^3)*d^3)*e^2*f + (3*C*b^3*c^2*
d - (2*C*a*b^2 + 5*B*b^3)*c*d^2 - (2*C*a^2*b - 10*B*a*b^2 - 15*A*b^3)*d^3)*
e*f^2 + (8*C*b^3*c^3 - (7*C*a*b^2 + 10*B*b^3)*c^2*d - (2*C*a^2*b - 10*B*a*b
^2 - 15*A*b^3)*c*d^2 - (2*C*a^3 - 5*B*a^2*b + 30*A*a*b^2)*d^3)*f^3)*sqrt(b*
d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*
c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*
c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2
+ (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3
), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*d^3*e^2*f
+ (7*C*b^3*c*d^2 - (3*C*a*b^2 + 10*B*b^3)*d^3)*e*f^2 + (8*C*b^3*c^2*d - (3*
C*a*b^2 + 10*B*b^3)*c*d^2 - (2*C*a^2*b - 5*B*a*b^2 - 15*A*b^3)*d^3)*f^3)*sq
rt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2
*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3
*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^
3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c
^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c
*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
 (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3)
, 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^4*f^4)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x
)
[Out] int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

3.75 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 387

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$
$$-\frac{2\sqrt{-bc+ad}(2aCdf-b(3Bdf-2C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+\frac{2\sqrt{-bc+ad}(aCf(de-cf)-b(3df(Be-Af)-Ce(2de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] 2/3*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/3*(2*a*C*d*f-b*(3*B
*d*f-2*C*(c*f+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(
f*x+e)^(1/2)/b^2/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3
*(a*C*f*(-c*f+d*e)-b*(3*d*f*(-A*f+B*e)-C*e*(c*f+2*d*e)))*EllipticF(d^(1/2)*
(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(a*d-b*c)^
(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/d^(3/2)
/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1629, 164, 115, 114, 122, 121}

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$
$$-\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\cdots}{\sqrt{\cdots}}\right.\right.}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$
$$+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\,(-2aCdf+3bBdf-2bC(cf+de))\,E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) + (2*Sqrt[-(b*c)
+ a*d]*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c)
+ a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*d^(3/2)*f^2*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(B*e - A
*f) - a*C*f*(d*e - c*f) - b*C*e*(2*d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c - a*
d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(3*b^2*d^(3/2)*f^2
*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rubi steps

$$\text{integral} = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\int \frac{-\frac{1}{2}b(bcCe+aCde+acCf-3Abdf)+\frac{1}{2}b(3bBdf-2aCdf-2bC(de+cf))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3b^2df}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{(3bBdf - 2aCdf - 2bC(de+cf))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{3bdf^2} - \frac{(3bdf(Be-Af) - aCf(de-cf) - bCe(2de+cf))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3bdf^2}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\left((3bdf(Be-Af) - aCf(de-cf) - bCe(2de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}\,dx}{3bdf^2\sqrt{c+dx}} + \frac{\left((3bBdf - 2aCdf - 2bC(de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\right)\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{3bdf^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}(3bBdf - 2aCdf - 2bC(de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{\left((3bdf(Be-Af) - aCf(de-cf) - bCe(2de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{b\cdots}}}{3bdf^2\sqrt{c+dx}\sqrt{e+fx}}$$

$$= \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}(3bBdf - 2aCdf - 2bC(de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{-bc+ad}(3bdf(Be-Af) - aCf(de-cf) - bCe(2de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\cdots}}{\sqrt{-b\cdots}}\right.\right.}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.49 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= \frac{\sqrt{a+bx}\left(2b^2Cdf(c+dx)(e+fx) - \frac{2b^2(-3bBdf+2aCdf+2bC(de+cf))(c+dx)(e+fx)}{a+bx} + 2i\sqrt{-a+\frac{bc}{d}}df(3bBdf - 2aC\cdots\right.}{\cdots}$$

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a*
C*d*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a +
 (b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqr
t[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[
I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f
)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*d*(C*e
 - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*
x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (
b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f^2*Sqrt[c
 + d*x]*Sqrt[e + f*x])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.59

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(\frac{2C\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+acfx+adex+bcex+ace}}{3bdf}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}acf+\frac{1}{2}ade+\frac{1}{2}bce\right)}{3bdf}\right)\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\cdots}}{\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
2/3*C/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2*(A-2/3*C/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d)*
((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(
1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e
)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+
2*(B-2/3*C/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*(
(x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*Ellipti
cE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF((
(x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(3\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}Cb^2d^2f^2+(2\,Cb^2d^2e^2+(Cb^2cd+(Cab-3\,Bb^2)d^2)ef+(2\,Cb^2c^2+(Cab-\right.}{}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*C*b^2*d^2*f^2 + (2*C*b^2*d
^2*e^2 + (C*b^2*c*d + (C*a*b - 3*B*b^2)*d^2)*e*f + (2*C*b^2*c^2 + (C*a*b -
3*B*b^2)*c*d + (2*C*a^2 - 3*B*a*b + 9*A*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstr
assPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d
 + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2
*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d
*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*b^2*d^2*e*f + (2*C*b^2*c*d
+ (2*C*a*b - 3*B*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^
2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f
^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
- 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2
- (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2
), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d -
4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)
/(b*d*f))))/(b^3*d^3*f^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

3.76 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 422

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$-\frac{2(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\,E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2(aC(de-cf)-b(cCe-Bcf+Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/
(b*x+a)^(1/2)-2*(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f+C*d*e))*Ell
ipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1
/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/f/(-a*f+b*e)/d^(1/2)/(a
*d-b*c)^(1/2)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2*(a*C*(-c*f+d*e)-
b*(A*d*f-B*c*f+C*c*e))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*
f+b*e))^(1/2)/b^2/f/d^(1/2)/(a*d-b*c)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1628, 164, 115, 114, 122, 121}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}f\sqrt{c+dx}\sqrt{ad-bc}(be-af)\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aC(de-cf)-b(Adf-Bcf+cCe)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}} - \frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (2*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*
e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*
e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*(b*e - a*f)*Sqrt[c + d*x]*Sqr
t[(b*(e + f*x))/(b*e - a*f)]) - (2*(a*C*(d*e - c*f) - b*(c*C*e - B*c*f + A*
d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Ellip
ticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b^2*Sqrt[d]*Sqrt[-(b*c) + a*d]*f*Sqrt[c + d*x]*Sqrt[e + f*
x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2\int \frac{-\frac{b^2Bce+a^2C(de+cf)-ab(cCe+Bde+Bcf-Adf)}{2b}+\frac{1}{2}\left(-\frac{2a^2Cdf}{b}-b(cCe+Adf)+a(Cde+cCf+Bdf)\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{(bc-ad)(be-af)}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{(aC(de-cf) - b(cCe - Bcf + Adf))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{b(bc-ad)f} + \frac{\left(\frac{2a^2Cdf}{b} + b(cCe + Adf) - a(Cde + cCf + Bdf)\right)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{(bc-ad)f(be-af)}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} + \frac{\left((aC(de-cf) - b(cCe - Bcf + Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{b(bc-ad)f\sqrt{c+dx}} + \frac{\left(\left(\frac{2a^2Cdf}{b} + b(cCe + Adf) - a(Cde + cCf + Bdf)\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\right)\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{(bc-ad)f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\right.}{b^2\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} + \frac{\left((aC(de-cf) - b(cCe - Bcf + Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}}{b(bc-ad)f\sqrt{c+dx}\sqrt{e+fx}}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}} - \frac{2(2a^2Cdf + b^2(cCe + Adf) - ab(Cde + cCf + Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\right.}{b^2\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2(aC(de-cf) - b(cCe - Bcf + Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}F\left(\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.27 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(-b^2(Ab^2+a(-bB+aC))(c+dx)(e+fx)+\frac{b^2(2a^2Cdf+b^2(cCe+Adf)-a\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (2*(-(b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)*(e + f*x)) + (b^2*(2*a^2*C*d
*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*(c + d*x)*(e + f*x)
)/(d*f) + (I*(b*c - a*d)*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e +
c*C*f + B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/(Sqrt[-a + (b*c)/d]*d) + (I*b*(-(b*c
) + a*d)*(a*C*(d*e - c*f) + b*(c*C*e - B*d*e + A*d*f))*(a + b*x)^(3/2)*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)
])/(Sqrt[-a + (b*c)/d]*d)))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]*Sqrt
[c + d*x]*Sqrt[e + f*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. 2(382) = 764.

Time = 3.34 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.86

method	result
elliptic	$\frac{\sqrt{(bx+a)(dx+c)(fx+e)}\left(-\frac{2\left(bdf\,x^2+bcfx+bdex+bce\right)\left(b^2A-abB+C\,a^2\right)}{\left(a^2df-acfb-abde+b^2ce\right)b^2\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^2+bcfx+bdex+bce\right)}}+\frac{2\left(\frac{Bb-Ca}{b^2}+\frac{(adf-bcf-bde)\left(b^2A-abB+C\,a^2\right)}{b^2\left(a^2df-acfb-abde+b^2ce\right)}+\cdots\right.}{\sqrt{bdf\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2*
(A*b^2-B*a*b+C*a^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B
*b-C*a)/b^2+1/b^2*(a*d*f-b*c*f-b*d*e)*(A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)+(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2*(A*b^2
-B*a*b+C*a^2))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/
2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*
f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f
+c/d)/(-e/f+a/b))^(1/2))+2*(C/b+1/b*d*f*(A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b)
)^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2
+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c
/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))
^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1240, normalized size of antiderivative = 2.94

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] -2/3*(3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
+ e)*d^2*f^2 + ((C*a*b^3*c*d - C*a^2*b^2*d^2)*e^2 + (C*a*b^3*c^2 + (2*C*a^2
*b^2 - 3*B*a*b^3)*c*d - (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*d^2)*e*f - (C*a
^2*b^2*c^2 + (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*c*d - (2*C*a^4 - B*a^3*b -
 2*A*a^2*b^2)*d^2)*f^2 + ((C*b^4*c*d - C*a*b^3*d^2)*e^2 + (C*b^4*c^2 + (2*C
*a*b^3 - 3*B*b^4)*c*d - (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*d^2)*e*f - (C*a*b
^3*c^2 + (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*c*d - (2*C*a^3*b - B*a^2*b^2 - 2
*A*a*b^3)*d^2)*f^2)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (
b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2),
-4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a
*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b
*d*f)) + 3*sqrt(b*d*f)*((C*a*b^3*c*d - C*a^2*b^2*d^2)*e*f - (C*a^2*b^2*c*d
- (2*C*a^3*b - B*a^2*b^2 + A*a*b^3)*d^2)*f^2 + ((C*b^4*c*d - C*a*b^3*d^2)*e
*f - (C*a*b^3*c*d - (2*C*a^2*b^2 - B*a*b^3 + A*b^4)*d^2)*f^2)*x)*weierstras
sZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2
*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*
e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*
```

```
b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInve
rse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d
^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^
2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b
*d*e + (b*c + a*d)*f)/(b*d*f))))/((a*b^5*c*d^2 - a^2*b^4*d^3)*e*f^2 - (a^2*
b^4*c*d^2 - a^3*b^3*d^3)*f^3 + ((b^6*c*d^2 - a*b^5*d^3)*e*f^2 - (a*b^5*c*d^
2 - a^2*b^4*d^3)*f^3)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

3.77 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 642

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+\frac{2(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf)))\sqrt{c+}}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}}$$

$$-\frac{2\sqrt{d}(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf)))\sqrt{}}{3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$-\frac{2(a^2Cd(de-cf)-b^2(3c^2Ce-3Bcde+2Ad^2e+Acdf)+ab(3(c^2C+Ad^2)f-Bd(de+2cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{}}{3b^2\sqrt{d}(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

```
[Out] -2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(3/2)+2/3*(2*a^3*C*d*f+a*b^2*(-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(
3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^(
1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(1/2)-2/3*(2*a^3*C*d*f+a*b^2*(-4*A
*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f
+d*e)))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a
*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)/b^2/(a*d
-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/3*(a^
2*C*d*(-c*f+d*e)-b^2*(A*c*d*f+2*A*d^2*e-3*B*c*d*e+3*C*c^2*e)+a*b*(3*(A*d^2+
C*c^2)*f-B*d*(2*c*f+d*e)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),
((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/
(-a*f+b*e))^(1/2)/b^2/(a*d-b*c)^(3/2)/(-a*f+b*e)/d^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 157, 164, 115, 114, 122, 121}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(a^2Cd(de-cf)+ab(3f(Ad^2+c^2C)-Bd(2cf+de))-b^2(Acdf+2Ad^2e-3Bcde+}{3b^2\sqrt{d}\sqrt{c+dx}\sqrt{e+fx}(ad-bc)^{3/2}(be-af)} - \frac{2\sqrt{d}\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bc}{3b^2\sqrt{c+dx}(ad-bc)^{3/2}(be-af)^2\sqrt{\frac{b(e+fx)}{be-af}}} + \frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-}{3b\sqrt{a+bx}(bc-ad)^2(be-af)^2} - \frac{2\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(3/2)) + (2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*
c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)^2*(b*e - a*f)^2*Sqrt
[a + b*x]) - (2*Sqrt[d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A
*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqr
t[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*(
-(b*c) + a*d)^(3/2)*(b*e - a*f)^2*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a
*f)]) - (2*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e + 2*A*d^2*e +
A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*Sqrt[(b*(c + d*x)
)/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(3*b^2*
Sqrt[d]*(-(b*c) + a*d)^(3/2)*(b*e - a*f)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&& !LtQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

Rule 122

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} - \frac{2\int \frac{-\frac{a^2C(de+cf)-ab(3cCe+Bde+Bcf-3Adf)+b^2(3Bce-2A(de+cf))}{2b}+\frac{1}{2}\left(-3bcCe+3aCde+3acCf+Abdf-aBdf-\frac{2a^2Cdf}{b}\right)x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{3(bc-ad)(be-af)}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de +}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}} + \frac{4\int \frac{-\frac{a^3Cdf(de+cf)-b^3ce(3cCe-Adf)+ab^2\left(6c^2Cef+Ad^2ef+cd\left(6Ce^2-4Bef+Af^2\right)\right)-a^2b\left(C\left(3d^2e^2+5cdef+3c^2f^2\right)+df(3Adf-2B(de+cf))}{4b}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3(bc-ad)^2(be-af)^2}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de +}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}} - \frac{(d(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de -}{3b(bc-ad)^2(be-af)^2} - \frac{(a^2Cd(de-cf) - b^2(3c^2Ce - 3Bcde + 2Ad^2e + Acdf) + ab(3(c^2C + Ad^2)f - Bd(de + 2cf)))}{3b(bc-ad)^2(be-af)}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de+c}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}} - \frac{\left((a^2Cd(de-cf) - b^2(3c^2Ce - 3Bcde + 2Ad^2e + Acdf) + ab(3(c^2C + Ad^2)f - Bd(de+2cf))\right.}{3b(bc-ad)^2(be-af)\sqrt{c+dx}} - \frac{\left(d(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de+\right.}{3b(bc-ad)^2(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de+c}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}} - \frac{2\sqrt{d}(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de}{3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+f}{be-a}}} - \frac{\left((a^2Cd(de-cf) - b^2(3c^2Ce - 3Bcde + 2Ad^2e + Acdf) + ab(3(c^2C + Ad^2)f - Bd(de+2cf))\right.}{3b(bc-ad)^2(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}} + \frac{2(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de+c}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}} - \frac{2\sqrt{d}(2a^3Cdf + ab^2(6cCe + Bde + Bcf - 4Adf) - b^3(3Bce - 2A(de+cf)) + a^2b(Bdf - 4C(de}{3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+f}{be-a}}} - \frac{2(a^2Cd(de-cf) - b^2(3c^2Ce - 3Bcde + 2Ad^2e + Acdf) + ab(3(c^2C + Ad^2)f - Bd(de+2cf)))}{3b^2\sqrt{d}(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(c+dx)(e+fx)\left(\left(Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+\left(-2a^3Cdf-ab^2(6cCe+B\right.\right.\right.}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C))
*(b*c - a*d)*(b*e - a*f) + (-2*a^3*C*d*f - a*b^2*(6*c*C*e + B*d*e + B*c*f -
 4*A*d*f) + b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(-(B*d*f) + 4*C*(d*e +
c*f)))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(2*a^3*C*d*f + a*b^2*
(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d*e + c*f)) + a^
2*b*(B*d*f - 4*C*(d*e + c*f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(2*a^3
*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d
*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c +
d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[
Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(
b*c - a*d)*(a^2*C*f*(d*e - c*f) + b^2*(3*c*C*e^2 + A*d*e*f + c*f*(-3*B*e +
2*A*f)) + a*b*(-3*C*d*e^2 + f*(2*B*d*e + B*c*f - 3*A*d*f)))*(a + b*x)^(3/2)
*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipt
icF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a
*d*f)])))/(3*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^(
3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. 2(588) = 1176.

Time = 4.56 (sec) , antiderivative size = 1249, normalized size of antiderivative = 1.95

method	result	size
elliptic	Expression too large to display	1249
default	Expression too large to display	13099

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2/3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3+a
*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2
-2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2/
b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*
d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)/((x+
a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*(C/b^2-1/3*d*f/b^2*(A*b^2-B
*a*b+C*a^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)+1/3/b^2*(a*d*f-b*c*f-b*d*e)*(
4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*d*e+3
*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)^2+1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)^2/b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*
a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e
))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/
(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+
b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+
a/b))^(1/2))+2/3/b*d*f*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B
*a*b^2*c*f-B*a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-
6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(e/f-c/d)*((x+e/f)/(e/f-
c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^
3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f
+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/
b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 2344, normalized size of antiderivative = 3.65

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(C*a^3*b^3 - A*a*b^5)*d^
2)*e*f - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (C*a^4*b^2 + 2*B*a^3*b^3 - 5*A*a^2*
b^4)*d^2)*f^2 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*C*a^2*b^4 - B*a*b^5 - 2*A*
b^6)*d^2)*e*f - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (2*C*a^3*b^3 + B*a
^2*b^4 - 4*A*a*b^5)*d^2)*f^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
+ ((9*C*a^2*b^4*c^2 - 3*(4*C*a^3*b^3 + B*a^2*b^4)*c*d + (5*C*a^4*b^2 + B*a^
3*b^3 + 2*A*a^2*b^4)*d^2)*e^2 - (3*(4*C*a^3*b^3 + B*a^2*b^4)*c^2 - (13*C*a^
4*b^2 + 11*B*a^3*b^3 + A*a^2*b^4)*c*d + (5*C*a^5*b + 4*B*a^4*b^2 + 5*A*a^3*
b^3)*d^2)*e*f + ((5*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (5*C*a^5*b +
```

```
4*B*a^4*b^2 + 5*A*a^3*b^3)*c*d + (2*C*a^6 + B*a^5*b + 5*A*a^4*b^2)*d^2)*f^
2 + ((9*C*b^6*c^2 - 3*(4*C*a*b^5 + B*b^6)*c*d + (5*C*a^2*b^4 + B*a*b^5 + 2*
A*b^6)*d^2)*e^2 - (3*(4*C*a*b^5 + B*b^6)*c^2 - (13*C*a^2*b^4 + 11*B*a*b^5 +
 A*b^6)*c*d + (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*a*b^5)*d^2)*e*f + ((5*C*a^2*
b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*a*b^5)*c*d
+ (2*C*a^4*b^2 + B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*f^2)*x^2 + 2*((9*C*a*b^5*c^2
 - 3*(4*C*a^2*b^4 + B*a*b^5)*c*d + (5*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*d^
2)*e^2 - (3*(4*C*a^2*b^4 + B*a*b^5)*c^2 - (13*C*a^3*b^3 + 11*B*a^2*b^4 + A*
a*b^5)*c*d + (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*e*f + ((5*C*a^3
*b^3 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A*a^2*b^
4)*c*d + (2*C*a^5*b + B*a^4*b^2 + 5*A*a^3*b^3)*d^2)*f^2)*x)*sqrt(b*d*f)*wei
erstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*
b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 +
a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3
*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(
3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*sqrt(b*d*f)*((3*(2*C*a^3*b^
3 - B*a^2*b^4)*c*d - (4*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*d^2)*e*f - ((4
*C*a^4*b^2 - B*a^3*b^3 - 2*A*a^2*b^4)*c*d - (2*C*a^5*b + B*a^4*b^2 - 4*A*a^
3*b^3)*d^2)*f^2 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*C*a^2*b^4 - B*a*b^5 - 2*
A*b^6)*d^2)*e*f - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (2*C*a^3*b^3 + B
*a^2*b^4 - 4*A*a*b^5)*d^2)*f^2)*x^2 + 2*((3*(2*C*a^2*b^4 - B*a*b^5)*c*d - (
4*C*a^3*b^3 - B*a^2*b^4 - 2*A*a*b^5)*d^2)*e*f - ((4*C*a^3*b^3 - B*a^2*b^4 -
 2*A*a*b^5)*c*d - (2*C*a^4*b^2 + B*a^3*b^3 - 4*A*a^2*b^4)*d^2)*f^2)*x)*weie
rstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d
 + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2
*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstras
sPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d +
 a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d
^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f
*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/((a^2*b^7*c^2*d - 2*a^3*b^6*c*d^2 +
a^4*b^5*d^3)*e^2*f - 2*(a^3*b^6*c^2*d - 2*a^4*b^5*c*d^2 + a^5*b^4*d^3)*e*f^
2 + (a^4*b^5*c^2*d - 2*a^5*b^4*c*d^2 + a^6*b^3*d^3)*f^3 + ((b^9*c^2*d - 2*a
*b^8*c*d^2 + a^2*b^7*d^3)*e^2*f - 2*(a*b^8*c^2*d - 2*a^2*b^7*c*d^2 + a^3*b^
6*d^3)*e*f^2 + (a^2*b^7*c^2*d - 2*a^3*b^6*c*d^2 + a^4*b^5*d^3)*f^3)*x^2 + 2
*((a*b^8*c^2*d - 2*a^2*b^7*c*d^2 + a^3*b^6*d^3)*e^2*f - 2*(a^2*b^7*c^2*d -
2*a^3*b^6*c*d^2 + a^4*b^5*d^3)*e*f^2 + (a^3*b^6*c^2*d - 2*a^4*b^5*c*d^2 + a
^5*b^4*d^3)*f^3)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(c + d*x)*sqrt(e + f*x)),
 x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,(a+b\,x)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

3.78 $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1116

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2(2a^3Cdf+ab^2(10cCe+Bde+Bcf-8Adf)-b^3(5Bce-4A(de+cf))+3a^2b(Bdf-2C(de+cf)))\sqrt{}}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+\frac{2(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+8Af}{}$$

$$+\frac{2\sqrt{d}(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+8}{}$$

$$+\frac{2\sqrt{d}(a^3Cdf(de-cf)+b^3(8Ad^2e^2-cde(10Be-3Af)+c^2(15Ce^2-5Bef+4Af^2))+ab^2(d^2e(2Be-19}{}$$

```
[Out] -2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e
)/(b*x+a)^(5/2)+2/15*(2*a^3*C*d*f+a*b^2*(-8*A*d*f+B*c*f+B*d*e+10*C*c*e)-b^3
*(5*B*c*e-4*A*(c*f+d*e))+3*a^2*b*(B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(2*a^4*C*d^2*f^2+a^
3*b*d*f*(3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*
(8*A*f^2-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*C
*e)-c*d*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*d^
2*e^2)+d*f*(23*A*d*f-7*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b
*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*(2*a^4*C*d^2*f^2+a^3*b*d*f*(3*B*d*f-7
*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*(8*A*f^2-10*B*e*f+
15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*C*e)-c*d*(23*A*f^2-
33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*d^2*e^2)+d*f*(23*A*d
```

```
*f-7*B*(c*f+d*e))))*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+
b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1
/2)/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(
1/2)+2/15*(a^3*C*d*f*(-c*f+d*e)+b^3*(8*A*d^2*e^2-c*d*e*(-3*A*f+10*B*e)+c^2*
(4*A*f^2-5*B*e*f+15*C*e^2))+a*b^2*(d^2*e*(-19*A*f+2*B*e)-c^2*f*(-B*f+20*C*e
)-c*d*(11*A*f^2-27*B*e*f+10*C*e^2))-3*a^2*b*(d*f*(-5*A*d*f+3*B*c*f+2*B*d*e)
-C*(3*c^2*f^2+c*d*e*f+d^2*e^2)))*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^
(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))*d^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/
2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^2/(d*x+c)^(1
/2)/(f*x+e)^(1/2)
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 1116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {1628, 157, 164, 115, 114, 122, 121}

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2\sqrt{c+dx}\sqrt{e+fx}(Ab^2-a(bB-aC))}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf))a^3-b^2(C(3d^2e^2-13cdfe+3c^2f^2)+df(23Adf-7B(de+cf)}{}$$

$$+\frac{2\sqrt{d}(Cdf(de-cf)a^3-3b(df(2Bde+3Bcf-5Adf)-C(d^2e^2+cdfe+3c^2f^2))\,a^2+b^2(-f(20Ce-Bf)c}{}$$

$$+\frac{2(2Cd^2f^2a^4+bdf(3Bdf-7C(de+cf))a^3-b^2(C(3d^2e^2-13cdfe+3c^2f^2)+df(23Adf-7B(de+cf)))\,a}{}$$

$$+\frac{2(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf)))\sqrt{c}}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*(
b*e - a*f)*(a + b*x)^(5/2)) + (2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e + B
*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*C*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(15*b*(b*c - a*d)^2*(b*e - a*f)^2*
(a + b*x)^(3/2)) + (2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(
10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c
^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
15*b*(b*c - a*d)^3*(b*e - a*f)^3*Sqrt[a + b*x]) + (2*Sqrt[d]*(2*a^4*C*d^2*f
^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B
*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e -
```

```
23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a
^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e +
 c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sq
rt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])
/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^3*Sqrt[c + d*x]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]) + (2*Sqrt[d]*(a^3*C*d*f*(d*e - c*f) + b^3*(8*A*d^2*e^2 - c*
d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2)) + a*b^2*(d^2*e*(
2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2 - 27*B*e*f + 11*A*f^
2)) - 3*a^2*b*(d*f*(2*B*d*e + 3*B*c*f - 5*A*d*f) - C*(d^2*e^2 + c*d*e*f + 3
*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(15*b^2*(-(b*c) + a*d)^(5/2)*(b*e - a*f)^2*Sqrt[c + d
*x]*Sqrt[e + f*x])
```

<u>Rule 114</u>

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

<u>Rule 115</u>

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

<u>Rule 121</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[A
rcSin[Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(
b*e - a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x,
 e + f*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

<u>Rule 122</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
```

```
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 164

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}} - \frac{2\int \frac{-\frac{a^2C(de+cf)-ab(5cCe+Bde+Bcf-5Adf)+b^2(5Bce-4A(de+cf))}{2b}+\frac{1}{2}\left(-5bcCe+5aCde+5acCf+3Abdf-3aBdf-\frac{2a^2Cdf}{b}\right)x}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx}{5(bc-ad)(be-af)}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de+cf)) + 3a^2b(Bdf - 2C(de}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+ \frac{4\int \frac{\frac{a^3Cdf(de+cf)+b^3\left(8Ad^2e^2-cde(10Be-7Af)+c^2\left(15Ce^2-10Bef+8Af^2\right)\right)+ab^2\left(d^2e(2Be-19Af)-2c^2f(5Ce-Bf)-cd\left(10Ce^2-28Bef+19\right.\right.}{4b}}{(a+}}{15(bc}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de+cf)) + 3a^2b(Bdf - 2C(de}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+ \frac{2(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de+cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10B}{}$$

$$- \frac{8\int \frac{\frac{df\left(a^4Cdf(de+cf)+b^4ce(5Bce-4A(de+cf))-ab^3\left(4Ad^2e^2-cde(4Be+9Af)+c^2\left(25Ce^2-4Bef+4Af^2\right)\right)-a^2b^2\left(d^2e(Be-11Af)-c^2f(26Ce-\right.\right.}{8b}}{}}{}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de+cf)) + 3a^2b(Bdf - 2C(de}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+ \frac{2(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de+cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10B}{}$$

$$- \frac{(d(a^3Cdf(de-cf) + b^3(8Ad^2e^2 - cde(10Be - 3Af) + c^2(15Ce^2 - 5Bef + 4Af^2)) + ab^2(d^2e(}{}$$

$$- \frac{(d(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de+cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10}{}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de+cf)) + 3a^2b(Bdf - 2C(de}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+ \frac{2(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de+cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10B}{}$$

$$- \frac{\Big(d(a^3Cdf(de-cf) + b^3(8Ad^2e^2 - cde(10Be - 3Af) + c^2(15Ce^2 - 5Bef + 4Af^2)) + ab^2(d^2e}{}$$

$$- \frac{\Big(d(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de+cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 1}{}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{5b(bc - ad)(be - af)(a + bx)^{5/2}}$$
$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de + cf)) + 3a^2b(Bdf - 2C(de\ldots}{15b(bc - ad)^2(be - af)^2(a + bx)^{3/2}}$$
$$+ \frac{2(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de + cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10B\ldots}{\ldots}$$
$$+ \frac{2\sqrt{d}(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de + cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 1\ldots}{\ldots}$$
$$- \frac{\left(d(a^3Cdf(de - cf) + b^3(8Ad^2e^2 - cde(10Be - 3Af) + c^2(15Ce^2 - 5Bef + 4Af^2)) + ab^2(d^2e(2\ldots\right.}{\ldots}$$

$$= -\frac{2(Ab^2 - a(bB - aC))\sqrt{c + dx}\sqrt{e + fx}}{5b(bc - ad)(be - af)(a + bx)^{5/2}}$$
$$+ \frac{2(2a^3Cdf + ab^2(10cCe + Bde + Bcf - 8Adf) - b^3(5Bce - 4A(de + cf)) + 3a^2b(Bdf - 2C(de\ldots}{15b(bc - ad)^2(be - af)^2(a + bx)^{3/2}}$$
$$+ \frac{2(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de + cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 10B\ldots}{\ldots}$$
$$+ \frac{2\sqrt{d}(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de + cf)) - b^4(8Ad^2e^2 - cde(10Be - 7Af) + c^2(15Ce^2 - 1\ldots}{\ldots}$$
$$+ \frac{2\sqrt{d}(a^3Cdf(de - cf) + b^3(8Ad^2e^2 - cde(10Be - 3Af) + c^2(15Ce^2 - 5Bef + 4Af^2)) + ab^2(d^2e\ldots}{\ldots}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.54 (sec) , antiderivative size = 1258, normalized size of antiderivative = 1.13

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{7/2}\sqrt{c + dx}\sqrt{e + fx}}\,dx =$$
$$-\frac{2\left(b^2\sqrt{-a + \frac{bc}{d}}(2a^4Cd^2f^2 + a^3bdf(3Bdf - 7C(de + cf)) - b^4(8Ad^2e^2 + cde(-10Be + 7Af) + c^2(15Ce^2 - \ldots\right.}{\ldots}$$

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x
]
```

```
[Out] (-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*
e + c*f)) - b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 - 10
*B*e*f + 8*A*f^2)) + a*b^3*(d^2*e*(-2*B*e + 23*A*f) - 2*c^2*f*(-5*C*e + B*f
) + c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) + a^2*b^2*(d*f*(7*B*d*e + 7*B*c*f
 - 23*A*d*f) + C*(-3*d^2*e^2 + 13*c*d*e*f - 3*c^2*f^2)))*(a + b*x)^2*(c + d
*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-
(b*B) + a*C))*(b*c - a*d)^2*(b*e - a*f)^2 + (b*c - a*d)*(b*e - a*f)*(-2*a^3
*C*d*f - a*b^2*(10*c*C*e + B*d*e + B*c*f - 8*A*d*f) + b^3*(5*B*c*e - 4*A*(d
*e + c*f)) + 3*a^2*b*(-(B*d*f) + 2*C*(d*e + c*f)))*(a + b*x) + (-2*a^4*C*d^
2*f^2 + a^3*b*d*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d^2*e*(2*B*e - 23*A
*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f - 23*A*f^2)) + b^4
*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^
2)) + a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + d*f*(23*A*d*f - 7*B
*(d*e + c*f))))*(a + b*x)^2) + I*(b*c - a*d)*f*(2*a^4*C*d^2*f^2 + a^3*b*d*f
*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) +
 c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a*b^3*(d^2*e*(-2*B*e + 23*A*f) - 2*
c^2*f*(-5*C*e + B*f) + c*d*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) + a^2*b^2*(d*f
*(7*B*d*e + 7*B*c*f - 23*A*d*f) + C*(-3*d^2*e^2 + 13*c*d*e*f - 3*c^2*f^2)))
*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a
 + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a
*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a^3*C*d*f*(-(d*e) + c*f) + a*b^
2*(d^2*e*(B*e - 11*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-20*C*e^2 + 27*B*e*
f - 19*A*f^2)) + b^3*(4*A*d^2*e^2 + c*d*e*(-5*B*e + 3*A*f) + c^2*(15*C*e^2
- 10*B*e*f + 8*A*f^2)) + 3*a^2*b*(d*f*(-3*B*d*e - 2*B*c*f + 5*A*d*f) + C*(3
*d^2*e^2 + c*d*e*f + c^2*f^2)))*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(d*(a +
b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c
)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (
b*c)/d]*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e +
f*x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2282 vs. $2(1054) = 2108$.

Time = 6.00 (sec) , antiderivative size = 2283, normalized size of antiderivative = 2.05

method	result	size
elliptic	Expression too large to display	2283
default	Expression too large to display	32154

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETUR
NVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c)*(f*x+e))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*(
-2/5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3+a
```

```
*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3
-2/15*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3*B*a^2*b*d*f-B*a*b^2*c*f-B*a*
b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f+6*C*a^2*b*d*e-10*C*a*b^2*c*e)
/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d
*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2-2/15*(b*d*f*x^2+b*c*f
*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/b^2*(23*A*a^2*b^2*d^2
*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2+7*A*b^4*c*d*e*f+
8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2
*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f-10*B
*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*C*a^3*b*d^2*e*f+3*C*a^2*b^
2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-10*C*
a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))
^(1/2)+2*(-1/15*d*f*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3*B*a^2*b*d*f-B*
a*b^2*c*f-B*a*b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f+6*C*a^2*b*d*e-1
0*C*a*b^2*c*e)/b^2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2+1/15/b^2*(a*d*f-b*c*
f-b*d*e)*(23*A*a^2*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^
4*c^2*f^2+7*A*b^4*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d
*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2
*e^2-10*B*b^4*c^2*e*f-10*B*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*
C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^
2-10*C*a*b^3*c^2*e*f-10*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)^3+1/15*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/b
^2*(23*A*a^2*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^4*c^2*
f^2+7*A*b^4*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d*f^2-7
*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2*e^2-1
0*B*b^4*c^2*e*f-10*B*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*C*a^3*
b*d^2*e*f+3*C*a^2*b^2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^2-10*C
*a*b^3*c^2*e*f-10*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2))*(e/f-c/d)*((x+e/f)/(e/
f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*
x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*Elli
pticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))+2/15/b*d*f*(
23*A*a^2*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2+
7*A*b^4*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d*f^2-7*B*a
^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2*e^2-10*B*
b^4*c^2*e*f-10*B*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*C*a^3*b*d^
2*e*f+3*C*a^2*b^2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^2-10*C*a*b
^3*c^2*e*f-10*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)^3*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x
+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*
d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),
((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e
/f+c/d)/(-e/f+a/b))^(1/2))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.05 (sec) , antiderivative size = 5108, normalized size of antiderivative = 4.58

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Too large to display}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="maxima")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algori
thm="giac")
[Out] integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,(a+b\,x)^{7/2}\,\sqrt{c+d\,x}}\,dx$$

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```